EFFECTS OF THE LOW LYING DIRAC MODES ON EXCITED HADRONS IN LATTICE QCD*

L.Ya. Glozman, C.B. Lang, M. Schröck

Institut für Physik, FB Theoretische Physik, Universität Graz
8010 Graz, Austria

(Received September 3, 2012)

Chiral symmetry breaking in Quantum Chromodynamics is associated with the low lying spectral modes of the Dirac operator according to the Banks–Casher relation. Here we study how removal of a variable number of low lying modes from the valence quark sector affects the masses of the ground states and first excited states of baryons and mesons in two flavor lattice QCD.

DOI:10.5506/APhysPolBSupp.5.1001
PACS numbers: 11.15.Ha, 12.38.Gc

1. Introduction

In [1, 2] the effect of removing the lowest Dirac eigenmodes from valence quark propagators which enter meson correlators has been studied. It has been found that the exponential decay signal of the pion correlator vanishes whereas those of the scalar, vector and axial vector currents became essentially better. Moreover, the removal of the lowest Dirac eigenmodes has been shown to restore the chiral symmetry in accordance with the Banks–Casher relation: the masses of the lowest vector and axial vector states became degenerate. Including more particles and excitations above the ground state is a crucial step in understanding the destiny of confinement upon artificial chiral symmetry restoration. In our recent work [3], the b_1 meson, the nucleon and Delta baryons of both parities as well as the first excited states of most of the hadrons under investigation have been included. Besides showing the existence of bound states, *i.e.*, confined quarks after having artificially restored the chiral symmetry, this work gives insights in the origin of the Δ–N mass splitting and its relation to chiral symmetry.

* Presented by M. Schröck at the Workshop "Excited QCD 2012", Peniche, Portugal, May 6–12, 2012.

2. Method

In order to study the effects of Dirac low modes on ground and excited states of hadrons, we modify the valence quark propagators which enter the interpolating fields of the particles. To be explicit, we construct *reduced* quark propagators

$$S_{\mathrm{red}(k)} = S - S_{\mathrm{lm}(k)} \equiv S - \sum_{i \le k} \mu_i^{-1} |v_i\rangle\langle v_i|\gamma_5 \,, \tag{1}$$

where S is the standard (untruncated) quark propagator and the low mode part $S_{\mathrm{lm}(k)}$ is explicitly constructed from the eigenvalues μ_i and eigenvectors $|v_i\rangle$ of the Hermitian Dirac operator $\gamma_5 D$. The reduction parameter k in (1) gives the number of low eigenmodes of the Dirac operator that have been removed from the quark propagator. We will adopt reduced quark propagators with $k = 0, 2, 4, 8, 12, 16, 20, 32, 64, 128$.

These truncated quark propagators can subsequently be used as an ingredient for the construction of hadron interpolators. In this way, we can study the evolution of hadron masses as a function of the number of the subtracted lowest Dirac eigenmodes. By increasing the number of the subtracted eigenmodes we gradually remove the chiral condensate of the vacuum and thus artificially restore the chiral symmetry. In [4] the latter has been demonstrated on basis of the momentum space quark propagator (in Landau gauge), where the dynamically generated mass of the quark propagator has been shown to disappear as more and more Dirac low modes were excluded.

Note that k itself is not the relevant truncation scale since k has to scale with the lattice volume when keeping the physics constant. Instead, we introduce a cutoff parameter σ such that all μ_k with $|\mu_k| < \sigma$ are excluded at reduction level σ.

3. The setup

3.1. Dirac operator

For our work we use the so-called chirally improved (CI) Dirac operator [5, 6] which is an approximate solution to the Ginsparg–Wilson equation and, therefore, offers better chiral properties compared to the Wilson Dirac operator while requiring about one order of magnitude less computation time than the chirally exact overlap operator.

3.2. Gauge configurations

Our investigation has been carried out on 161 gauge field configurations [7, 8] that were generated for two degenerate dynamical light CI fermions with a corresponding pion mass $m_\pi = 322(5)$ MeV. The lattice size is $16^3 \times 32$ and the lattice spacing $a = 0.144(1)$ fm.

4. Results

Here we list the hadrons under study, give details of the calculation and finally, present the evolution of the hadron masses under Dirac low mode truncation.

4.1. Mesons

We restrict ourselves to the study of isovector mesons of spin 1. Isoscalars require the inclusion of disconnected diagrams which are too costly with the CI Dirac operator. The scalar meson a_0 and the pseudoscalar pion under Dirac low mode removal have been investigated in [1, 2]. Therefore, the studied nonexotic channels are the J^{PC} combinations 1^{--} (ρ), 1^{++} (a_1) and 1^{+-} (b_1).

The variational method [9, 10] combined with standard interpolating fields and three different kinds of quark sources (narrow and wide Gaussian shape sources [11, 12] and a derivative source) enable us to extract some information about excited states.

Diagonalisation of the cross-correlation matrix for lattice interpolators with the appropriate quantum numbers results in eigenvalues with exponential decay $\exp(-E_n t)$. More details of the calculation can be found in [3], where we also show sample plots for all particles of the eigenvalues and plots of the eigenvector components and the effective masses including fit ranges and values. From the latter the energy values can be determined from exponential fits to the eigenvalues. For the a_1 and b_1 we could only extract the values of the ground state masses reliably, whereas for the ρ we found a clear signal for the two lowest lying states and consequently performed fits to these two states.

In Fig. 1, we show the masses of the mesons under study as a function of the truncation level σ (see Sec. 2) in units of the ρ-mass at each truncation level. We also give the corresponding truncation index k on the upper abscissa of the plot.

As was already explored in [1], the masses of the ρ and a_1 approach each other when subtracting all eigenmodes with a mass smaller or equal to approximately two bare quark masses (in our study the bare quark mass is $\approx$ 15 MeV) which indicates restoration of the $\mathrm{SU(2)_R} \times \mathrm{SU(2)_L}$ chiral symmetry. Furthermore, the additional degeneracy of the ρ and a_1 with the ρ' state hints to a higher symmetry which has the chiral symmetry as a subgroup. The fact that the b_1 mass remains clearly heavier tells us not only that the $\mathrm{U(1)_A}$ symmetry remains broken but also gives strong evidence in the meson sector that we are still confronted with confined particles instead of two free quarks traveling next to each other.

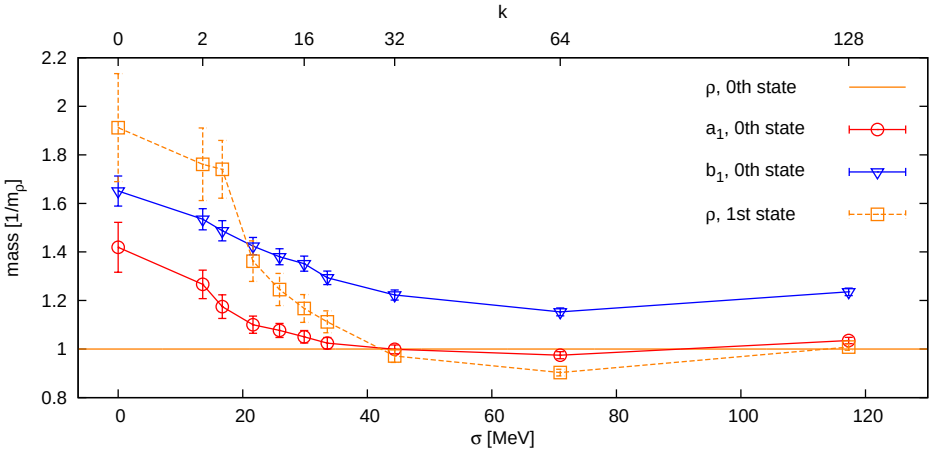


Fig. 1. Evolution of the meson masses under Dirac low mode removal. Masses in units of the ρ-mass at the corresponding truncation level.

4.2. Baryons

We analyze the nucleon and Δ baryons with positive and negative parity. For the interpolators, we use two different Gaussian smeared quark sources [3]. For the nucleon, we adopt interpolating fields with three different Dirac structures. We use parity projection for all baryons and Rarita–Schwinger projection for the Δ [8]. For all details of the calculation, we refer again to [3].

In Fig. 2, we show the mass values of the ground and first excited states of all baryons as a function of the truncation level. We find the nucleons of positive and negative parity ($J^{\mathrm{P}} = \frac{1}{2}^{+}$ and $J^{\mathrm{P}} = \frac{1}{2}^{-}$) to become degenerate upon Dirac low mode removal which reveals them as (would be) chiral partners. But not only the ground states of the two nucleons end up having the same mass, we also see again hints to a higher symmetry which relates the ground and first excited states of the nucleon of both parities.

On the other hand, the first excitations of the Delta with both parities ($J^{\mathrm{P}} = \frac{3}{2}^{+}$ and $J^{\mathrm{P}} = \frac{3}{2}^{-}$) remain clearly higher than the corresponding ground state masses which again manifests the existence of confined particle states after having artificially restored the chiral symmetry.

The last observation is that the Δ–N splitting gets reduced by about a factor 2 once the lowest Dirac modes are excluded. This allows to draw important conclusions: the Δ–N splitting is usually attributed to the hyperfine spin–spin interaction between the valence quarks via either the spin–spin color-magnetic interaction [13, 14] or the flavor–spin interaction related to the spontaneous chiral symmetry breaking [15]. When restoring the chiral

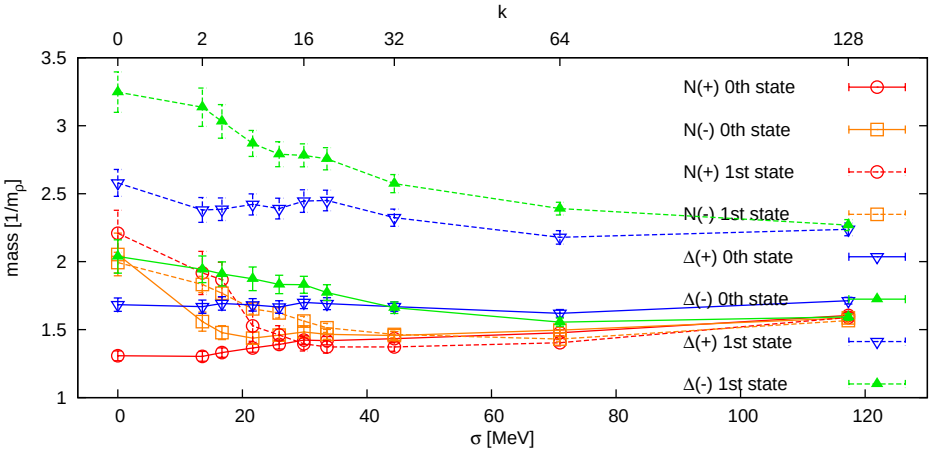


Fig. 2. Evolution of the baryon masses under Dirac low mode removal. Masses in units of the ρ-mass at the corresponding truncation level.

symmetry the effective flavor–spin quark–quark interaction becomes impossible. The color-magnetic interaction remains. Thus, our result suggests that both these mechanisms are of equal importance to the Δ–N splitting.

5. Conclusions

We removed a varying number of the lowest Dirac eigenmodes from the valence quark sector and constructed hadron correlators out of these modified quark propagators.

In the meson sector, we found clear evidence for the restoration of the chiral symmetry; the ρ and a_1 mesons became degenerate. Moreover, the degeneracy of the ρ and ρ' states hints to the existence of a higher symmetry. The mass value of the b_1 remains clearly larger which tells us that the $\mathrm{U(1)_A}$ symmetry is still broken. Furthermore, the existence of (distinguished) particle states after having restored the chiral symmetry shows that confinement remains intact.

In the baryon sector, we studied the nucleon and Δ with positive and negative parity. Both nucleon ground states and their first excitations were found to become degenerate which again hints to a higher symmetry than simply $\mathrm{SU(2)_R \times SU(2)_L}$. The first excitations of Δ remain larger than the corresponding ground state masses which supports our argument that confinement persists under the artificial restoration of chiral symmetry. Lastly, the Δ–N splitting reduces to roughly half its value from which we can conclude that the color-magnetic and the flavor–spin interactions between valence quarks are of equal importance.

Support by the Research Executive Agency (REA) of the European Union under Grant Agreement PITN-GA-2009-238353 (ITN STRONGnet) and by the Austrian Science Fund (FWF) through grant DK W1203-N16 is gratefully acknowledged.

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