PHENOMENOLOGY OF DILATON IN A CHIRAL LINEAR SIGMA MODEL WITH VECTOR MESONS*

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In the framework of the $\mathrm{U(2)_R} \times \mathrm{U(2)_L}$ symmetric linear sigma model with (axial)vector mesons generalized by including a dilaton field, we study the phenomenology of the scalar–isoscalar resonances below 2 GeV. It turns out that in our favoured scenario, the resonance $f_0(1370)$ is predominantly a $\bar{q}q$ state and $f_0(1500)$ is predominantly a glueball state. Additionally, we are able to calculate the value of the gluon condensate, which is in agreement with lattice QCD results.

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1. Introduction

One of the interesting issues in particle physics is the overpopulation in the scalar–isoscalar channel, $I^G(J^{\mathrm{PC}}) = 0^+(0^{++})$ in the energy region below 2 GeV. In this region, there are currently five states listed by PDG [1]. Below 1 GeV, there are two resonances: $f_0(500)$ and $f_0(980)$. Above 1 GeV, there are the three resonances $f_0(1370), f_0(1500)$ and $f_0(1710)$. The nature of these resonances is not completely understood up to the present day. In this work, we consider an $N_f = 2$ [2, 3] effective Lagrangian with two scalar–isoscalar states, where one of them is a quark–antiquark state, $|\bar{n}n\rangle \equiv \left(\bar{u}u + \bar{d}d\right)/\sqrt{2}$, and the other one is a scalar glueball, $|G\rangle \equiv gg$. The experimental verification of glueballs is extremely challenging due to the mixing with ordinary $\bar{q}q$ mesons but the proof of their existence would be a further important achievement in QCD. Furthermore, the proper candidate for a glueball should possess the following features: *(i)* Due to the 'democratic' coupling of the gluons to all kinds of quarks, the glueball should be flavour blind. *(ii)* The decay width should be rather narrow because the

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large-N_c behaviour shows that glueball decays scales as N_c^{-2}, thus they are stronger suppressed than decays of ordinary mesonic $\bar{q}q$ states, which only scale as N_c^{-1}. The flavour blind decay behaviour of the resonance $f_0(1500)$ with a mass of $M_{f_0(1500)} = (1505 \pm 6)$ MeV and its narrow decay width, $\Gamma_{f_0(1500)} = (109 \pm 7)$ MeV [1], make it a proper scalar glueball candidate. The main aim of this study is to investigate the mixing between the scalar–isoscalar glueball and the scalar–isoscalar quark–antiquark state in order to make some statements about the nature of the scalars–isoscalars below the mass of 2 GeV.

As shown in Ref. [3] on which this proceeding is based on, our most successful scenario is realized by the assignment: $|G\rangle \cong f_0(1500)$ and $|\bar{q}q\rangle \cong f_0(1370)$. Moreover, due to the new available determination of the resonance $f_0(500)$ [1] ($M_{f_0(500)} = (400\text{–}550)$ MeV and $\Gamma_{f_0(500)} = (400\text{–}700)$ MeV) we tested again the following two alternative scenarios: (1) $|G\rangle \cong f_0(1500)$ and $|\bar{q}q\rangle \cong f_0(500)$ and (2) $|G\rangle \cong f_0(1710)$ and $|\bar{q}q\rangle \cong f_0(500)$ but both of them are still inconsistent with the experimental data and hence not favoured. Additionally, by the use of the phenomenology of mesons only, we are capable to calculate the value of the gluon condensate, $\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle$ which agrees to the lattice QCD results.

2. The model

The effective model under study is based on the chiral symmetry $U(N_f)_\mathrm{R} \times U(N_f)_\mathrm{L}$ and on the trace anomaly of QCD. It is composed of the quark–antiquark Lagrangian (see Ref. [2]) and of the dilaton Lagrangian describing the trace anomaly (see Ref. [3] and references therein). The latter one reads

$$\mathcal{L}_\mathrm{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right), \tag{1}$$

where G is the scalar dilaton/glueball field [4] and Λ an energy scale. The minimum of the potential in Eq. (1) is given by $G_0 = \Lambda$, and after shifting the dilaton field, $G \to G_0 + G$, a massive particle occurs, which corresponds to the scalar glueball. According to lattice QCD [5] its mass is about $m_G \sim 1.6$ GeV. The logarithmic term breaks the dilatation symmetry, $x^\mu \to \lambda^{-1} x^\mu$, explicitly and this leads to the divergence of the corresponding current: $\partial_\mu J^\mu_\mathrm{dil} = T^\mu_{\mathrm{dil},\,\mu} = -\frac{1}{4} m_G^2 \Lambda^2$. In the chiral limit (neglecting the $U(1)_\mathrm{A}$ anomaly) the energy scale Λ in Eq. (1) is the only dimensionful parameter of the effective model. Furthermore, we require that the effective Lagrangian must be finite for every finite value of the gluon condensate G_0. This implies that in the chiral limit all other terms have dimension $\left[E^4\right]$ in order to ensure dilatation invariance. Thus, the full effective Lagrangian reads:

$$\mathcal{L}=\mathcal{L}_{\rm dil}+{\rm Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi)-m_0^2\left(\frac{G}{G_0}\right)^2\Phi^\dagger\Phi-\lambda_2\left(\Phi^\dagger\Phi\right)^2\right]-\lambda_1\left(\left[{\rm Tr}\Phi^\dagger\Phi\right]\right)^2$$
$$+c_1\left[\det(\Phi)-\det\left(\Phi^\dagger\right)\right]^2+{\rm Tr}\left[H\left(\Phi^\dagger+\Phi\right)\right]-\frac{1}{4}{\rm Tr}\left[(L^{\mu\nu})^2+(R^{\mu\nu})^2\right]$$
$$+\frac{m_1^2}{2}\left(\frac{G}{G_0}\right)^2{\rm Tr}\left[(L^\mu)^2+(R^\mu)\right]^2+\frac{h_1}{2}{\rm Tr}\left[\Phi^\dagger\Phi\right]{\rm Tr}\left[L_\mu L^\mu+R_\mu R^\mu\right]$$
$$+h_2{\rm Tr}\left[\Phi^\dagger L_\mu L^\mu\Phi+\Phi R_\mu R^\mu\Phi^\dagger\right]+2h_3{\rm Tr}\left[\Phi R_\mu\Phi^\dagger L^\mu\right], \qquad (2)$$

where the (pseudo)scalar and the left- and the right-handed (axial)vector d.o.f. are organized in multiplets. The explicit form of these multiplets in the case of $N_f = 2$ is the following: $\Phi = (\sigma + i\eta_{\rm N})\, t^0 + (\vec{a}_0 + i\vec{\pi})\cdot\vec{t}$, $L^\mu = (\omega^\mu + f_1^\mu)\, t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu)\cdot\vec{t}$ and $R^\mu = (\omega^\mu - f_1^\mu)\, t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu)\cdot\vec{t}$, where t^0, $\vec{t}$ are the corresponding generators. The assignment of the fields in (2) is as follows: The pseudoscalar fields $\vec{\pi}$ and $\eta_{\rm N} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ with $m_{\eta_{\rm N}} = 716$ MeV [2, 6] represent the pions [1] and the nonstrange part of the η and η' mesons, respectively. The scalar–isoscalar field, $\sigma \equiv \left(\bar{u}u + \bar{d}d\right)/\sqrt{2}$, represents the nonstrange quark–antiquark state. It turns out that the resonance $f_0(1370)$ is favoured to be a predominantly $\bar{q}q$ state [2]. Therefore, we identify σ with the resonance $f_0(1370)$, but we also assign it to the resonance $f_0(500)$ in order to test all possible scenarios. Corresponding to the study of Ref. [2], the scalar–isovector field $\vec{a}_0$ represent the resonance $a_0(1450)$. Finally, the vector fields ω^μ and $\vec{\rho}^{\,\mu}$ are assigned to the $\omega(782)$ and $\rho(770)$, respectively, and the axialvector fields f_1^μ and $\vec{a}_1^{\,\mu}$ to the $f_1(1285)$ and $a_1(1260)$, respectively [1]. Note, the mass of $a_1(1260)$ given by PDG is only an estimate. According to Ref. [7], we fixed the mass of m_{a_1} to 1050 MeV. After shifting the scalar–isoscalar fields $\sigma = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $G = gg$ by their vacuum expectation values, $\sigma \to \sigma + \phi$ and $G \to G + G_0$, a bilinear mixing term $\sim \sigma G$ in (2) occurs. This required a diagonalization of the corresponding effective Lagrangian (realized by a SO(2) rotation) in order to obtain the physical fields σ' and G'

$$\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma \\ G \end{pmatrix}, \qquad (3)$$

where $\theta = \frac{1}{2}\arctan\left[-4\,\frac{\phi}{G_0}\,\frac{m_0^2}{M_G^2 - M_\sigma^2}\right]$ is the quarkonium-glueball mixing angle.

3. Results and discussion

The model in Eq. (2) contains ten free parameters: m_0, λ_1, λ_2, m_1, g_1, c_1, h, $\tilde{h} = h_1 + h_2 + h_3$, m_G, $\Lambda = \sqrt{11}\,C^2/(2m_G)$, where C represents the gluon condensate. Once we used the masses of π, $\rho(770)$, m_{η_N}, and m_{a_1} as well as the pion decay constant f_π, we are left with four free parameters: C, m_1, M_σ and m_G. We obtained them by a χ^2 analysis using the five experimental quantities of Table I (details in Ref. [3]).

TABLE I

Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$.

Quantity	Fit [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 26	1200–1500
$M_{G'}$	1505 ± 6	1505 ± 6
$G' \to \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \to \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \to K\bar{K}$	9.3 ± 1.7	9.37 ± 1.69

3.1. Scenario with $G' \equiv f_0(1500)$ and $\sigma' \equiv f_0(1370)$

Our best fit is obtained for the assignment $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$. We used as input for the χ^2 analysis the following quantities according to [1]: masses of the resonances $f_0(1500)$ and $f_0(1370)$ (for which we used the mean value $M^{\rm ex}_{\sigma'} = (1350 \pm 150)$ MeV due to the wide mass range of this resonance) and the three well-known decay widths of $f_0(1500)$: $\Gamma_{f_0(1500)\to\pi\pi}$, $\Gamma_{f_0(1500)\to\eta\eta}$ and $\Gamma_{f_0(1500)\to K\bar{K}}$ (see Table I).

The value of the quarkonium-glueball mixing angle is $\theta = (29.7 \pm 3.6)^\circ$. This implies that the resonance $f_0(1500)$ consists to 76% of a glueball and to the remaining 24% of a quark–antiquark state. In the case of $f_0(1370)$, we obtain an inverted situation. An important outcome of our fit is the value of the gluon condensate, $C = (699 \pm 40)$ MeV, which is in agreement with lattice QCD results [8]. Note that the gluon condensate is an essential quantity of QCD and we obtained its numerical value by use of experimental data. Further consequences and predictions are given in Table II. The decay of $f_0(1500)$ into 4π through the intermediate state of $\rho\rho$ mesons is calculated by using the ρ spectral function. Our result is about half of the experimental one. We expect that the intermediate state consisting of two $f_0(500)$ resonances also contributes in this decay channel but this resonance is not yet included in the model. The decays of the resonance $f_0(1370)$ are in agreement with the experimental data regarding the full decay width: $\Gamma_{f_0(1370)} = (200\text{–}500)$ MeV [1], where our result is around $\Gamma_{\sigma'} \simeq 360$ MeV.

Note that the inclusion of the (axial)vector d.o.f. was crucial in order to obtain the presented results [2, 9]. The artificial decoupling of (axial)vector states would generate a, by far too wide, $f_0(1370)$ state. For this reason, the glueball-quarkonium mixing scenario above 1 GeV has been previously studied only in phenomenological models with flavour symmetry [10, 11] but not in the context of chirally invariant models. In Ref. [3], we also investigated the scenario $\{\sigma', G'\} = \{f_0(1370),\ f_0(1710)\}$ and we have found that it is not favoured by experimental data.

TABLE II

Further results regarding the $\sigma' \equiv f_0(1370)$ and $G' \equiv f_0(1500)$ decays.

Quantity	Fit [MeV]	Experiment [MeV]
$G' \to \rho\rho \to 4\pi$	30	54.0 ± 7.1
$G' \to \eta\eta'$	0.6	2.1 ± 1.0
$\sigma' \to \pi\pi$	284 ± 43	—
$\sigma' \to \eta\eta$	72 ± 6	—
$\sigma' \to K\bar{K}$	4.6 ± 2.1	—
$\sigma' \to \rho\rho \to 4\pi$	0.09	—

3.2. Scenarios with $\sigma' \equiv f_0(500)$

We have tested the assignments $\{\sigma', G'\} = \{f_0(500),\ f_0(1500)\}$ and $\{\sigma', G'\} = \{f_0(500),\ f_0(1710)\}$ using the new available experimental data of the $f_0(500)$ resonance [1]. We used for the calculation the mean value of its mass, $M_{\sigma'}^{\rm ex} = (475 \pm 75)$ MeV. In both assignments, the mixing angle turns out to be small ($\lesssim 13^\circ$) and this implies that the state $f_0(500)$ is almost a pure quarkonium. The problem of these scenarios is that the decay into two pions is too narrow, $\Gamma_{\sigma' \to \pi\pi} \lesssim 180$ MeV (as already found in Ref. [3]), in comparison to the experimental one, $\Gamma_{f_0(500) \to \pi\pi} = (400\text{–}700)$ MeV. We thus confirm our result in Ref. [3] that the scenarios with the resonance $f_0(500)$ as a quarkonium state are not favoured.

4. Conclusions and outlook

We have used a chiral linear sigma model with (axial)vector mesons and a scalar glueball to study the phenomenology of the scalar–isoscalar states below 2 GeV. The best agreement with the present experimental data is reached when the resonance $f_0(1500)$ is predominantly identified with a glueball state, $|G\rangle \equiv gg$ and $f_0(1370)$ with a quark–antiquark state, $|\bar{n}n\rangle \equiv \left(\bar{u}u + \bar{d}d\right)/\sqrt{2}$. Scenarios in which $f_0(500)$ is predominantly a quark–antiquark state show discrepancies with the experiment.

Ongoing works are the full inclusion of strangeness, $N_f = 3$ [12] and eventually the inclusion of a nonet of tetraquarks [13]. This may enable us to describe a general mixing scenario of all five scalar–isoscalar states below the mass of 2 GeV listed by PDG [1].

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