SOME PROPERTIES OF TWO NAMBU–JONA-LASINIO-TYPE MODELS WITH INPUTS FROM LATTICE QCD*

P. Costa, C.A. de Sousa, M.C. Ruivo, O. Oliveira, P.J. Silva

Centro de Física Computacional, Departamento de Física
Universidade de Coimbra, 3004-516 Coimbra, Portugal

H. Hansen

IPNL, Université de Lyon/Université Lyon 1, CNRS/IN2P3
4 rue E. Fermi, 69622 Villeurbanne Cedex, France

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We investigate the phase diagram of the so-called Polyakov–Nambu–Jona-Lasinio (PNJL) model at finite temperature and nonzero chemical potential. The calculations are performed in the light and strange quark sectors (u, d, s) which includes the 't Hooft instanton induced interaction term that breaks the axial symmetry, and the quarks are coupled to the (spatially constant) temporal background gauge field. On one hand, a special attention is paid to the critical end point (CEP). The strength of the flavor-mixing interaction alters the CEP location, since when it becomes weaker the CEP moves to low temperatures and can even disappear. On the other hand, we also explore the connection between QCD, a nonlocal Nambu–Jona-Lasinio type model and the Landau gauge gluon propagator. Possible links between the quenched gluon propagator and low energy hadronic phenomenology are investigated.

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1. The phase diagram in the context of the PNJL model

Chiral symmetry breaking and confinement are two of the most important features of quantum chromodynamics (QCD). Chiral models like the Polyakov–Nambu–Jona-Lasinio (PNJL) model have been successful in explaining the dynamics of spontaneous breaking of chiral symmetry and its

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restoration at high temperatures and densities/chemical potentials. The PNJL model also plays an interesting role in the investigation of the QCD phase structure. Understanding the properties of matter at finite temperatures and densities is one of the most important goals from both the theoretical and experimental point of view. For example, the critical end point of QCD, proposed at the end of the eighties, is still a very important subject of discussion nowadays: indeed its existence and location is one of the main goals in SPS at CERN and in RHIC at BNL [1].

The NJL model describes interactions between constituent quarks, giving the correct chiral properties; static gluonic degrees of freedom are then introduced in the NJL Lagrangian, through an effective gluon potential in terms of Polyakov loops, with the aim of taking into account features of both chiral symmetry breaking and deconfinement. The coupling of the quarks to the Polyakov loop leads to the reduction of the weight of quark degrees of freedom as the critical temperature is approached from above, which is interpreted as a manifestation of confinement and is essential to reproduce lattice results.

Our calculations are performed in the framework of an extended $\mathrm{SU}(3)_f$ PNJL Lagrangian, which includes the 't Hooft instanton induced interaction term that breaks the $\mathrm{U_A}(1)$ symmetry, and the quarks are coupled to the (spatially constant) temporal background gauge field Φ [2, 3]

$$\begin{aligned}\mathcal{L} &= \bar{q}(i\gamma^\mu D_\mu - \hat{m})q + \tfrac{1}{2}\, g_S \sum_{a=0}^{8} \left[(\,\bar{q}\,\lambda^a\, q\,)^2 + (\,\bar{q}\, i\,\gamma_5\,\lambda^a\, q\,)^2 \right] \\ &\quad + g_D\, \{\det\left[\bar{q}\,(1+\gamma_5)\,q\right] + \det\left[\bar{q}\,(1-\gamma_5)\,q\right]\} - \mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)\,. \end{aligned} \tag{1}$$

The covariant derivative is defined as $D^\mu = \partial^\mu - iA^\mu$, with $A^\mu = \delta^\mu_0 A_0$ (Polyakov gauge); in Euclidean notation $A_0 = -iA_4$. The strong coupling constant g is absorbed in the definition of $A^\mu(x) = g\mathcal{A}^\mu_a(x)\frac{\lambda_a}{2}$, where $\mathcal{A}^\mu_a$ is the $(\mathrm{SU}(3)_c)$ gauge field and λ_a are the (color) Gell-Mann matrices.

The effective potential for the (complex) field Φ adopted in our parametrization of the PNJL model reads

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{a\,(T)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1 - 6\bar{\Phi}\Phi + 4\left(\bar{\Phi}^3+\Phi^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right], \tag{2}$$

where

$$a\,(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad\text{and}\quad b(T) = b_3\left(\frac{T_0}{T}\right)^3 . \tag{3}$$

The parameters of the effective potential $\mathcal{U}$ are given by $a_0 = 3.51$, $a_1 = -2.47$, $a_2 = 15.2$ and $b_3 = -1.75$. When quarks are added, the parameter T_0, the critical temperature for the deconfinement phase transition (that manifests itself as a breaking of the center symmetry) within a pure gauge approach, was fixed to 270 MeV, according to lattice findings. This choice ensures an almost exact coincidence between chiral crossover and deconfinement at zero chemical potential, as observed in lattice calculations.

The parameters of the NJL sector are: $m_u = m_d = 5.5\,\mathrm{MeV}$, $m_s = 140.7\,\mathrm{MeV}$, $g_S \Lambda^2 = 3.67$, $g_D \Lambda^5 = -12.36$ and $\Lambda = 602.3\,\mathrm{MeV}$, which are fixed to reproduce the values of the coupling constant of the pion, $f_\pi = 92.4\,\mathrm{MeV}$, and the masses of the pion, the kaon, the η and η', respectively, $M_\pi = 135\,\mathrm{MeV}$, $M_K = 497.7\,\mathrm{MeV}$, $M_\eta = 514.8\,\mathrm{MeV}$ and $M_{\eta'} = 960.8\,\mathrm{MeV}$ [4].

The inclusion of the Polyakov loop effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$, that can be seen as an effective pressure term mimicking the gluonic degrees of freedom of QCD, is required to get the correct Stefan–Boltzmann limit. Indeed, in the NJL model the ideal gas limit is far to be reached due to the lack of gluonic degrees of freedom.

In Fig. 1 (left panel), we present the phase diagram of the PNJL model. As the temperature increases the chiral transition is first order and persists up to the CEP. At the CEP the chiral transition becomes a second order one. The location of the CEP is found at $T^{\mathrm{CEP}} = 155.80\,\mathrm{MeV}$ and $\mu^{\mathrm{CEP}} = 290.67\,\mathrm{MeV}$ ($\rho_B^{\mathrm{CEP}} = 1.87\rho_0$). For temperatures above the CEP there is a crossover whose location is calculated making use of $\partial^2 \langle \bar{q}q \rangle / \partial T^2 = 0$, *i.e.* the inflection point of the quark condensate $\langle \bar{q}q \rangle$.

The transition to the deconfinement is given by $\partial^2 \Phi / \partial T^2 = 0$, and is represented by the hatched (magenta) line. The surrounding shaded area that limits the region where the crossover takes place is determined by the extremes of the susceptibility $\partial \Phi / \partial T$.

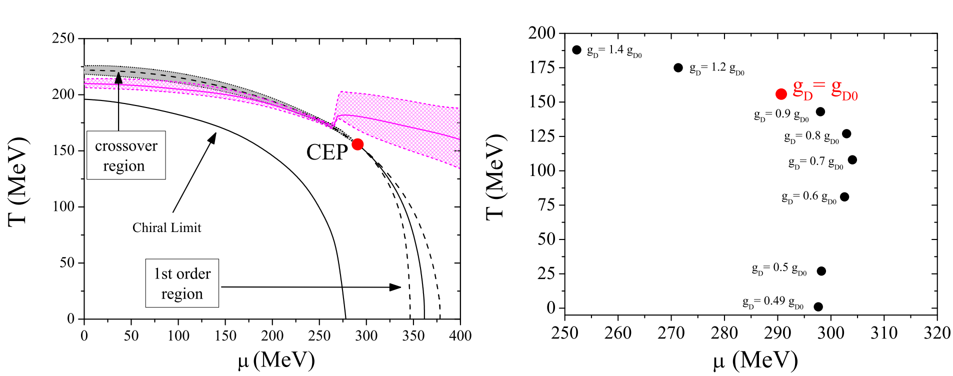

Fig. 1. Phase diagram in the SU(3) PNJL model: the location of the CEP is found at $T^{\mathrm{CEP}} = 155.80\,\mathrm{MeV}$ and $\mu^{\mathrm{CEP}} = 290.67\,\mathrm{MeV}$ (see details in the text). Dependence of the location of the CEP on the strength of the 't Hooft coupling constant g_D.

Due to the importance of the location of the CEP from the experimental point of view, let us investigate the influence of other parameters which can lead to a significant change in the CEP's localization.

It is well known that the $\mathrm{U_A}(1)$ anomaly has big influence on the behavior of several observables, so it is demanding to investigate possible changes in the location of the CEP in the (T, μ) plane when the anomaly strength is modified. The axial $\mathrm{U_A}(1)$ symmetry is broken explicitly by instantons, leaving a $\mathrm{SU}(\mathrm{N}_f)\otimes \mathrm{SU}(\mathrm{N}_f)$ symmetry which determines the chiral dynamics. Since instantons are screened in a hot or dense environment, the $\mathrm{U_A}(1)$ symmetry may be effectively restored in matter. So, the change of the $\mathrm{U_A}(1)$ anomaly strength has a strong influence on the localization of the CEP in the (T, μ) plane.

In Fig. 1 (right panel), we show the location of the CEP for several values of g_D compared to the results for g_{D_0}, the value used for the vacuum. As already pointed out by Fukushima in [2], the location of the CEP depends on the value of g_D. The results show that, in the framework of this model, the existence or not of the CEP is determined by the strength of the anomaly coupling, the CEP getting closer to the μ axis as g_D decreases. As the strength of the flavor-mixing interaction becomes weaker, the CEP moves to low temperatures and can even disappear.

2. Low energy physics and the gluon propagator

In this section, we explore the connection between QCD, a nonlocal Nambu–Jona-Lasinio type model and the Landau gauge gluon propagator [5].

The interaction between quarks and gluons in QCD reads

$$\mathcal{L}_{\overline{\psi}\psi A} = g\,\overline{\psi}\,\gamma^\mu\,A^a_\mu\,\frac{\lambda^a}{2}\,\psi\,. \tag{4}$$

Expanding the term containing $\mathcal{L}_{\overline{\psi}\psi A}$ up to g^2 and integrating the gluon fields (see [5] for details), the theory becomes an effective nonlocal fermionic theory

$$\begin{aligned} S\left[\,\overline{\psi},\psi\right] = &\int d^4x d^4y \Big\{ \overline{\psi}(y)\,\delta(y-x)\,\left(i\gamma^\mu\partial_\mu - m\right)\psi(x) \\ &+\frac{g^2}{8}\,J(x,y)D(x-y)J(y,x) - \frac{g^2}{8}\,J_5(x,y)D(x-y)J_5(y,x)\Big\} \end{aligned} \tag{5}$$

with $J(x,y)=\overline{\psi}(x)\psi(y)$ and $J_5(x,y)=\overline{\psi}(x)\gamma_5\psi(y)$ and $D(x,y)$ is the gluon propagator form factor.

First principles calculations of the gluon propagator have been performed using lattice QCD and DSE (see for example [6] and references therein). The momentum space propagator

$$D\left(p^2\right) = Z\,\frac{\left(p^2\right)^{2\kappa-1}}{\left(p^2+\Lambda_{\mathrm{QCD}}^2\right)^{2\kappa}} \tag{6}$$

is able to describe both the scaling ($\kappa > 0.5$) and decoupling ($\kappa = 0.5$) infrared DSE solutions and the lattice data up to $p \sim 800$ MeV; Λ_{QCD} stands for an infrared mass scale.

Let us define the dimensionless form factor in momentum space as

$$f\left(p^2\right) = \Lambda^2 D\left(p^2\right) = \frac{\Lambda^2}{p^2}\left(\frac{p^2}{p^2+\Lambda_{\mathrm{QCD}}^2}\right)^{2\kappa}\theta(\Lambda - p)\,. \tag{7}$$

The constant Z in Eq. (6) will be included in the definition of the coupling constant G, which multiplies the quark currents of the nonlocal theory. G carries the dimension of a length squared. In $f(p^2)$, Λ is the cut-off. In a first step, we assume $\Lambda_{\mathrm{QCD}} = \Lambda$. The form factor $f(p^2)$ is shown in Fig. 2 (left panel) together with typical form factors considered in the literature.

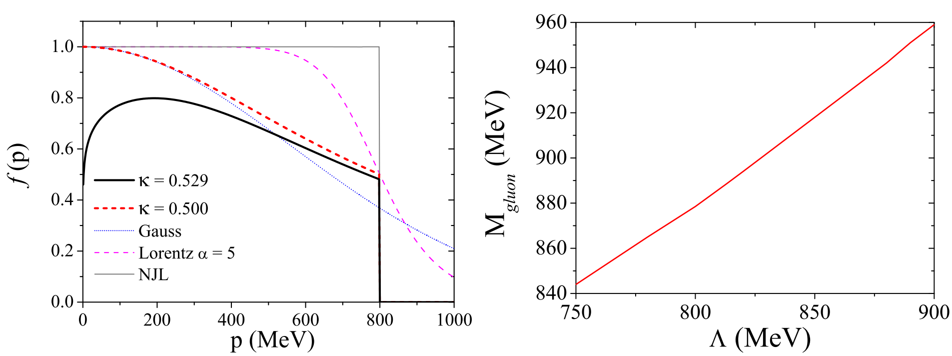

Fig. 2. Form factors as a function of p. The figure includes typical form factors used in previous studies. $M_{\mathrm{gluon}}(\Lambda)$ required to reproduce the experimental $\Gamma_{\pi\to\gamma\gamma}$.

Demanding the nonlocal model to reproduce the experimental values for M_π, f_π and $\Gamma_{\pi\to\gamma\gamma}$, with a cut-off $\Lambda = \Lambda_{\mathrm{QCD}} = 800\,\mathrm{MeV}$, we obtain $m_q = 4.205\,\mathrm{MeV}$, $-\langle\bar{q}q\rangle^{1/3} = 271.1\,\mathrm{MeV}$, $G\Lambda^2 = 7.491$ and $\kappa = 0.529$. The presented results favor $\kappa > 0.5$. Now we investigate the decoupling type of propagator

$$D\left(p^2\right) = \frac{Z}{p^2 + M_{\mathrm{gluon}}^2}\,, \tag{8}$$

where $M_{\rm gluon}$ takes the role of $\Lambda_{\rm QCD}$ and can be interpreted as an effective gluon mass. Requiring the model to reproduce the same experimental quantities as before, we find $M_{\rm gluon}$ within typical values found in the literature, but with a strong dependence with the cut-off. In fact, $M_{\rm gluon}$ is a linear function of Λ — see Fig. 2, right panel. In what concerns the quark condensate, the model shows that $\langle \bar{q}q \rangle$ increases with $M_{\rm gluon}$, *i.e.* with the cut-off Λ. In order to reproduce the experimental value of the condensate, *i.e.* to have $(-\langle \bar{q}q \rangle)^{1/3} = 270\,{\rm MeV}$, it turns out that the gluon mass is $M_{\rm gluon} = 878\,{\rm MeV}$ for $\Lambda = 800\,{\rm MeV}$. Therefore, we conclude that low energy physics does not distinguish between the so-called decoupling and scaling solutions of the Dyson–Schwinger equations. This result means that, provided that the model parameters are chosen appropriately, one is free to choose any of the above scenarios.

Finally, it is interesting to note that the model considered here is chiral invariant and satisfies the GMOR relation at the 1% level of precision.

As future work, we will generalize the results to nonzero temperature (this requires modeling the gluon propagator at finite temperature by a functional form compatible with both Dyson–Schwinger and lattice QCD results) which will allow us to investigate the meson properties at finite temperature as probes for the chiral symmetry restoration.

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