PHENOMENOLOGY OF AXIAL-VECTOR MESONS FROM AN EXTENDED LINEAR SIGMA MODEL*

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We discuss the phenomenology of the axial-vector mesons within a three-flavour Linear Sigma Model containing scalar, pseudoscalar, vector and axial-vector degrees of freedom.

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1. Introduction

A correct description of axial-vector mesons is important in Quantum Chromodynamics (QCD) for several reasons. The lightest measured axial-vector meson — the $a_1(1260)$ resonance — is experimentally ambiguous already in vacuum: according to listings of the Particle Data Group (PDG [1]), the decay width of $a_1(1260)$ possesses values between 250 MeV and 600 MeV. There is only one class of mesons distinct from axial-vectors with similar values of decay widths: the scalar mesons, the theoretical and experimental description of which is famously ambiguous (see, *e.g.*, Refs. [2–5]). The large decay width renders experimental as well as theoretical determination of the properties of $a_1(1260)$ rather problematic.

Additionally, it is well-known that the QCD Lagrangian with N_f massless quark flavours possesses an exact $\mathrm{SU}(N_f)_\mathrm{A}$ axial symmetry. Consequently, a conserved axial-vector current of the form $\bar{q}_f \gamma^\mu \gamma_5 t^i q_f$ arises, where q_f denotes a quark flavour, γ^μ and γ_5 are Dirac matrices and t^i represent

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generators of the chiral $\mathrm{SU}(N_f)_\mathrm{L} \times \mathrm{SU}(N_f)_\mathrm{R}$ chiral group with N_f flavours, $i = 1, ..., N_f^2 - 1$. An axial rotation of this current leads to a conserved vector current of the form $\bar{q}_f \gamma^\mu t^i q_f$. Identifying (putatively) the latter two currents with the $a_1(1260)$ and $\rho(770)$ mesons, respectively, leads to the assertion that the latter two resonances should be degenerate in vacuum. The opposite is observed due to the Spontaneous Breaking of the Chiral Symmetry [6]; however, the mentioned degeneration is expected to be restored at finite temperatures and densities (the so-called chiral transition). A viable theoretical description of this postulated high-temperature phenomenon requires a satisfactory description of axial-vectors already in vacuum — and such a description is an objective of this article. (Note that claims have been made [7] that the vector current is actually a mixture of two rotated chiral-partner fields, an axial-vector and a pseudovector one. As a first approximation, we will neglect this possibility.)

In this article, we present a Linear Sigma Model containing scalar, pseudoscalar, vector and axial-vector mesons both in the non-strange and strange sectors (extended Linear Sigma Model or eLSM [3, 5, 8–10]). The model contains only $\bar{q}q$ states [3–5] rendering it appropriate to study not only general features (masses/decays) of mesons but also their structure — if a physical resonance can be accommodated within our model, then it possesses the $\bar{q}q$ structure. This criterion is important for scalars [4, 8] but also for axial-vectors considering the claims that, *e.g.*, $a_1(1260)$ represents a meson–meson molecule rather than a quarkonium [11]. Consequently, this paper will consider the issue whether the $a_1(1260)$ meson can be accommodated within eLSM, *i.e.*, if a_1 can be described as (predominantly) a $\bar{q}q$ state.

The outline of the paper is as follows. In Sec. 2 we present the three-flavour Lagrangian with vector and axial-vector mesons. Consequences of a global fit of observables for all states present in the model except scalar isosinglets and K_1 are presented in Sec. 3. We provide our conclusions in Sec. 4.

2. The model

The Lagrangian of the Extended Linear Sigma Model with the chiral $\mathrm{U}(3)_\mathrm{L} \times \mathrm{U}(3)_\mathrm{R}$ symmetry (eLSM) reads [3, 5, 8–10]:

$$\begin{aligned}
\mathcal{L} &= \mathrm{Tr}\left[(D^\mu \Phi)^\dagger (D^\mu \Phi)\right] - m_0^2 \,\mathrm{Tr}\left(\Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \,\mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 \\
&- \frac{1}{4} \mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L^\mu)^2 + (R^\mu)^2\right] + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] \\
&+ c_1 \left(\det \Phi - \det \Phi^\dagger\right)^2 + i \frac{g_2}{2} \left(\mathrm{Tr}\left\{L_{\mu\nu}\left[L^\mu, L^\nu\right]\right\} + \mathrm{Tr}\left\{R_{\mu\nu}\left[R^\mu, R^\nu\right]\right\}\right) \\
&+ \frac{h_1}{2} \mathrm{Tr}\left(\Phi^\dagger \Phi\right) \mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + h_2 \,\mathrm{Tr}\left[|\Phi R^\mu|^2 + |L^\mu \Phi|^2\right] + 2h_3 \,\mathrm{Tr}\left(\Phi R_\mu \Phi^\dagger L^\mu\right) ,
\end{aligned} \tag{1}$$

where

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_{\mathrm{N}}+a_0^0)+i(\eta_{\mathrm{N}}+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_{\mathrm{N}}-a_0^0)+i(\eta_{\mathrm{N}}-\pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_{\mathrm{S}} + i\eta_{\mathrm{S}} \end{pmatrix} \qquad (2)$$

is a matrix containing the scalar and pseudoscalar degrees of freedom, $L^\mu = V^\mu + A^\mu$ and $R^\mu = V^\mu - A^\mu$ are, respectively, the left-handed and the right-handed matrices containing vector and axial-vector degrees of freedom with

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{\mathrm{N}}+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_{\mathrm{N}}-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_{\mathrm{S}} \end{pmatrix}^\mu , \quad A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1\mathrm{N}}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1\mathrm{N}}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1\mathrm{S}} \end{pmatrix}^\mu \qquad (3)$$

and $\Delta = \mathrm{diag}(\delta_{\mathrm{N}}, \delta_{\mathrm{N}}, \delta_{\mathrm{S}})$ describes the explicit breaking of the chiral symmetry in the (axial-)vector channel (in terms of masses of u, d and s quarks, $\delta_{\mathrm{N}} \sim m_{u,d}^2$ and $\delta_{\mathrm{S}} \sim m_s^2$; isospin symmetry for non-strange quarks has been assumed). Explicit symmetry breaking in the (pseudo)scalar sector is described by $\mathrm{Tr}[H(\Phi + \Phi^\dagger)]$ with the constant matrix $H = 1/2\,\mathrm{diag}(h_{0\mathrm{N}}, h_{0\mathrm{N}}, \sqrt{2}h_{0\mathrm{S}})$. Additionally, $D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA^\mu[T_3, \Phi]$ is the covariant derivative; $L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu[T_3, L^\mu]\}$ and $R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu[T_3, R^\mu]\}$ are, respectively, the left-handed and right-handed field strength tensors, A^μ is the electromagnetic field, T_3 is the third generator of the SU(3) group and the term $c_1(\det\Phi - \det\Phi^\dagger)^2$ describes the $\mathrm{U}(1)_{\mathrm{A}}$ anomaly [12].

We assign the field $\vec{\pi}$ to the pion; η_{N} and η_{S} are assigned, respectively, to the pure non-strange and the pure strange counterparts of the η and η' mesons. The fields ω_{N}^μ, $\vec{\rho}^\mu$, $f_{1\mathrm{N}}^\mu$ and $\vec{a}_1^\mu$ are assigned to the $\omega(782)$, $\rho(770)$, $f_1(1285)$ and $a_1(1260)$ mesons, respectively. We also assign the K fields to the kaons; the ω_{S}^μ, $f_{1\mathrm{S}}^\mu$ and $K^{\star\mu}$ fields correspond to the $\varphi(1020)$, $f_1(1420)$ and $K^\star(892)$ mesons, respectively. Assignment of the K_1^μ field is, unfortunately, not as clear since this state can be assigned either to the $K_1(1270)$ or to the $K_1(1400)$ resonances. This is further discussed in Sec. 3.

The isoscalar fields σ_{N} and σ_{S} mix in the Lagrangian (1) originating two mixed states; these states, together with the non-strange isovector state $\vec{a}_0$ and the scalar kaon $K_0^\star$, can be assigned to resonances below or above 1 GeV in the physical spectrum [1]; results from our model prefer the latter assignment [3, 5, 8, 10]. As a consequence, we assign our $\vec{a}_0$ and $K_0^\star$ states to $a_0(1450)$ and $K_0^\star(1430)$, respectively.

Spontaneous chiral-symmetry breaking requires a shift of the fields $\sigma_{\rm N}$ and $\sigma_{\rm S}$ by their respective vacuum expectation values $\phi_{\rm N}$ and $\phi_{\rm S}$. We then observe that mixing terms containing axial-vectors and pseudoscalars and $K^\star$ and $K_0^\star$ arise in the Lagrangian; these are removed as described in Refs. [3, 5, 9]. Subsequently, renormalisation coefficients need to be introduced for the pseudoscalar fields and $K_0^\star$ (more details in Refs. [3, 5, 9]).

Lagrangian (1) contains 14 parameters: λ_1, λ_2, c_1, $h_{0\rm N}$, $h_{0\rm S}$, h_1, h_2, h_3, m_0^2, g_1, g_2, m_1, $\delta_{\rm N}$, $\delta_{\rm S}$. Parameters $h_{0\rm N}$ and $h_{0\rm S}$ are determined from the extremum condition for the potential obtained from Eq. (1). Parameter $\delta_{\rm N}$ is set to zero throughout this paper because the explicit breaking of the chiral symmetry is small in the non-strange quark sector. The other 11 parameters are calculated from a global fit including 21 observables: f_π, f_K, m_π, m_K, m_η, $m_{\eta'}$, m_ρ, $m_{K^\star}$, $m_{\omega_{\rm S}\equiv\varphi(1020)}$, $m_{f_{1\rm S}\equiv f_1(1420)}$, m_{a_1}, $m_{a_0\equiv a_0(1450)}$, $m_{K_0^\star\equiv K_0^\star(1430)}$, $\Gamma_{\rho\to\pi\pi}$, $\Gamma_{K^\star\to K\pi}$, $\Gamma_{\phi\to KK}$, $\Gamma_{a_1\to\rho\pi}$, $\Gamma_{a_1\to\pi\gamma}$, $\Gamma_{f_1(1420)\to K^\star K}$, $\Gamma_{a_0(1450)}$, $\Gamma_{K_0^\star(1430)\to K\pi}$ (data from Ref. [1]). Note that the observables entering the fit allow us to determine only linear combinations $m_0^2+\lambda_1(\phi_{\rm N}^2+\phi_{\rm S}^2)$ and $m_1^2+h_1\left(\phi_{\rm N}^2+\phi_{\rm S}^2\right)/2$ rather than parameters m_0, m_1, λ_1 and h_1 by themselves. It is nonetheless possible to calculate axial-vector masses and decay widths (explicit formulas in Refs. [3, 5]) — see Sec. 3.

3. Fit results

Results for the observables from our best fit are presented in Table I. The fit yields $\chi^2 = 12.33$, *i.e.* $\chi^2/(21 \text{ observables } -11 \text{ parameters}) = 1.23$. Table I demonstrates a remarkable correspondence of our results with experimental data in all meson channels. In particular, our fit yields $m_{a_1} = (1186\pm 6)$ MeV, $\Gamma_{a_1\to\rho\pi} = (549\pm 43)$ MeV and $\Gamma_{a_1\to\pi\gamma} = (0.66\pm 0.01)$ MeV. Note that $\Gamma_{a_1(1260)\to\pi\gamma} = (0.64\pm 0.25)$ MeV [1] is the only experimental information regarding the $a_1(1260)$ meson which is not an estimate; our result for $\Gamma_{a_1\to\pi\gamma}$ is within the experimental interval and, being constrained by abundant experimental input, it even produces an error for $\Gamma_{a_1(1260)\to\pi\gamma}$ that is smaller than the one quoted by PDG. Thus results in the non-strange axial-vector channel justify the interpretation of $a_1(1260)$ as (predominantly) a $\bar{q}q$ state.

Our fit allows us to calculate mass and decay width of the strange axial-vector state K_1 as a prediction. We obtain $m_{K_1} = 1282$ MeV, $\Gamma_{K_1\to K^\star\pi} = 205$ MeV, $\Gamma_{K_1\to\rho K} = 44$ MeV and $\Gamma_{K_1\to\omega K} = 15$ MeV. The mass is very close to the PDG value $m_{K_1(1270)} = (1272\pm 7)$ MeV [1]. However, the full decay width is 264 MeV, while the PDG data read $\Gamma_{K_1(1270)} = (90\pm 20)$ MeV and $\Gamma_{K_1(1400)} = (174\pm 13)$ MeV. Our result is, therefore, approximately three times too large when compared to the data for $K_1(1270)$ and approximately 50% too large when compared to the data for $K_1(1400)$, even with errors omitted from the calculation. These results demonstrate the necessity to include a pseudovector $I(J^{\rm PC}) = 1(1^{+-})$ nonet into our model and implement its mixing with the already present axial-vector nonet.

TABLE I

Best-fit results for masses and decay widths compared with experiment. Central values of observables are from Ref. [1]. Errors of observables are considered according to the criterion maximum (5%, experimental error). The reason is that already isospin-breaking effects in the physical hadron mass spectrum are of the order of 5% [for instance the difference between the charged and neutral pion masses or the masses of $a_1(1260)$ and $f_1(1285)$] but are completely neglected in our model. We have, therefore, artificially increased the experimental errors to 5% if the actual error is smaller or used the experimental values if the error is larger than 5%.

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3±0.7	92.2±4.6
f_K	106.9±0.6	110.4±5.5
m_π	141.0±5.8	137.3±6.9
m_K	485.6±3.0	495.6±24.8
m_η	509.4±3.0	547.9±27.4
$m_{\eta'}$	962.5±5.6	957.8±47.9
m_ρ	783.1±7.0	775.5±38.8
$m_{K^\star}$	885.1±6.3	893.8±44.7
m_ϕ	975.1±6.4	1019.5±51.0
m_{a_1}	1186±6	1230±62
$m_{f_1(1420)}$	1372.5±5.3	1426.4±71.3
m_{a_0}	1363±1	1474±74
$m_{K_0^\star}$	1450±1	1425±71
$\Gamma_{\rho\to\pi\pi}$	160.9±4.4	149.1±7.4
$\Gamma_{K^\star\to K\pi}$	44.6±1.9	46.2±2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34±0.14	3.54±0.18
$\Gamma_{a_1\to\rho\pi}$	549±43	425±175
$\Gamma_{a_1\to\pi\gamma}$	0.66±0.01	0.64±0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6±39.9	43.9±2.2
Γ_{a_0}	266±12	265±13
$\Gamma_{K_0^\star\to K\pi}$	285±12	270±80

4. Conclusions

We have presented an extended Linear Sigma Model containing (axial-) vector mesons (eLSM). We have performed a global fit of masses and decay widths from which we have drawn two conclusions: *(i)* the non-strange axial-vector meson $a_1(1260)$ can be accommodated as a $\bar{q}q$ state within our model and *(ii)* a correct description of the strange axial-vector states $K_1(1270)$ and

$K_1(1400)$ requires the implementation of mixing between an axial-vector and a pseudovector nonet. The latter also represents an outlook for further investigation of axial-vector mesons; the model can, however, also be applied to further study other mesons in vacuum and at finite temperatures and densities (see Refs. [3, 5] for details).

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