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# $X(3872)$ AS A $D\bar{D}^*$ MOLECULE BOUND BY QUARK EXCHANGE FORCES*

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The Bethe–Salpeter equation for the T-matrix of $D^0\bar{D}^{*0}$ scattering is solved with a meson–meson potential that results from 2nd order Born approximation of quark exchange processes. This potential turns out to be complex and energy dependent due to the pole contribution from the coupling to the intermediate $J/\psi$–$\rho$ meson pair propagator. As a consequence, a bound state with a mass close to 3.872 GeV occurs in the $J/\psi$–$\rho$ continuum. This result suggests that quark exchange forces may provide the solution to the puzzling question for the origin of the interaction which leads to a binding of $D$ and $\bar{D}^*$ mesons in the $X(3872)$ state.

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## 1. Introduction

The $X(3872)$ resonance was detected by Belle [1] by examining the invariant mass distribution of particles produced in the decay of $B^+$ into $K^+\ \pi^+\ \pi^-\ J/\psi$. This sighting was later confirmed by BaBar [2]. Even though the particle composition is still under discussion, this resonance is likely to be a $D\bar{D}^*$ bound state with binding energy below 1 MeV [3]. Some attempts have been presented to explain $X(3872)$ based on a T-matrix approach taking into account the neutral and charged $D$ meson channels to properly study the isospin violation in $X$ decay to $J/\psi + \pi^+ + \pi^-$ or $J/\psi + \pi^+ + \pi^- + \pi^0$ [4, 5].

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In this contribution, we present the derivation of a potential for the $D\bar{D}^*$ interaction which is based on an extension of the separable quark exchange interaction in the $D\bar{D}^* \to J/\psi + \rho$ channel [6] to the 2nd order Born approximation [7, 8]. An elucidation of the nature of the $X(3872)$ state is important also from the point of view of quark-gluon plasma search since it has been conjectured that in a heavy-ion collision the $c\bar{c}$ state in *statu nacendi* may be subject to important modifications due to its coupling to the $X(3872)$ resonance at the continuum threshold [9, 10]. A generalization [11] of the Matsui model [12] has been applied to the description of the threshold-like structure in the very precise data from In–In collisions at CERN-NA60 [10]. Its relationship to the theory of the plasma Hamiltonian for heavy quarkonia [13, 14] is currently being worked out [15]. As a partial result of these studies, we discuss here the role of quark exchange processes as a possible binding mechanism leading to the $X(3872)$ as a $D\bar{D}^*$ molecule.

## 2. T-matrix approach

The Bethe–Salpeter equation for a T-matrix description of $D\bar{D}^*$ scattering is written in the ladder approximation as [8]

$$T\left(a, a', z\right) = U^{(2)}\left(a, a'\right) + \sum_{a''} U^{(2)}\left(a, a''\right)\; G^0_{2D}\left(a'', z\right)\; T\left(a'', a', z\right) \tag{1}$$

which follows from the diagrammatic representation depicted in Fig. 1.

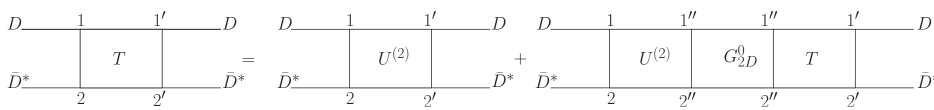


Fig. 1. Diagrammatic representation of Bethe–Salpeter equation for T-matrix of $D\bar{D}^*$ scattering.

For convenience we introduced the shorthand notation $a = \boldsymbol{p}_1, \boldsymbol{p}_2$, $a' = \boldsymbol{p}_{1'}, \boldsymbol{p}_{2'}$, $a'' = \boldsymbol{p}_{1''}, \boldsymbol{p}_{2''}$, $a''' = \boldsymbol{p}_{1'''}, \boldsymbol{p}_{2'''}$. Here $z$ refers to the energy of the scattering process described by (1).

For the interaction kernel $U^{(2)}$ we suggest here an extension of first Born order diagrams for quark exchange processes in meson–meson scattering [16, 17] to second order (see Fig. 2), where the first order process is given by a separable potential [17, 6]

$$\boldsymbol{U}\left(a, a'\right) = -\lambda\; L(a)\; R\left(a'\right)\,, \qquad \boldsymbol{U}^*\left(a', a\right) = -\lambda\; R\left(a'\right)\; L(a)\,, \tag{2}$$

where $L$ and $R$ are the meson form factors, dimensionless functions interpolating between 1 at zero relative momentum and 0 at high momentum. The amplitude $\lambda$ of the interaction potential has the dimension of the potential in momentum space, *i.e.* $[\lambda] = \mathrm{GeV}^{-2}$. The origin of such interaction can be explained by quark exchange forces [8]. With this ansatz the meson–meson potential in the 2nd Born approximation (Fig. 2) leads to a dynamic, separable potential

$$\begin{aligned} \boldsymbol{U}^{(2)}\left(a, a', z\right) &= \sum_{a'''} \boldsymbol{U}\left(a, a'''\right)\ G^0_{J_1}\left(a''', z\right)\ \boldsymbol{U}^*\left(a''', a'\right) \\ &= L(a)\ L\left(a'\right)\ \underbrace{\lambda^2 \sum_{a'''} R^2\left(a'''\right)\ G^0_{J_1}\left(a''', z\right)}_{\boldsymbol{V}(z)} \\ &= \mathrm{V}(z)\ L(a)\ L\left(a'\right)\ . \end{aligned} \tag{3}$$

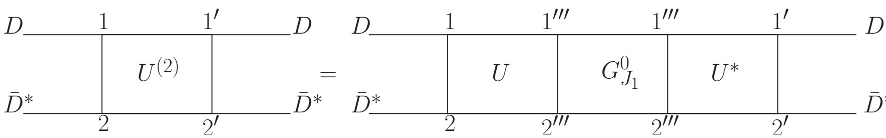


Fig. 2. Diagrammatic representation of the quark exchange interaction kernel for $D\bar{D}^*$ scattering. The subscript $J_1$ stands for the channel $J/\psi + \rho$.

For a separable kernel, the Bethe–Salpeter equation for the T-matrix has a solution in the form

$$T\left(a, a', z\right) = L(a)\ L\left(a'\right)\ t(P, z)\,, \tag{4}$$

where $P$ is the total conserved momentum of the pair $D\bar{D}^*$. By replacing (2), (3) and (4) into (1) we find the solution

$$t(P, z) = \frac{\boldsymbol{V}(z)}{1 - \underbrace{\boldsymbol{V}(z) \sum_{a''} L^2\left(a''\right)\ G^0_{2D}\left(a'', z\right)}_{B(P,z)}}\,. \tag{5}$$

In the following, we use the rest frame with vanishing total momentum ($P = 0$) and the relative momenta $p$, $p'$, where $\boldsymbol{p}_{1''} = p$, $\boldsymbol{p}_{2''} = -p$, $\boldsymbol{p}_{1'''} = p'$, $\boldsymbol{p}_{2'''} = -p'$.

The function $G^0_{2D}(p, z)$ stands, depending on the considered channel, for one of the nonrelativistic two-particle propagators $G^0_{D_1}(p, z)$ or $G^0_{J_1}(p, z)$, given by

$$G^0_{D_1}(p, z) = \frac{1}{E_{D_1} - \frac{p^2}{2\mu_{D_1}} \pm i\varepsilon}\,, \qquad G^0_{J_1}(p, z) = \frac{1}{E_{J_1} - \frac{p^2}{2\mu_{J_1}} \pm i\varepsilon}\,. \tag{6}$$

The abbreviations $D_1 = D^0 + \bar{D}^{*0}$ and $J_1 = J/\psi + \rho$ stand for the two-meson channels and $\mu_{D_1}$, $\mu_{J_1}$ for the corresponding reduced masses. In the same framework the binding energies are defined by

$$E_{D_1} = z - m_{D^0} - m_{\bar{D}^{*0}}\,, \qquad E_{J_1} = z - m_{J/\psi} - m_\rho\,. \tag{7}$$

We are interested in finding a resonance with a mass just below the threshold of the $D_1$ continuum, in the region, where $m_{J/\psi} + m_\rho \le z \le m_{D^0} + m_{\bar{D}^{*0}}$. The potential (3) turns out complex by considering the pole contribution of the two-particle propagator $G^0_{J_1}(p,z)$. This pole located at $p_{J_1} = \sqrt{2\mu_{J_1} E_{J_1}}$ provides the meson–meson potential with sufficient strength to allow the formation of a bound state. Thus integrating around the pole leads to

$$\begin{aligned}
\boldsymbol{V}(z) &= \lambda^2 \sum_{a'''} R^2\left(a'''\right)\, G^0_{J_1}\left(a''',z\right) = \frac{\lambda^2 \mu_{J_1}}{\pi^2} \lim_{\varepsilon\to 0} \int\limits_0^\infty dp\, \frac{p^2\, R^2(p)}{p^2_{J_1} - p^2 \pm i\varepsilon} \\
&= \frac{\lambda^2 \mu_{J_1}}{\pi^2} \int\limits_0^\infty dp\, p^2\, R^2(p) \left[\frac{\wp}{p^2_{J_1} - p^2} \mp i\pi\, \delta\left(p^2_{J_1} - p^2\right)\right] \\
&= \lambda^2 \Bigg[\underbrace{\frac{\mu_{J_1}}{\pi^2} \lim_{\varepsilon\to 0} \int\limits_0^\infty dp\, \frac{p^2\, R^2(p)\left(p^2_{J_1} - p^2\right)}{\left(p^2_{J_1} - p^2\right)^2 + \varepsilon^2}}_{A_{J_1}(z)} + i \underbrace{\left(\mp \frac{\mu_{J_1}\, p_{J_1}\, R^2(p_{J_1})}{2\pi}\right) \Theta(E_{J_1})}_{B_{J_1}(z)}\Bigg].
\end{aligned} \tag{8}$$

Similarly, the integration around $p_{D_1} = \sqrt{2\mu_{D_1} E_{D_1}}$ gives

$$\begin{aligned}
B(0,z) &= \boldsymbol{V}(z) \frac{\mu_{D_1}}{\pi^2} \int\limits_0^\infty dp\, \frac{p^2\, L^2(p)}{p^2_{D_1} - p^2 \pm i\varepsilon} \\
&= \boldsymbol{V}(z) \Bigg[\underbrace{\frac{\mu_{D_1}}{\pi^2} \lim_{\varepsilon\to 0} \int\limits_0^\infty dp\, \frac{p^2\, L^2(p)\left(p^2_{D_1} - p^2\right)}{\left(p^2_{D_1} - p^2\right)^2 + \varepsilon^2}}_{A_{D_1}(z)} \\
&\qquad + i \underbrace{\left(\mp \frac{\mu_{D_1}\, p_{D_1}\, L^2(p_{D_1})}{2\pi}\right) \Theta(E_{D_1})}_{B_{D_1}(z)}\Bigg].
\end{aligned} \tag{9}$$

Notice that the step function $\Theta(E_{D_1})$ cancels the term $B_{D_1}(z)$ for $E_{D_1} \le 0$.

## 3. Scattering phase shift

The scattering phase shift is determined as

$$\tan(\delta) = \frac{\mathrm{Im}[t(0,z)]}{\mathrm{Re}[t(0,z)]} = \frac{B_{J_1}(z) + \left(A^2_{J_1}(z) + B^2_{J_1}(z)\right) B_{D_1}(z)\lambda^2}{A_{J_1}(z) - \left(A^2_{J_1}(z) + B^2_{J_1}(z)\right) A_{D_1}(z)\lambda^2}\,. \quad (10)$$

The expression (10) is very useful to decide whether or not a bound state is located in the region $m_{J/\psi} + m_\rho \leq z \leq m_{D^0} + m_{\bar{D}^{*0}}$. This occurs when $B_{J_1}(z) \neq 0$. Therefore, the analytical continuation of the meson–meson interaction is crucial for explaining resonance formation. An instructive calculation is performed with a Lorentzian form factor (Yamaguchi potential with width parameter $\gamma$), using the integral $\int_0^\infty dx \frac{x^2}{(x^2+1)^2(x^2-\tilde{E})} = \frac{\pi}{4}\frac{1}{(1+\sqrt{-\tilde{E}})^2}$ with $\tilde{E} = \frac{2\mu E}{\gamma^2}$. Notice that for $E > 0$ the equality splits into real and imaginary parts, otherwise it is always real [18]. The result is

$$B_{J_1}(z) = \mp\frac{\mu_{J_1}\gamma}{2\pi}\frac{\sqrt{\tilde{E}_{J_1}}}{\left(1+\tilde{E}_{J_1}\right)^2}\Theta\left(\tilde{E}_{J_1}\right),$$

$$B_{D_1}(z) = \mp\frac{\mu_{D_1}\gamma}{2\pi}\frac{\sqrt{\tilde{E}_{D_1}}}{\left(1+\tilde{E}_{D_1}\right)^2}\Theta\left(\tilde{E}_{D_1}\right),$$

$$A_{J_1}(z) = -\frac{\mu_{J_1}\gamma}{4\pi}\begin{cases}\frac{1-\tilde{E}_{J_1}}{\left(1+\tilde{E}_{J_1}\right)^2} & \tilde{E}_{J_1} > 0\,,\\ \frac{1}{\left(1+\sqrt{-\tilde{E}_{J_1}}\right)^2} & \tilde{E}_{J_1} \leq 0\,,\end{cases}$$

$$A_{D_1}(z) = -\frac{\mu_{D_1}\gamma}{4\pi}\begin{cases}\frac{1-\tilde{E}_{D_1}}{\left(1+\tilde{E}_{D_1}\right)^2} & \tilde{E}_{D_1} > 0\,,\\ \frac{1}{\left(1+\sqrt{-\tilde{E}_{D_1}}\right)^2} & \tilde{E}_{D_1} \leq 0\,.\end{cases} \quad (11)$$

For a particular choice of $\lambda$ and $\gamma$ it is possible that the denominator of the expression (10) vanishes at threshold $z = m_{D^0} + m_{\bar{D}^{*0}} = 3.872\,\mathrm{GeV}$. The interesting features of these results are shown in Fig. 3, where the scattering phase shift reflects the production of a quasi-bound state by mean interaction of two $D$ mesons, $D^0$ and $\bar{D}^{*0}$, passing through the $J/\psi + \rho$ channel.

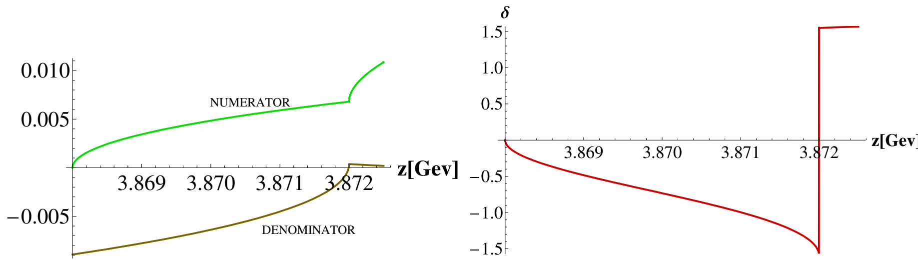


Fig. 3. Plots are done by choosing $\lambda = 20.3\,\mathrm{GeV}^{-2}$, $\gamma = 0.8\,\mathrm{GeV}$ and sign $(+)$ for imaginary parts. Left panel: Numerator and denominator of expression (10). Notice the behavior of the denominator just around the threshold of $D$ mesons which vanishes exactly at $z = 3.872\,\mathrm{GeV}$. Right panel: The sharp jump of the scattering phase shift by $\pi$ indicates that a quasi-bound state (resonance) with a mass of $z = 3.872\,\mathrm{GeV}$ appeared. A similar example of such characteristic behavior can be observed in the deuteron channel in nuclear matter: just above the critical temperature for Bose condensation of Cooper pairs a quasi-bound state appears in the continuum, see Fig. 6 of Ref. [19].

## 4. Conclusions

In this work, we have considered the solution of the Bethe–Salpeter equation for the T-matrix of $D^0\bar{D}^{*0}$ scattering which via quark exchange couples to the virtual propagation of a $J/\psi$–$\rho$ pair. The kernel of the Bethe–Salpeter equation is a complex meson–meson potential obtained by a 2nd order Born approximation to quark exchange. We have shown that this analytical continuation is crucial for the formation of a bound state in the region $m_{J/\psi} + m_\rho \leq z \leq m_{D^0} + m_{\bar{D}^{*0}}$. The dynamical nature of this complex potential provides a sufficient enhancement of the strength at the threshold which leads to the $X(3872)$ bound state, rather independent of the detailed dynamics of the model. In a next step, we consider the coupled channel problem, including charged $D$ meson states and the $J/\psi$–$\omega$ channel [8].

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