MAGNETIC MOMENT OF COOPER PAIRS IN MAGNETIZED COLOR SUPERCONDUCTIVITY*

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We discuss how the ground state of the three-flavor color superconducting phase in the presence of a magnetic field is enriched with the presence of an extra condensate related with the alignment of the magnetic moments of Cooper pairs of charged quarks. The new condensate enhances the condensation energy of pairs formed by charged quarks. We point out possible consequences of the new order parameter on the issue of the chromomagnetic instability that appears in color superconductivity at moderate density and for the planned low-temperature/high-density heavy-ion collision experiments.

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1. Introduction

Color superconductivity (CS) is the favored state of nuclear matter at high density and low temperature [1]. It is expected that those extreme conditions can exist in the high dense cores of compact stars. Compact stars, on the other hand, can exhibit very strong magnetic fields, as for instance, the so-called magnetars, which can have surface magnetic fields as large as 10^{14}–10^{15} G and inner fields estimated of the order of 10^{18}–10^{20} G [2].

An important feature of spin-zero color superconductivity is that although the color condensate has non-zero electric charge, there is a linear combination of the photon A_μ and a gluon G^8_μ that remains massless [3], $\widetilde{A}_\mu = \cos\theta\, A_\mu - \sin\theta\, G^8_\mu$. In the CFL phase the mixing angle θ is sufficiently small ($\sin\theta \sim e/g \sim 1/40$). Thus, the penetrating field in the color superconductor is mostly formed by the photon with only a small gluon admixture. The field $\widetilde{A}_\mu$ plays the role of an in-medium or rotated electromagnetic

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field, as the color condensate is neutral with respect to the corresponding rotated charge. The unbroken $\widetilde{\mathrm{U}}(1)$ symmetry corresponding to the long-range rotated photon is generated by $\tilde{Q} = Q \times 1 + 1 \times T_8/\sqrt{3}$, where Q is the conventional electromagnetic charge of quarks and T_8 is the 8th Gell-Mann matrix. Using the representation of matrices $Q = \mathrm{diag}(-1/3, -1/3, 2/3)$ for (s, d, u) flavors and $T_8 = \mathrm{diag}(-1/\sqrt{3}, -1/\sqrt{3}, 2/\sqrt{3})$ for (b, g, r) colors, the $\tilde{Q}$ charges of different quarks are (in units of $\tilde{e} = e\cos\theta$) $(0, 0, -, 0, 0, -, +, +, 0)$ corresponding to $(s_b, s_g, s_r, d_b, d_g, d_r, u_b, u_g, u_r)$.

The less symmetric realization of the CFL pairing that occurs in the presence of a magnetic field, is known as the Magnetic CFL (MCFL) phase [4]. Similarly to the CFL phase [3], the MCFL has a locking between flavor and color transformations, but unlike the CFL, where the symmetry breaking pattern is $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}} \times \mathrm{U}(1)_B \to \mathrm{SU}(3)_{\mathrm{C+L+R}}$, in the MCFL the symmetry breaking pattern is $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_B \times \mathrm{U}^{(-)}(1)_A \longrightarrow \mathrm{SU}(2)_{\mathrm{C+L+R}}$. Both phases are similar in that they lock color and flavor and have no Meissner effect for an in-medium magnetic field. However, they have important differences too. The MCFL has five Goldstone bosons, all of which are neutral with respect to the rotated charge. This is in contrast with the CFL phase that has nine Goldstone bosons, some of which are charged. Hence, in the MCFL phase, as well as in the CFL one, the fermion excitations are gapped, and the gluon fields acquire masses thanks to the Meissner–Anderson–Higgs mechanism, but the symmetry breaking that gives rise to MCFL leaves a smaller number of Nambu–Goldstone fields, all of which are neutral with respect to the rotated electric charge [4,5]. Hence, the MCFL phase behaves as an insulator, as it has no low-energy charged excitations at zero temperature.

In the MCFL phase the symmetry reduction is also manifested in the fact that the ground state is characterized by two antisymmetric gaps Δ and Δ_H, instead of just one, as in the regular CFL case. Even though all the diquarks are neutral with respect to the rotated electromagnetic charges, they can be formed either by a pair of neutral or by a pair of charged quarks with opposite rotated charges. As shown in [4], the gap Δ only gets contributions from pairs of neutral quarks, while Δ_H has contributions from both charged and neutral quarks, thus it can directly feel the background field through the minimal coupling of the charged quarks with $\tilde{H}$. At fields large enough that only the lowest Landau level (LLL) is occupied, the field substantially modifies the density of states of the charged quarks and the energy gap Δ_H becomes significantly enhanced by the penetrating field [4]. At moderate magnetic fields (henceforth when we say magnetic field, we actually mean the rotated magnetic field) the energy gaps exhibit oscillations with respect to $\tilde{e}\tilde{H}/\mu^2$ [6], owed to the de Haas–van Alphen effect.

Recently, it was also found that in the MCFL phase a new condensate has to materialize [7]. The new condensate is associated with the magnetic moment of the Cooper pairs. It should be noticed that the Cooper pairs formed by charged quarks have a non-zero magnetic moment, since the quarks in the pair not only have the opposite charge but also opposite spin. Hence, as found in Ref. [7], the magnetic moment of this kind of pairs leads, in principle, to a non-zero average magnetic moment for the system, which in turn would be reflected in the existence of an extra condensate $\varDelta_M$ in addition to the $\varDelta$ and $\varDelta_H$ gaps. We will concentrate in this report on the mechanism behind the generation of this new condensate.

The presence of a magnetic field breaks the spatial rotational symmetry O(3) to the subgroup of rotations O(2) about the axis parallel to the field. This symmetry breaking opens new attractive pairing channels through the new Fierz identities that were not available in the CFL phase [7]. One of these channels has Dirac structure $\varDelta_M \sim \mathcal{C}\gamma_5\gamma^1\gamma^2$. A condensate with this structure corresponds to a magnetic moment condensate. Because it does not break any symmetry that has not already been broken by the gaps $\varDelta$ and $\varDelta_H$, a magnetic moment condensate is not symmetry-protected and, in principle, should exist.

2. Gap structure of the MCFL phase

An external magnetic field (assumed here to be along the z-direction) introduces a normalized tensor $\widehat{F}^{\mu\nu} = \widehat{F}^{\mu\nu}/|\tilde{H}|$, with $\mu,\nu = 1,2$. Once this extra tensor is available in the theory, the metric tensor can be separated into transverse $g_{\perp}^{\mu\nu} = \widehat{F}^{\mu\rho}\widehat{F}_{\rho}^{\nu}$ and longitudinal $g_{\parallel}^{\mu\nu} = g^{\mu\nu} - g_{\perp}^{\mu\nu}$ components. Next, the finite density introduces yet another normalized vector in the theory, the four-velocity u_μ, which in the rest frame reduces to $u_\mu = (1,0,0,0)$. With these structures, the four-fermion interaction term in the system Lagrangian density

$$\mathcal{L}_{\rm int} = -G\left(\bar{\psi}\varGamma_\mu^a\psi\right)\left(\bar{\psi}\varGamma_a^\mu\psi\right) , \tag{1}$$

with quark-gluon vertex $\varGamma_\mu^a = \gamma_\mu\lambda^a$ (where λ^as are the Gell-Mann matrices for color SU(3) group), has Dirac contraction given by $\gamma^\mu\gamma_\mu = [au_\mu u_\nu + bg_{\mu\nu}^{\perp} + c(g_{\mu\nu}^{\parallel} - u_\mu u_\nu)]\gamma^\mu\gamma^\nu$. Hence, one can write in the rest frame the four-fermion interaction (1) as three distinct terms

$$\begin{aligned}\mathcal{L}_{\rm int} = &- g_E\left(\bar{\psi}\gamma_0\lambda^a\psi\right)\left(\bar{\psi}\gamma_0\lambda^a\psi\right) - g_M^{\perp}\left(\bar{\psi}\gamma^{\perp}\lambda^a\psi\right)\left(\bar{\psi}\gamma_{\perp}\lambda^a\psi\right) \\ &- g_M^3\left(\bar{\psi}\gamma^3\lambda^a\psi\right)\left(\bar{\psi}\gamma_3\lambda^a\psi\right) .\end{aligned} \tag{2}$$

Using a Fierz transformation (see Appendix A of Ref. [7]), one can verify that the breaking of the Lorentz and rotational symmetries give rise to new particle–particle channels of interaction, and, in particular, to $(\gamma^5\sigma^{ab}\mathcal{C})$ $(\mathcal{C}\sigma_{ab}\gamma^5)$ (with $\mathcal{C} = i\gamma^2\gamma^0$ and $\sigma^{ab} = \frac{1}{2}[\gamma^a,\gamma^b]$, where $a,b = 1,2$), which is the one that can lead to a magnetic-moment condensate along the z-axis $\Delta_M = \langle\psi^T\mathcal{C}\Sigma^3\gamma^5\psi\rangle$, with $\Sigma^3 = \sigma^{12}$, being the spin operator.

It will be natural to expect that only diquark pairs formed by charged quarks with opposite rotated charges and opposite spins, so having a net magnetic moment, will contribute to the magnetic-moment condensate $\Delta_M = \langle\psi^T\mathcal{C}\Sigma^3\gamma^5\psi\rangle$. Then, because of the symmetric nature of $\sigma_{ab}\gamma_5\mathcal{C}$ under transposition in Dirac, this condensate will be a spin-1 condensate symmetric in *Dirac*. Second, because we want to guarantee the strongest attractive channel, we choose it to be antisymmetric in color. Finally, to ensure the total antisymmetry required by Pauli principle, it should be symmetric in flavor. Basing on this considerations, we proposed in [7] that the gap matrix in the presence of a magnetic field takes the form

$$\begin{aligned}\Phi^{\alpha\beta}_{ij} &= \Delta\epsilon^{\alpha\beta3}\epsilon_{ij3} + \Delta_H\left(\epsilon^{\alpha\beta1}\epsilon_{ij1} + \epsilon^{\alpha\beta2}\epsilon_{ij2}\right)\\ &\quad+\Delta_M\left[\epsilon^{\alpha\beta1}\left(\delta_{i2}\delta_{j3}+\delta_{i3}\delta_{j2}\right)+\epsilon^{\alpha\beta2}\left(\delta_{i1}\delta_{j3}+\delta_{i3}\delta_{j1}\right)\right], \qquad (3)\end{aligned}$$

where α,β and i,j denote the color and flavor indices respectively.

The magnetic-moment condensate gap Δ_M is different from the case of the conventional gaps Δ and Δ_H, which are antisymmetric in both color and flavor. We can also check that the gap structure (3) satisfies the same symmetry than that of the MCFL ansatz [4]. That is, the $\mathrm{SU(2)_{C+L+R}}$ symmetry, which requires the invariance of the gap $\Phi^{\alpha\beta}_{ij}$ under simultaneous flavor $(1 \leftrightarrow 2)$ and color $(1 \leftrightarrow 2)$ exchanges.

It is also important to point out that the magnetic-moment gap, being symmetric under transposition in Dirac, has to be a spin-1 condensate. As shown in Appendix B of Ref. [7], the specific spin-1 condensate, we are considering, has a zero-spin projection ($M_\mathrm{S} = 0$) along the field direction. Thus, it corresponds to a symmetric wave function associated to pairs formed by quarks with opposite charges and spins, and consequently with net magnetic moment different from zero.

3. Gap equations and numerical solutions

The system free-energy in the zero-temperature limit is

$$\Omega = -\int\limits_{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{1}{2}\sum_{j=1}^{2}|\varepsilon_j| - \sum_{l=0}^{n_{\tilde{B}}}\int\limits_{\Lambda}\frac{dp_2dp_3}{(2\pi)^3}|\varepsilon^c| + \frac{\Delta^2+2\Delta_H^2+2\Delta_M^2}{G_0}, \qquad (4)$$

where Λ is the energy cutoff of the NJL effective theory and $n_{\widetilde{B}} = I[\Lambda^2/2\widetilde{e}\widetilde{B}]$, with $I[...]$ denoting the integer part of the argument. In (4), ε_j and ε^c are the energy modes of the neutral quarks and charged quarks, respectively, given by $\varepsilon_1 = \pm\sqrt{(p\pm\mu)^2+\Delta^2}$, with degeneracy $(d=6)$ and

$$\varepsilon_2 = \pm\left[2\left(\Delta_H^2+\Delta_M^2\right)+\frac{1}{2}\Delta^2+\left(p_3^2+p_\perp^2\right)+\mu^2\right.$$
$$\left.\pm\frac{1}{2}\sqrt{\Delta_a^2\Delta_b^2+16\left[2\Delta_M^2 p_\perp^2+\mu^2\left(p_\perp^2+p_3^2\right)\right]\pm 8\mu\sqrt{\Delta_b^2\left(\Delta_a^2 p_3^2+\Delta^2 p_\perp^2\right)}}\right]^{1/2}, \tag{5}$$

for neutral quarks with degeneracy $(d=2)$. In (5), we defined $\Delta_a^2 = \Delta^2 + 8\Delta_M^2$ and $\Delta_b^2 = \Delta^2 + 8\Delta_H^2$ The dispersion relations for charged quarks in higher LLs $(l \geq 0)$ are

$$\varepsilon^c = \pm\sqrt{2\tilde{e}\tilde{B}l+\Delta_M^2+\Delta_H^2+\mu^2+p_3^2\pm 2\sqrt{2\tilde{e}\tilde{B}l\left(\Delta_M^2+\mu^2\right)+\left(\mu p_3\pm\Delta_M\Delta_H\right)^2}}\,. \tag{6}$$

with degeneracy $(d=4)$ and $p_\perp^2 = p_1^2+p_2^2$ defined. We want to call attention to the double sign in front of Δ_M in (6). This is reflecting the breaking of the spin degeneracy for the higher LL modes of the charged quasi-particles due to the presence of the magnetic-moment condensate Δ_M. Hence, the dispersion relation for the LLL cannot be the limit $(l=0)$ of (6), since the LLL has not spin degeneracy (see Appendix C in [7]).

A stable phase must minimize the free energy with respect to the variation of the three gap parameters, Δ, Δ_H and Δ_M. This gives rise to the gap equations

$$\frac{\partial\Omega}{\partial\Delta} = \frac{\partial\Omega}{\partial\Delta_H} = \frac{\partial\Omega}{\partial\Delta_M} = 0\,. \tag{7}$$

These gap equations are quite complicated, even in the strong-magnetic-field limit where only the contribution from the LLL is important, and can only be solved numerically.

From the graphical representation of Δ_M in Fig. 1, it can be noticed that its value remains relatively small up to magnetic-field values of the order of μ^2. This can be explained taking into account that mainly the Cooper pairs formed by particles in the LLL contribute to the magnetic moment condensate. Hence, the increase of Δ_M at $\widetilde{e}\widetilde{H} \simeq \mu^2$ takes place when the LLL is most populated.

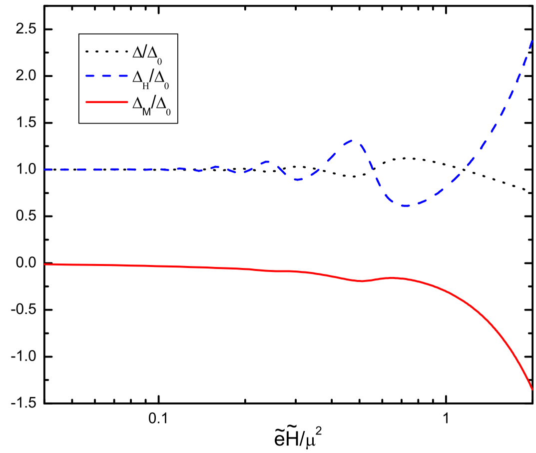

Fig. 1. The three gaps of the MCFL phase as a function of $\tilde{e}\tilde{H}/\mu^2$ for $\mu = 500\,\mathrm{MeV}$. They are scaled with respect to the CFL gap $\Delta_0 = 25\,\mathrm{MeV}$.

The difference found between the other Δ's gaps and Δ_M is also in agreement with the fact that the induced expectation value of the magnetic moment is a quantity purely generated by the magnetic field (*i.e.* $\Delta_M = 0$ at $\widetilde{H} = 0$). On the other hand, taking into account that the relevant scale for the generation of the other Δ's gaps is the energy at the Fermi surface (*i.e.* the chemical potential), then, only when the magnitude of the magnetic field is as large as the chemical potential, the induced average magnetic moment becomes as large as the other gaps. Another important consequence of the generation of the average magnetic moment is that its presence strengthens the gap Δ_H in the sufficiently strong-magnetic-field region, as can be checked by comparing our results in Fig. 1 with those of Ref. [6].

This last fact makes the new parameter Δ_M of particular relevance for the potential realization of the MCFL phase in the core of magnetars. Any effect that can augment the effective gap magnitude will contribute to stabilize the phase by pushing the emergence of the chromomagnetic instability to smaller regions of densities. Since magnetars have the strongest surface fields, they should also have the strongest fields in the core, so they are the best candidates for the realization of the magnetic CFL phase studied in this paper. The larger value of the gap Δ_H at strong fields will be also reflected in a larger critical temperature for this color superconducting phase, a property that opens yet another possibility for the realization of the MCFL state because the conditions of high densities, low temperatures, and strong magnetic fields will likely coexist in the planned low-temperature/high-density heavy-ion collision experiments at NICA@JNIR, CBM@FAIR and low-energy RIHC [8].

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