NLO CORRECTIONS FOR LARGE MULTIPLICITY PROCESSES*

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In this contribution we explain the so-called unitarity method for the calculation of Next-to-Leading Order perturbative corrections with high-multiplicity final states.

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1. Introduction

Theory predictions play a very important role in the particle physics experiments at current hadron colliders. They are not only needed for comparison with the signal one aims at measuring, but are also required throughout the analysis for the modelling of the numerous backgrounds that make most experimental measurements very challenging. In many cases, the signal-to-background ratio is close to or below unity, so that a valuable measurement is only possible with a reliable understanding of the background processes.

At hadron colliders the main tool for obtaining theory predictions is perturbation theory in the strong coupling, which is applicable thanks to asymptotic freedom. The first approximation for a theoretical prediction is a tree-level calculation. This type of computation is highly automated and there exist many programs for their calculation. Unfortunately, these predictions are, in general, very crude and suffer from many uncertainties, such as a large dependence on the unphysical factorisation and renormalisation scales. This dependence is due to the truncation of the perturbation series, the dependence would be cancelled by the higher order terms which get

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neglected. At tree level, this dependence is monotonic and therefore, tree-level predictions cannot provide a quantitative prediction for the absolute normalisation of cross sections. The problem gets worse with each additional power of the coupling constant obtained as one increases the number of jets in the process. This fact is illustrated in Table I. Next-to-Leading Order (NLO) is the first order at which the scale dependence of the coupling constant is partly counter-balanced by the scale dependence of the matrix elements. Therefore, NLO is the first order at which a quantitatively reliable prediction can be provided. NLO corrections are typically large and can, in addition, affect shapes of distributions significantly, yielding a better description which is needed to extrapolate backgrounds from a control region to the signal region. Reliable theory predictions can also be used to 'convert' the measurement of a process into an estimate for the contribution of another.

TABLE I

Dependence of the cross sections on the renormalisation and factorisation scales. The numbers are for the cross section of a W boson accompanied by up to four jets. They are taken from Table I of Ref. [1], where more details can be found.

# of jets	LO % scale dep.	NLO % scale dep.
1	9%	4.5%
2	28%	5.2%
3	47%	7.8%
4	64%	8.4%

2. NLO corrections

To compute NLO corrections, one has to compute two different pieces in addition to the tree-level calculation. The two parts, called the real and the virtual parts can be computed independently but they are both separately divergent. When combined, the divergences cancel, yielding a physically meaningful result.

2.1. Real part

The real part of the NLO corrections accounts for the emission of an additional parton into the final state. The real part displays infrared divergences when the emitted particle is either collinear to another final state parton or if that particle is emitted with a soft momentum. To allow for the numerical evaluation of the phase-space integration of the real part, its divergences have to be regulated.

2.2. Virtual part

The virtual part is currently the bottleneck of the complete automation of the calculation or NLO cross sections and observables. The standard method to compute the virtual corrections involves computing all Feynman diagrams associated with the process. This results for complicated processes in a very large number of terms for the one-loop amplitude

$$\mathcal{A} = \int dl \sum_{\text{many terms}} \frac{\mathcal{N}(l, p_i)}{\prod_j P_j(l, p_i)} \,. \tag{1}$$

These terms are called tensor integrals and integrals with $\mathcal{N} = 1$ are called scalar integrals. $P_j(l, p_i)$ are propagators and $\mathcal{N}$ is a numerator function which depends on the loop momentum l and on the external momenta. Each of these tensor integrals can be written in terms of scalar integrals using Passarino–Veltman reduction. This step usually involves solving very large systems of equations and results in very large and potentially numerically unstable expressions for the coefficients of the scalar integrals. The final answer for the one-loop amplitudes takes the form

$$\mathcal{A} = R + \sum d_i I_i^{(4)} + \sum c_i I_i^{(3)} + \sum b_i I_i^{(2)} \,, \tag{2}$$

where $I^{(n)}$ is an n-point scalar integral and the sums run over all possible propagator combinations. The scalar integrals are well known and process independent. R is the so-called rational part that does not contain any logarithm or polylogarithms. For simplicity, we will assume that the internal propagators are massless and, therefore, no tadpole integrals are present.

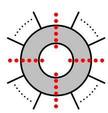
Recently, a new set of approaches has emerged that takes advantage of the knowledge that the final answer can be written in terms of a basis of scalar integrals and aims at computing their coefficient directly avoiding any computationally intensive integral reduction.

2.3. Unitarity-based methods

The unitarity methods use the general factorisation properties of the amplitude in combination with the reduction at the integrand level [2] as a tool to compute coefficients of the scalar amplitudes in Eq. (2). This section gives a sketch of this method, for a review, see Ref. [3]. The unitarity-based methods use the so-called generalised unitarity cuts which have the effect of replacing multiple propagators under the loop integral with delta-functions

$$\frac{1}{P^2} \to 2\pi i \delta\left(P^2\right) \,. \tag{3}$$

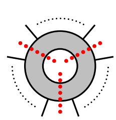
Unitarity cuts can be seen as projectors that project onto everything that contains all the propagators of the cut. The cuts are applied on both sides of Eq. (2). If we apply a quadruple cut on the right-hand side of Eq. (2), the cut operation on the right-hand side will single out the coefficient of the single box scalar integral that has the four propagators of the cut. What is the effect of the quadruple cut on the left-hand side of Eq. (2)? The effect of a quadruple cut in four dimensions is to freeze the loop integral, since four denominators are transformed into delta-functions, all integration variables in the integrand are fixed by the four cut conditions. In addition, the factorisation properties of the amplitude ensure that the integrand factorises into a product of the four tree-level amplitudes that are singled out at the four corners of the quadruple cut, with their external legs along the cut propagators evaluated at a value of the loop momentum that satisfies the four cut conditions



$$= \sum_{l_i} A_1(l_i)A_2(l_i)A_3(l_i)A_4(l_i)\,, \qquad (4)$$

where the sum is over all solutions of the loop momenta that satisfy the four cut conditions. The advantage of expressions derived in this way compared to those obtained by the traditional reduction methods, beyond the simplicity of their calculation, is that they are much more compact. This is partly owed to the fact that all ingredients are on-shell tree amplitudes and, therefore, do not carry any gauge information which usually clutters the coefficients obtained by the standard method while cancelling in the final answer. Such compact expressions are numerically very stable.

In a triple cut, one chooses three propagators to promote to delta-functions. Since the loop integration is four-dimensional and we have three conditions imposed by the cut, we will be left with a one-dimensional integral. The left-hand side of Eq. (2) will have the schematic form



$$= \int dt J(t)A_1(t)A_2(t)A_3(t) \qquad (5)$$

after a triple cut has been performed. Again, the integrand splits up into a product of tree amplitudes due to the factorisation properties of the one-loop amplitude. The right-hand side of Eq. (2) will get two types of contributions when a triple cut is applied. There will be a contribution containing the coefficient of the triangle scalar integral that has exactly the three propagators chosen for the cut and also contributions from box scalar integrals that have the three propagators chosen for the triple cut, along with an additional propagator without cut. This additional propagator contains a pole in the remaining free loop momentum parameter. So with a proper choice of parametrisation for the loop momentum and the knowledge of the parametric form of the integrand in this parametrisation it is possible to disentangle the triangle coefficient and the box contributions in the triple cut [4]. In a numerical approach, one can take advantage of the analytic form of the integrand on the complex plane to subtract the box poles, as these can be computed as outlined above, from the triple cut and compute the triangle coefficient [5].

A similar strategy can be used to compute the bubble coefficients. More details can be found in [4, 5].

3. Applications

Due to both the high statistics and the relative simplicity of its signature the production of a W boson accompanied with jets is a very important process at a hadron collider. The production of a vector boson can be used as a mean of calibrating the underlying event and the jet energy scale. As these processes are among the best understood, both experimentally and theoretically, they are widely used as a validation or testing ground for new tools and methods. As an illustration of the applicability of unitarity methods, we show results for $W+4$ jets using the BlackHat library [5] for the virtual matrix elements and SHERPA [6–9] for the remaining pieces.

Fig. 1 presents the transverse momentum distribution of the first, second, third and fourth jet in $W + 4$ jets events. A detailed list of the cuts, jet algorithm, approximations and calculation setup can be found in Ref. [1].

The dashed (blue) curve is the leading order prediction, the solid (black) one the NLO result. In the lower pane the ratio with respect to the NLO result is taken. In addition, the scale variation bands are displayed as hatched grey (orange) for the LO and in grey for the NLO result. They are obtained as the envelope of the variation by factors of $1/2$, $1/\sqrt{2}$, 1, $\sqrt{2}$ and 2 around the central scale value.

One can see that is a shape difference between the leading order and the NLO predictions. The scale variation is, as expected, much smaller at NLO that it is at LO.

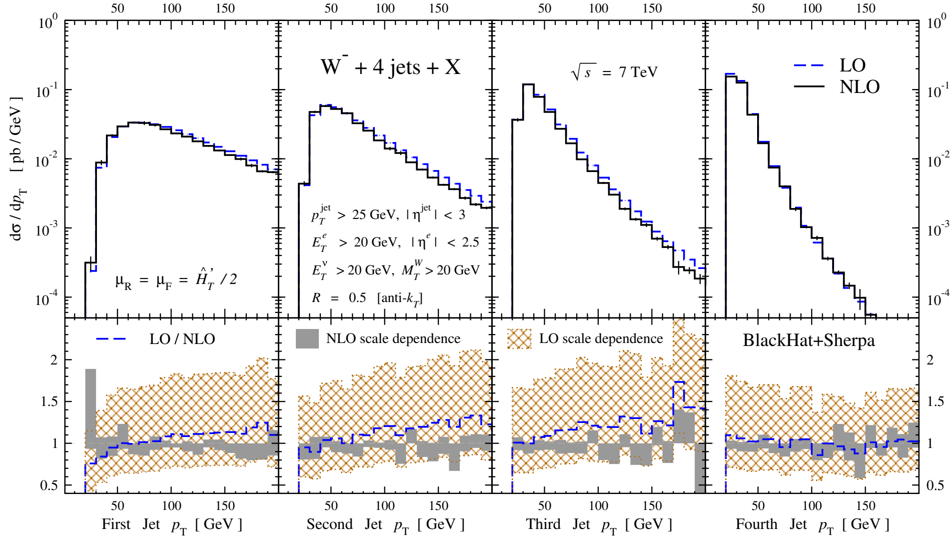


Fig. 1. $P_{\rm t}$ distribution for the first, second, third and fourth jets in $W+4$ jets events. Details on the setup of the calculation can be found in Ref. [1]

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REFERENCES

[1] C. Berger *et al.*, *Phys. Rev. Lett.* **106**, 092001 (2011).
[2] G. Ossola, C.G. Papadopoulos, R. Pittau, *Nucl. Phys.* **B763**, 147 (2007).
[3] H. Ita, *J. Phys.* **A44**, 454005 (2011).
[4] D. Forde, *Phys. Rev.* **D75**, 125019 (2007).
[5] C. Berger *et al.*, *Phys. Rev.* **D78**, 036003 (2008).
[6] T. Gleisberg *et al.*, *J. High Energy Phys.* **0402**, 056 (2004).
[7] T. Gleisberg *et al.*, *J. High Energy Phys.* **0902**, 007 (2009).
[8] F. Krauss, R. Kuhn, G. Soff, *J. High Energy Phys.* **0202**, 044 (2002).
[9] T. Gleisberg, F. Krauss, *Eur. Phys. J.* **C53**, 501 (2008).