IS THE $X(3872)$ A MOLECULE?*

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Because of the controversial $X(3872)$ meson's very close proximity to the $D^0\overline{D}^{*0}$ threshold, this charmonium-like resonance is often considered a meson–meson molecule. However, a molecular wave function must be essentially of a meson–meson type, *viz.* $D^0\overline{D}^{*0}$ in this case, with no other significant components. We address this issue by employing a simple two-channel Schrödinger model, in which the $J^{\rm PC}=1^{++}$ $c\bar{c}$ and $D^0\overline{D}^{*0}$ channels can communicate via the $^3P_0$ mechanism, mimicked by string breaking at a sharp distance a. Thus, wave functions and their probabilities are computed, for different bound-state pole positions approaching the $D^0\overline{D}^{*0}$ threshold from below. We conclude that at the PDG $X(3872)$ mass and for reasonable values of a, *viz.* 2.0–3.0 GeV^{-1}, the $c\bar{c}$ component remains quite substantial and certainly not negligible, despite accounting for only about 6–10% of the total wave-function probability, owing to the naturally long tail of the $D^0\overline{D}^{*0}$ component.

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The $X(3872)$ charmonium-like meson is by now a very well established resonance [1]. It was first observed in 2003, by the Belle Collaboration [2], in the decay $B^\pm \to K^\pm\pi^+\pi^- J/\psi$, with significance in excess of 10σ. Since then, it has been confirmed by several collaborations, *viz.* Belle, BaBar,

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CDF, D0, CLEO, and, more recently, by LHCb (see the 2012 PDG [1] listings for references). The PDG summary table lists the $X(3872)$ as an isoscalar state with positive C-parity, from the observed $\gamma J/\psi$ decay, but unknown J and P, having an average mass $m = 3871.68 \pm 0.17$ MeV$/c^2$ and a width $\Gamma < 1.2$ MeV$/c^2$. The two most likely $J^{\rm PC}$ assignments are 1^{++} and 2^{-+}, while the observed hadronic decay modes are $\rho^0 J/\psi$, $\omega J/\psi$, $D^0\overline{D}^{*0}$, and $D^0\overline{D}^0\pi^0$. Henceforth, we shall denote $D^0\overline{D}^{*0}$ simply by D^0D^{*0}.

Meson spectroscopists have been puzzled by the $X(3872)$ because of its low mass as compared to predictions of conventional quark models, as well as its remarkable proximity to the D^0D^{*0} threshold, being "bound" by only 0.15 MeV [1]. This has led to a plethora of model descriptions of the $X(3872)$, *viz.* as a $c\bar{c}$ state, meson–meson (*MM*) molecule, tetraquark, or hybrid meson. For a number of reviews on the many different approaches and the experimental situation, see [3]. Recently, we have described [4] the $X(3872)$ as a regular but "unquenched" 1^{++} ($^3P_1$) charmonium meson, whose physical mass is dynamically shifted about 100 MeV downwards from the bare $2\,^3P_1$ $c\bar{c}$ state due to its strong coupling to the S-wave D^0D^{*0} and $D^{\pm}D^{*\mp}$ channels, besides several other OZI-allowed and OZI-forbidden ($\rho^0 J/\psi$, $\omega J/\psi$) channels. Thus, the observed hadronic $X(3872)$ properties were well reproduced [4].

Nevertheless, the closeness of the $X(3872)$ to the D^0D^{*0} threshold seems to favour a molecular interpretation [5]. In the latter paper, it is stated that, whatever the original mechanism generating the resonance, a near-threshold bound state will always have a molecular structure. This implies that the *MM* component of the wave function, *i.e.*, D^0D^{*0}, should be the only relevant one. Here, we shall study this issue in a simplified, coordinate-space version of the model employed in [4], restricted to the most important channels, *viz.* $c\bar{c}$ and D^0D^{*0}. Note that even if the $X(3872)$ is essentially a molecule, it will mix with $c\bar{c}$ states having the same quantum numbers.

Now, we turn to the two-channel model used in [6], with parameters adjusted for the $X(3872)$. Consider a coupled $q\bar{q}$–M_1M_2 system, with the $q\bar{q}$ pair confined through a harmonic-oscillator (HO) potential, whereas the two mesons M_1, M_2 are free. The correponding 2×2 radial Schrödinger equation is given by Eq. (1), with the Hamiltonians (2) and (3). Here, $\mu_{c,f}$ is the reduced mass in either channel, $m_q = m_{\bar{q}}$ the constituent quark mass, l_c, l_f the orbital angular momenta, and ω the HO frequency:

$$\begin{pmatrix} h_c & V \\ V & h_f \end{pmatrix} \begin{pmatrix} u_c \\ u_f \end{pmatrix} = E \begin{pmatrix} u_c \\ u_f \end{pmatrix} , \tag{1}$$

$$h_c = \frac{1}{2\mu_c}\left(-\frac{d^2}{dr^2} + \frac{l_c(l_c+1)}{r^2}\right) + \frac{1}{2}\mu_c\omega^2 r^2 + m_q + m_{\bar{q}}\, , \tag{2}$$

$$h_f = \frac{1}{2\mu_f}\left(- \frac{d^2}{dr^2} + \frac{l_f(l_f+1)}{r^2}\right) + M_1 + M_2 \,. \tag{3}$$

Note that we use here relativistic definitions for the *MM* reduced mass μ_f and relative momentum k, even below threshold, contrary to [6], though this is practically immaterial for the $X(3872)$. At some "string-breaking" distance a, transitions between the two channels are described by an off-diagonal point-like potential with strength g

$$V = \frac{g}{2\mu_c a}\delta(r-a)\,. \tag{4}$$

Continuity and twice integrating Eqs. (1)–(3) yields the boundary conditions

$$u_c'(r\uparrow a) - u_c'(r\downarrow a) + \tfrac{\lambda}{a}u_f(a) = u_f'(r\uparrow a) - u_f'(r\downarrow a) + \tfrac{\lambda\mu_f}{a\mu_c}u_c(a) = 0\,, \tag{5}$$

$$u_c(r\uparrow a) = u_c(r\downarrow a) \qquad \text{and} \qquad u_f(r\uparrow a) = u_f(r\downarrow a)\,. \tag{6}$$

A general solution to this problem is given by Eqs. (7) and (8) for the confined and the *MM* state, respectively. The two-component function $u(r) = (u_c(r), u_f(r))$ is related to the radial wave function as $u(r) = rR(r)$:

$$u_c(r) = \begin{cases} A_c F_c(r) & r < a\,, \\ B_c G_c(r) & r > a\,, \end{cases} \tag{7}$$

$$u_f(r) = \begin{cases} A_f J_{l_f}(kr) & r < a\,, \\ B_f\left[J_{l_f}(kr)k^{2l_f+1}\cot\left(\delta_{l_f}(E)\right) - N_{l_f}(kr)\right] & r > a\,. \end{cases} \tag{8}$$

Now, $F_c(r)$ vanishes at the origin and $G_c(r)$ falls off exponentially for $r\to\infty$. Defining then $z = \mu\omega r^2$ and

$$\nu = \frac{E-2m_c}{2\omega} - \frac{l_c+3/2}{2}\,, \tag{9}$$

we get

$$F(r) = \frac{1}{\Gamma(l+3/2)}z^{(l+1)/2}e^{-z/2}\phi(-\nu, l+3/2, z)\,, \tag{10}$$

$$G(r) = -\tfrac{1}{2}\Gamma(-\nu)rz^{l/2}e^{-z/2}\psi(-\nu, l+3/2, z)\,. \tag{11}$$

Here, the functions ϕ and ψ are the confluent hypergeometric functions of first and second kind, respectively, and the Γ function acts as a normalising function. The functions J and N in Eq. (8) are defined in terms of the spherical Bessel and Neumann functions j, n, *i.e.*, $J_l(kr) = k^{-l}rj_l(kr)$

and $N_l(kr) = k^{l+1} r n_l(kr)$. From the boundary conditions (5), (6) and the explicit wave-function expressions in Eqs. (7), (8), we obtain

$$\begin{aligned} G_c'(a)F_c(a) - F_c'(a)G_c(a) &= \frac{g}{a} J_{l_f}(ka) F_c(a) \frac{A_f}{B_c}\,, \\ J_{l_f}'(ka)N_{l_f}(ka) - J_{l_f}(ka)N_{l_f}'(ka) &= \frac{g}{a}\frac{\mu_f}{\mu_c} J_{l_f}(ka) F_c(a) \frac{A_c}{B_f}\,. \end{aligned} \tag{12}$$

Using next the Wronskian relations

$$W\left(F_c(a), G_c(a)\right) = \lim_{r\to a}\left[F_c(r)G_c'(r) - F_c'(r)G_c(r)\right] = 1\,, \tag{13}$$

$$W\left(N_{l_f}(ka), J_{l_f}(ka)\right) = \lim_{r\to a}\left[N_{l_f}(kr)J_{l_f}'(kr) - N_{l_f}'(kr)J_{l_f}(kr)\right] = -1$$

yields

$$A_f B_f = -\frac{\mu_f}{\mu_c} A_c B_c \tag{14}$$

and

$$\frac{A_f}{B_f} = -\left[\frac{g^2}{a^2}\frac{\mu_f}{\mu_c} J_{l_f}^2(ka) F_c^2(a)\right]^{-1} \frac{B_c}{A_c}\,. \tag{15}$$

Finally, with the expression for the MM scattering wave function $u_f(r)$ (second line in Eq. (8)), the final result for $\cot\delta_{l_f}(E)$ is obtained, reading

$$\cot\left(\delta_{l_f}(E)\right) = -\left[g^2 \frac{\mu_f}{\mu_c} k j_{l_f}^2(ka) F_c(a) G_c(a)\right]^{-1} + \frac{n_{l_f}(ka)}{j_{l_f}(ka)}\,. \tag{16}$$

Now, in the present $X(3872)$ model, there is only one scattering channel, *viz.* for the $D^0 D^{*0}$ system. Thus, poles in the S-matrix, which represent possible resonances, bound states, or virtual states, are given by the simple relation $\cot\delta_{l_f}(E) = i$. On the other hand, the solutions to the two-component radial wave function (7), (8) are then fully determined by relations (14) and (15), up to an overall normalisation constant.

Next, we apply this formalism to the coupled $c\bar{c}$–$D^0 D^{*0}$ system. In the confined channel, the $c\bar{c}$ system is in a $2\,{}^3P_1$ state, and so $l_c = 1$, whereas the $D^0 D^{*0}$ channel has $l_f = 0$. In Table I, we give the fixed parameters of the model, with the HO frequency ω and the constituent charm mass as in [7],

TABLE I

Fixed model parameters [7] and $D^0 D^{*0}$ threshold.

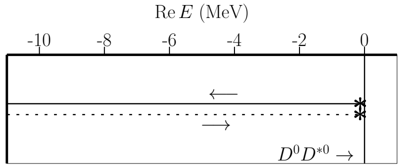
Parameter	ω	m_c	m_{D^0}	$m_{D^{*0}}$	$m_{D^0} + m_{D^{*0}}$
Value (MeV)	190	1562	1864.84	2006.97	**3871.81**

being unaltered ever since. However, the radial quantum number ν in Eq. (9) varies as a function of the energy, and therefore will generally be non-integer, becoming even complex for resonance poles. The parameter that determines such variations is the coupling g. In the uncoupled case, *i.e.*, for $g = 0$, one recovers the bare $^3P_1$ HO spectrum, with energies $(3599 + 2n\omega)$ MeV $(n = 0, 1, 2, \ldots)$. The only other free parameter is the string-breaking distance a. Now we try to find S-matrix poles as a function of the coupling g and for two reasonable values of a, *viz.* 2.0 and 3.0 GeV^{-1} ($\approx$0.4 and 0.6 fm). Searching near the D^0D^{*0} threshold, a dynamical pole is found, either on the first Riemann sheet, corresponding to a bound state, or on the second one, which represents a virtual state (see Ref. [4], second paper). These results are presented in Table II and Fig. 1.

TABLE II

Bound and virtual states near the D^0D^{*0} threshold.

a (GeV^{-1})	g	Pole (MeV)	Type of bound state
2.0	1.133	3871.68	virtual
2.0	1.150	3871.81	virtual
2.0	1.153	3871.81	real
2.0	1.170	3871.68	real
3.0	2.097	3871.68	virtual
3.0	2.144	3871.81	virtual
3.0	2.150	3871.81	real
3.0	2.199	3871.68	real



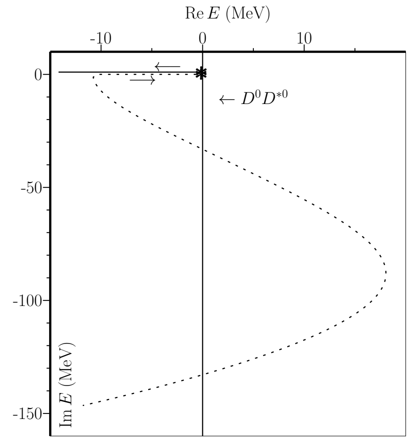


Fig. 1. Dynamical real (solid) and virtual (dashed) pole trajectories for a = 2.0 GeV^{-1} (left) and a = 3.0 GeV^{-1} (right). The arrows indicate pole movement for increasing g. The PDG [1] $X(3872)$ mass is labelled by $*$. Also see Table II.

Note that the dynamical pole arises from the D^0D^{*0} continuum and is not connected to the bare $2\,^3P_1$ $c\bar{c}$ state at 3979 MeV, contrary to the situation in [4] (first paper). For our study here, this is of little consequence.

Finally, we depict the normalised two-component wave-function $R(r)$ in Fig. 2, evaluated for the PDG [1] $X(3872)$ mass of 3871.68 MeV. One clearly sees the P-wave behaviour of the $c\bar{c}$ component, whereas D^0D^{*0} is in an S-wave. Moreover, the $c\bar{c}$ admixture is certainly not negligible, despite the low total probablities of 6.13% and 10.20%, for $a = 2$ GeV^{-1} and $a = 3$ GeV^{-1}, respectively, which are logical because of the very long tail of the D^0D^{*0} component; also see [8]. Soon we will publish more detailed work.

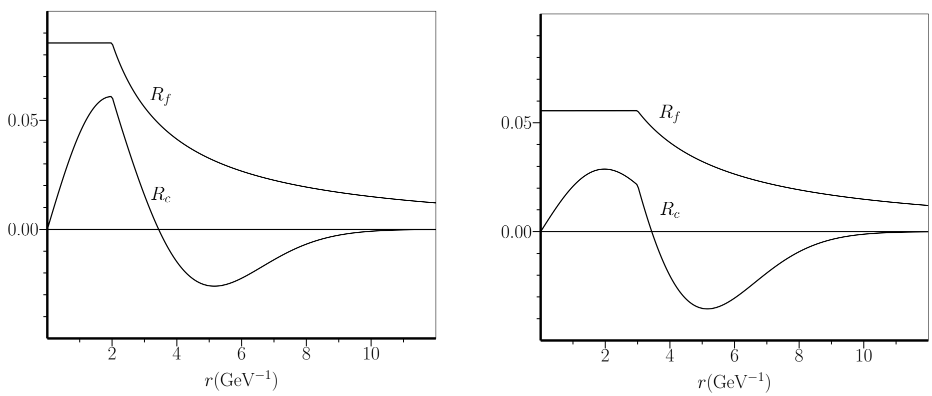


Fig. 2. Radial wave-functions for $E = 3871.68$ MeV and $g = 1.170$, $g = 2.199$ for $a = 2.0$ GeV^{-1} (left) and $a = 3.0$ GeV^{-1} (right). Also see Table II.

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