VECTOR SPECTRAL FUNCTION IN THE DECAY OF THE τ LEPTON*

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We present the vector spectral function for the τ lepton calculated in the framework of the extended Linear Sigma Model. The τ decays weakly with intermediate ρ and a_1 meson states. Electroweak interaction was introduced into the model by requiring invariance under $\mathrm{U(1)_Y} \times \mathrm{SU(2)_L}$ gauge transformations. We determine the mass of the ρ meson and consequently the decay width that yield the best description of the data and obtain slightly smaller values with respect to the values given by the Particle Data Group.

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1. Introduction

The phenomenology of the low energy hadronic resonances is still not fully understood (see Refs. [1, 2] and references therein). Deep inelastic scattering of leptons and hadrons has led to the idea that hadrons are composite objects made from quarks and gluons. In the high energy region the running coupling α_S becomes small enough for a perturbative expansion, while in the low energy region an expansion in the pion momentum within the framework of chiral perturbation theory may be applied. In the region of $\simeq 1\,\mathrm{GeV}$ hadrons are the observed physical quantities. Therefore, it is natural to construct phenomenological models based on hadronic degrees of freedom where we use the symmetry principles known from the Lagrangian of Quantum Chromodynamics.

The Linear Sigma Model we use here is based on a linear realisation of chiral symmetry which is known from the QCD Lagrangian in the limit of vanishing quark masses. The $N_f = 2$ hadronic degrees of freedom in this

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work are the low lying scalar, pseudoscalar, vector and axial-vector meson isosinglets and isotriplets. With this model it is possible to investigate the structure of hadrons by calculating scattering lengths, decay widths, and meson masses.

The quarks inside mesons are not only subject to strong interaction but also to electromagnetic and weak interactions. As a lepton, the τ is not subject to strong interaction. However, it decays weakly into a τ-neutrino and a charged weak boson. This process is well known from the Standard Model of Particle Physics. The W boson can, in turn, create a quark–antiquark pair. Thus, including the weak interaction into our hadronic model gives us insight into the nature of processes that involve strong and weak interactions.

2. Extended Linear Sigma Model with electroweak interactions

The *extended Linear Sigma Model* [2, 3] describes (pseudo-) scalar and (axial-) vector mesons on the basis of a spontaneously and explicitly broken global chiral $\mathrm{U}(2)_\mathrm{L} \times \mathrm{U}(2)_\mathrm{R} = \mathrm{U}(2)_\mathrm{A} \times \mathrm{U}(2)_\mathrm{V}$ symmetry. The $\mathrm{U}(2)_\mathrm{A} \times \mathrm{U}(2)_\mathrm{V}$ symmetry is explicitly broken to $\mathrm{SU}(2)_\mathrm{V}$ by the non-vanishing quark masses and the $\mathrm{U}(1)_\mathrm{A}$ anomaly. Spontaneous symmetry breaking generates the mass difference between the chiral partners, *e.g.*, $\pi \to \sigma$, by a non-vanishing vacuum expectation value of the σ.

Scalar and pseudoscalar mesons are described by the matrix-valued field

$$\Phi = S + iP = (\sigma_0 + i\eta)t_0 + (\vec{a}_0 + i\vec{\pi})\,\vec{t}\,,$$

where $t_0 = 1/2\mathbb{1}, \vec{t} = 1/2\vec{\sigma}$ and σ_i, $i = 1, 2, 3$ are the Pauli matrices. Vector and axial-vector mesons are described by

$$V^\mu = \omega^\mu t_0 + \vec{\rho}^\mu \vec{t}, \qquad A^\mu = f_1^\mu t_0 + \vec{a}_1 \vec{t}.$$

They are included into the Lagrangian by means of left- and right-handed fields which are defined as

$$L^\mu = V^\mu + A^\mu\,, \qquad R^\mu = V^\mu - A^\mu\,.$$

Under chiral transformations the meson fields transform as follows

$$\Phi \to U_\mathrm{L} \Phi U_\mathrm{R}^\dagger\,, \qquad L^\mu \to U_\mathrm{L} L^\mu U_\mathrm{L}^\dagger\,, \qquad R^\mu \to U_\mathrm{R} R^\mu U_\mathrm{R}^\dagger\,.$$

Then, the $N_f = 2$ Lagrangian reads

$$\begin{aligned}
\mathscr{L} &= \mathrm{Tr}\left[(D^\mu \Phi)^\dagger (D^\mu \Phi)\right] + \delta \frac{g\cos\theta_\mathrm{C}}{2}\,\mathrm{Tr}[W_{\mu\nu}L^{\mu\nu}] + \bar{\delta}\frac{e}{2}\,\mathrm{Tr}[R_{\mu\nu}B^{\mu\nu}] - m_0^2\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right] \\
&-\lambda_1\,\mathrm{Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right] - \lambda_2\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 - \frac{1}{4}\,\mathrm{Tr}\left[(L^{\mu\nu})^2 - (R^{\mu\nu})^2\right] + \frac{m_1}{2}\,\mathrm{Tr}\left[(L^\mu)^2 - (R^\mu)^2\right] \\
&+\frac{m_1}{2}\,\mathrm{Tr}\left[(L^\mu)^2 - (R^\mu)^2\right] + \frac{1}{4}\,\mathrm{Tr}\left[(W^{\mu\nu})^2 + (B^{\mu\nu})^2\right] + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] \\
&+c_1\left(\det\Phi - \det\Phi^\dagger\right)^2 + \mathscr{L}_3 + \mathscr{L}_4 + \mathscr{L}_5\,. \qquad (1)
\end{aligned}$$

The determinant term $c_1(\det\Phi - \det\Phi^\dagger)^2$ corresponds to the $\mathrm{U}(1)_\mathrm{A}$ anomaly while the term $\mathrm{Tr}[H(\Phi+\Phi^\dagger)]$ generates the explicit breaking of the $\mathrm{U}(2)_\mathrm{L} \times \mathrm{U}(2)_\mathrm{R}$ symmetry to $\mathrm{SU}(2)_\mathrm{V}$ due to the bare quark masses. The interaction between scalar and vector mesons is contained in the covariant derivative (3) by a minimal coupling term. The terms $\mathscr{L}_3, \mathscr{L}_4$, and $\mathscr{L}_5$ contain chirally invariant three and four point interactions between scalars and vectors which are not relevant in the following. Scalar and vector fields transform under a $\mathrm{U}(1)_\mathrm{Y} \times \mathrm{SU}(2)_\mathrm{L}$ gauge transformation according to

$$\Phi \to U_\mathrm{L}\Phi U_\mathrm{Y}^\dagger\,, \qquad L^\mu \to U_\mathrm{L} L^\mu U_\mathrm{L}^\dagger\,, \qquad R^\mu \to U_\mathrm{Y} R^\mu U_\mathrm{Y}^\dagger\,,$$

with the transformations

$$U_\mathrm{L} = \exp\{-i\theta_\mathrm{L}^a(x^\mu)t_a\}\,, \qquad U_\mathrm{Y} = \exp\{i\theta_\mathrm{Y}(x^\mu)t_3\}\,. \qquad (2)$$

With these transformations the covariant derivative with the interactions between scalar mesons and weak bosons reads

$$\begin{aligned}
D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ig\cos\theta_\mathrm{C}(W_1^\mu t_1 + W_2^\mu t_2)\Phi \\
&\quad -ie[A^\mu, \Phi] - ig\cos\theta_\mathrm{W}\left(Z^\mu\Phi + \tan^2\theta_\mathrm{W}\Phi Z^\mu\right)\,. \qquad (3)
\end{aligned}$$

The bare fields W^μ and B^μ have been rotated by the Weinberg angle θ_W into the physical electroweak interaction fields A^μ and Z^μ. The weakly interacting eigenstates q' are related to the strongly interacting eigenstates q by the Cabibbo–Kobayashi–Maskawa matrix ($N_f = 3 : d' = d\cos\theta_\mathrm{C} + s\sin\theta_\mathrm{C}$). The charged weak bosons induce the flavor transition between up and down quarks and, therefore, also in the two flavor model, we need an additional factor $\cos\theta_\mathrm{C}$ for each interaction between mesons and the charged weak bosons.

The field strength tensors for left- and right-handed fields in (1) are then defined as

$$\begin{aligned}
L^{\mu\nu} &\equiv \partial^\mu L^\nu - ie\,[A^\mu, L^\nu] - ig\cos\theta_\mathrm{C}\,[W_1^\mu t_1 + W_2^\mu t_2, L^\nu] \\
&\quad - \{\partial^\nu L^\mu - ie\,[A^\nu, L^\mu] - ig\cos\theta_\mathrm{C}\,[W_1^\nu t_1 + W_2^\nu t_2, L^\mu]\} \qquad (4)
\end{aligned}$$

and $R^{\mu\nu}$ similarly. In order to also obtain mixing terms between weak gauge bosons W and vector mesons ρ, we included the chirally and $\mathrm{U}(1)_{\mathrm{Y}}\times\mathrm{SU}(2)_{\mathrm{L}}$ invariant product of the electroweak field strength tensors with the field strength tensors for the left-handed and right-handed fields

$$\delta\frac{g\cos\theta_{\mathrm{C}}}{2}\,\mathrm{Tr}[W_{\mu\nu}L^{\mu\nu}]+\bar{\delta}\frac{e}{2}\,\mathrm{Tr}[R_{\mu\nu}B^{\mu\nu}]\,. \tag{5}$$

The vector and axial-vector spectral functions as measured by the ALEPH Collaboration are given in Ref. [4]. They are related to the Källen–Lehmann representations of vector and axial-vector spectral functions $\rho_{\mathrm{V}}(s)$ and $\rho_{\mathrm{A}}(s)$ by

$$v_1(s) \;=\; N_{\mathrm{V}}\frac{(2\pi^2)}{S_{\mathrm{EW}}}(\delta\, s)^2\frac{1}{s}\rho_{\mathrm{V}}(s)\,, \tag{6}$$

$$a_1(s) \;=\; N_{\mathrm{A}}\frac{(2\pi^2)}{S_{\mathrm{EW}}}(\delta\, s+g_1\phi_0^2)^2\frac{1}{s}\rho_{\mathrm{A}}(s)\,. \tag{7}$$

For a detailed discussion on spectral functions and their interpretation see Refs. [5, 6]. The couplings in the mixing terms between W bosons and ρ and a_1 mesons are obtained from our Lagrangian under the assumption of a point-like exchange of the weak charged bosons according to figure 1.

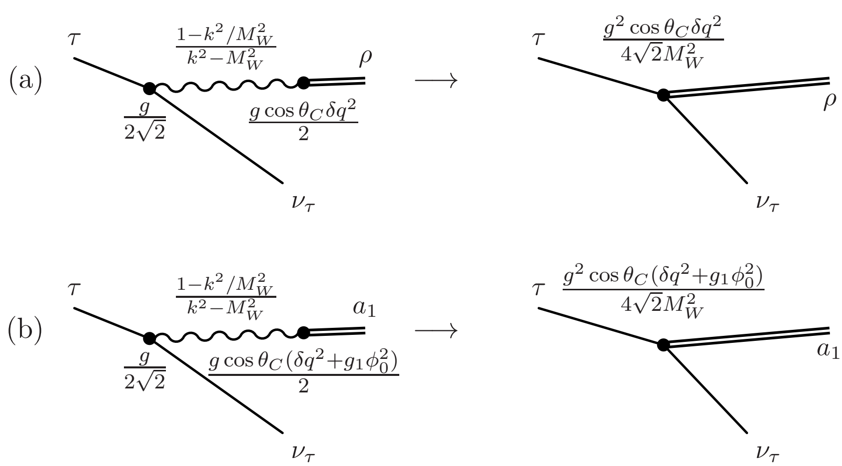

Fig. 1. Effective couplings of (a) $\tau\to\rho\nu_\tau$ and (b) $\tau\to a_1\nu_\tau$.

3. Vector channel results

The main contribution to the vector channel is the decay $\tau \to \nu_\tau \pi\pi^0$ which takes places via an intermediate ρ resonance (see figure 2). The ρ meson decays with a probability of almost 100% into the final two-pion state $\pi\pi^0$ and we can determine the vector–channel coupling δ.

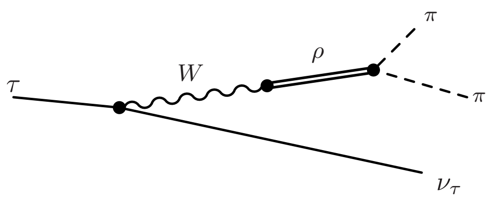

Fig. 2. The process $\tau \to W\nu_\tau \to \pi\pi^0$.

In terms of the $\rho \to \pi\pi^0$ decay width the momentum dependent parametrisation of the spectral density reads

$$v_1(s) = N_{\mathrm{V}} \frac{2\pi}{S_{\mathrm{EW}}} (\delta\, s)^2 \frac{1}{s} \frac{\sqrt{s}\; \mathit{\Gamma}_{\rho\to\pi\pi^0}(s)}{(s-m_\rho^2)^2 + \left(\sqrt{s}\; \mathit{\Gamma}_{\rho\to\pi\pi^0}(s)\right)^2} \,. \tag{8}$$

The decay width for $\rho \to \pi\pi^0$ has been calculated within the extended linear sigma model and reads [3]

$$\mathit{\Gamma}_\rho(s) = \frac{m_\rho^5}{48\pi m_{a_1}^2} \left[1 - \left(\frac{2m_\pi}{\sqrt{s}}\right)^2\right]^{\frac{3}{2}} \left[g_1 Z^2 \frac{g_2}{2}\left(1 - Z^2\right)\right]^2 \,. \tag{9}$$

In order to reproduce the line shape of the data it is necessary to shift the ρ mass about 2.9% from the PDG value $m_\rho = 0.77549\,\mathrm{GeV}$ [7] to $m_\rho = 0.75516\,\mathrm{GeV}$. With this shifted mass we obtain $\mathit{\Gamma}_\rho(m_\rho^2) = 0.14146\,\mathrm{GeV}$ for the full ρ decay with which is about 5.1% smaller than the value given by the PDG $\mathit{\Gamma}_\rho = 0.1491\,\mathrm{GeV}$ [7]. In Ref. [8] it was shown how the exact value of ρ mass and width depend on the parametrization that underlies the decay width. In their results the ρ mass is also shifted to slightly smaller values. The result for the PDG ρ mass and width in comparison with our value is shown in figure 3. It is seen that with two parameters, the weak vector channel coupling δ in (5) and the ρ mass, the calculated spectral density gives a very good description of the data.

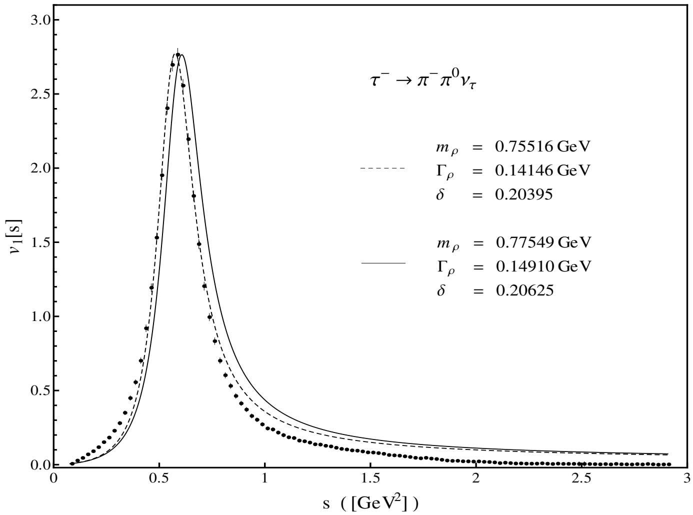

Fig. 3. Vector–channel spectral functions for $m_\rho = 0.77455$ GeV, $\Gamma_\rho = 0.1491$ GeV from the PDG and our result $m_\rho = 0.75516$ GeV, $\Gamma_\rho(m_\rho^2) = 0.1415$ GeV.

4. Conclusion and outlook

We have included weak interaction into the model in Ref. [3] by performing a $SU(2)_L \times U(1)_Y$ gauge transformation of the (pseudo-) scalar and (axial-) vector fields. In order to study the vector channel in the τ decay we calculated the vector spectral density from the interaction vertices in our Lagrangian and could see that with two free parameters we were able to describe the data. The full calculation of the axial-vector channel with all contributions is in progress.

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