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LANGEVIN DIFFUSION IN HOLOGRAPHIC PLASMAS*

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Within the context of the gauge/gravity duality, we discuss the framework to compute the spectral functions governing the generalized Langevin dynamics of a heavy quark propagating through a strongly coupled, non-conformal large-N plasma. Particular attention is focused on the definition of a spectral function that has the correct large-frequency fall-off and satisfies appropriate dispersion relations.

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1. Introduction

In the past ten years, the holographic gauge/gravity duality conjecture [1] has provided a new tool for the description of the non-perturbative physics of strongly coupled gauge theories, allowing observables to be computed in terms of the dynamics of a gravitational system in a higher-dimensional curved space-time (the *gravity dual* description).

In this context, much attention has been devoted to the study of the dynamics of hard probes propagating through a deconfined plasma, described in the dual picture by a higher-dimensional black hole. In particular, the problem of Langevin diffusion of a heavy quark through a deconfined plasma has been discussed in [2, 3] for a generic non-conformal theory admitting a gravity dual. Here, we will give an overview of this problem and summarise the results of [2, 3], to which the reader is referred for details.

2. The Generalized Langevin Process: 4D perspective

Under very general assumptions, the motion of a heavy quark through the deconfined QCD plasma[1] can be described by a *Generalized Langevin*

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[1] By *heavy*, we mean that the quark mass is much larger than the temperature of the plasma. This applies, for example, to the charm and bottom quarks produced in the RHIC and LHC fireballs.

Process, *i.e.* an integro-differential equation for the classical evolution of the particle position $X(t)$, of the form[2]

$$M_q \ddot{X}(t) = \int_{-\infty}^{+\infty} dt'\, G_{\mathrm{R}}\left(t-t'\right) X\left(t'\right) + \xi(t)\,. \tag{1}$$

Above, the first term on the right-hand side is a classical viscous force with retardation effects, governed by the kernel $G_{\mathrm{R}}(\tau)$, whereas the second term is a stochastic noise function with Gaussian correlation:

$$\langle \xi(t) \rangle = 0\,, \qquad \left\langle \xi(t)\xi\left(t'\right)\right\rangle = G_{\mathrm{sym}}\left(t-t'\right)\,. \tag{2}$$

Both functions G_{R} and G_{sym} can be, in principle, calculated as the correlators of the same operator $\mathcal{F}(t)$, which represents the instantaneous force that the thermal bath exerts on the particle

$$G_{\mathrm{R}}(t) = -i\theta(t)\,\langle\, [\mathcal{F}(t), \mathcal{F}(0)]\,\rangle\,, \qquad G_{\mathrm{sym}}(t) = -\frac{i}{2}\,\langle\, \{\mathcal{F}(t), \mathcal{F}(0)\}\,\rangle\,, \tag{3}$$

where now the brackets represent quantum expectation values in the ensemble describing the plasma.

It is instructive to write equation (1) in Fourier space, in terms of the frequency ω

$$-M_q\,\omega^2\, X(\omega) = G_{\mathrm{R}}(\omega) X(\omega) + \xi(\omega)\,, \qquad \left\langle \xi^2(\omega)\right\rangle = G_{\mathrm{sym}}(\omega)\,. \tag{4}$$

Dispersion relations allow to write $G_{\mathrm{R}}(\omega)$ in terms of its imaginary part, *i.e.* the *spectral density* $\rho(\omega)$

$$\mathrm{Im}\, G_{\mathrm{R}}(\omega) = -\pi\rho(\omega)\,, \qquad G_{\mathrm{R}}(\omega) = \int d\omega'\,\frac{\rho(\omega')}{\omega-\omega'}\,. \tag{5}$$

Furthermore, $G_{\mathrm{sym}}(\omega)$ may be obtained from $\rho(\omega)$ once the density matrix of the plasma is known.

When using this formulation, one must be careful to notice that the Fourier integrals exist only if $\rho(\omega)$ has an appropriate fall-off behavior at large ω. If this does not happen, the process is dominated by wild random kicks at short time separation. In order to be considered physical, the spectral density has to vanish sufficiently fast for large ω.

[2] Here and in the following sections, we consider, for simplicity of notation, the one-dimensional, non-relativistic case. In the non-relativistic case, all equations generalize straightforwardly to three dimensions, whereas more care is needed for the relativistic treatment, where the correlators become non-trivial matrices. See [2] for details.

The Green's functions governing the Langevin process can be computed at strong coupling via holography if we assume that, in this regime, the gauge theory admits at gravitational dual description. Our task in the following will be to outline the setup for this computation, and to describe how to obtain a physical spectral with the correct high frequency behavior.

3. The Generalized Langevin Process: the gravity dual perspective

3.1. Review of gauge/gravity duality

The *gauge/gravity duality*, or AdS/CFT correspondence, (see [1] for a review) is the conjectured equivalence between a gauge field theory in D flat space-time dimensions (*boundary theory*) and a string theory in a $(D+n)$-dimensional curved space-time (*bulk theory*). The field theory can be thought of as living on the boundary of the higher-dimensional space-time. The latter features a non-compact coordinate r which parametrizes the distance from the boundary and corresponds to the energy scale in the field theory dual.

In the limit, when the gauge theory has a large number of colors, and is in a regime of strong 't Hooft coupling, the string theory may be well approximated with its classical gravity limit. One of the main insights about this correspondence, which we summarize below, allows the computation of field theory correlators at strong coupling via a classical calculation on the gravity side [4]. The main ingredient for these calculations is the field/operator correspondence:

- To each boundary theory gauge-invariant operator $O(x)$, there corresponds a bulk field $\varPhi(x,r)$, whose boundary value represents a source for $O(x)$, *i.e.* a Lagrangian coupling in the boundary theory of the form
$$S_{\text{coupling}} = \int d^D x\, O(x)\varPhi(x, r=0)\,. \tag{6}$$
- Close to the boundary, the classical solution for the field $\varPhi(x,r)$ has an expansion of the form:
$$\varPhi(x,r) = A(x) r^{D-\varDelta} + B(x) r^{\varDelta}\,, \qquad \varDelta \equiv \dim[O]\,. \tag{7}$$
- The two-point function of the operator $O(x)$ is given in momentum space by
$$\langle O(p) O(-p)\rangle \propto B(p)/A(p)\,, \tag{8}$$
where $A(p)$, $B(p)$ are the Fourier transforms of the functions appearing in Eq. (7).

In the next subsection, we will identify the bulk field that corresponds to the operator $\mathcal{F}(t)$ which defines the Langevin correlators via Eq. (3).

We will be interested in the case when the bulk space-time is a five-dimensional general *asymptotically Anti-de Sitter (AdS) black hole*, with metric[3]

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right], \qquad i = 1,2,3\,. \tag{9}$$

The AdS asymptotics require that the space-time has a (conformal) boundary at $r = 0$, *i.e.*

$$r \to 0\,, \qquad f(r) \to 1\,, \qquad \log b(r) \sim \log\frac{\ell}{r} + \text{subleading}\,. \tag{10}$$

The presence of a bulk black hole means that there exists a regular horizon at $r = r_{\mathrm{h}} > 0$, where

$$f(r_{\mathrm{h}}) = 0\,, \qquad \dot{f}(r_{\mathrm{h}}) = 4\pi T_{\mathrm{h}}\,. \tag{11}$$

The 4D dual to such a system is an asymptotically conformal (in the UV) field theory in a deconfined plasma in equilibrium at temperature T_{h} [1].

3.2. The trailing string picture of quark diffusion

A heavy quark is represented in the gravity dual by the boundary endpoint, moving at constant speed v, of a *string* with finite tension, stretching from the boundary down into the interior [6]. The trailing string profile in the interior is found by solving for the minimal embedding of the string, *i.e.* the extremization of the $1+1$ dimensional Nambu–Goto action in the background (9). The solution is expressed in terms of the embedding coordinates

$$\vec{X}(r,t) = (vt + \xi(r))\vec{v}/v\,, \tag{12}$$

where $\xi(r)$ vanishes at $r = 0$ and represents the bending of the string away from the boundary. The non-trivial metric in the bulk causes momentum to flow from the endpoint into the horizon, and this produces a classical drag friction. The induced worldsheet metric is a $1+1$-dimensional black hole, with temperature $T_{\mathrm{s}} < T_{\mathrm{h}}$, and horizon located at the point r_{s}, where $f(r_{\mathrm{s}}) = v^2$.

As noted in [7], the fluctuations around the classical string solution, $\delta X(r,t)$ are the bulk field dual to the force operator $\mathcal{F}(t)$ that determines the Langevin correlators. Therefore, these correlators can be computed, and the Langevin dynamics derived, by defining fluctuations around the background (12)

[3] These backgrounds may be obtained as solutions of simple five-dimensional Einstein–Dilaton systems, see [5] and references therein.

$$\vec{X}(r,t) = (vt + \xi(r))\vec{v}/v + \delta\vec{X}(x,t)\,, \tag{13}$$

solving the linearized equation of motion for the bulk field $\delta\vec{X}(x,t)$, and applying the prescription (6)–(8). As explained in detailed in [2], this procedure allows to numerically obtain, for arbitrary temperature $T_{\rm h}$ and velocity v, the full frequency-dependent Langevin spectral density $\rho(\omega)$ which determines the correlator through Eq. (5). Furthermore, in [2], analytic expressions were obtained for the high-frequency limit, which controls the short-time behavior of the diffusion process and reads, in the non-relativistic case

$$\rho(\omega) \simeq \frac{\ell^2}{2\pi^2\ell_{\rm s}^2}\,\omega^3\,h\left(\frac{\sqrt{2}}{\omega}\right)\,, \qquad \omega\to\infty\,, \tag{14}$$

where ℓ is the asymptotic AdS length, $\ell_{\rm s}$ the string length, and the function $h(r)$ defines the details of the metric near the boundary

$$b(r) \sim \frac{\ell}{r}h(r)\,, \qquad r\to 0\,. \tag{15}$$

From (14) it is clear that the spectral density obtained in this fashion has a fast growth, $\rho\sim\omega^3$, for large ω, and it cannot directly be used to model the physical Langevin process, as discussed at the end of Sec. 1.

3.3. Dressed Langevin correlators

Now we address the question of defining holographically a spectral density that satisfies the correct fall-off behavior at large ω. In general, $\rho(\omega)$ will depend on temperature, but notice that the high frequency behavior (14) does not. Thus, if we subtract the same quantity at, say, $T=0$, this will cancel the leading term at all T. This property was used in [3] to propose a definition for the physical spectral density as

$$\rho^{(\rm phys)}(\omega) = \rho(\omega) - \rho^{(\rm vac)}(\omega)\,, \tag{16}$$

where the subtracted term $\rho^{(\rm vac)}(\omega)$ accounts for the dissipation in the zero-temperature vacuum, due to quantum fluctuations. As shown in [3], the expression (16) can be derived from a path integral computation, by introducing a *dressed* quark coordinate required to undergo no dissipation when propagating in vacuum.

The subtracted vacuum term $\rho^{(\rm vac)}(\omega)$ is computed by the same procedure outlined in Sec. 3.2, in a bulk solution with no black hole, *i.e.* as in (9) but with $f(r)=1$. The resulting $\rho^{(\rm vac)}$ is trivially temperature independent, and as shown in [3] the subtraction cancels the leading as well as the first subleading high-frequency behavior, leaving a spectral density with a physically acceptable fall-off

$$\rho^{(\mathrm{phys})}(\omega) \sim \omega^{-1}\,, \qquad \omega \to \infty\,. \tag{17}$$

As an illustration of this results, we display in Fig. 1 the subtracted spectral function in the simplest case, where the bulk metric is exactly AdS, *i.e.* $b(r) = \ell/r$. For more realistic applications, one can compute the same quantities in phenomenological gravity duals which are closer to QCD, for example those described in [5].

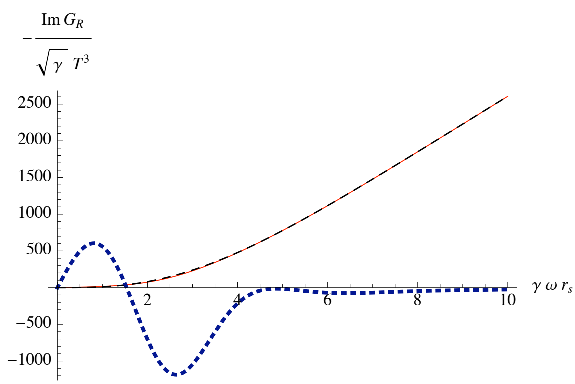

Fig. 1. The unsubtracted (red solid), vacuum (black dashed) and subtracted (blue dashed, scaled up by a factor 10^2) temperature-normalized spectral densities as a function of frequency, in the example of the AdS black hole bulk metric. The quark velocity is taken to be $v = 0.995c$.

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