OBSERVABLES IN THE 3 FLAVOR PNJL MODEL AND THEIR RELATION TO EIGHT QUARK INTERACTIONS* **

B. Hiller, J. Moreira, A.A. Osipov, A.H. Blin

Departamento de Física, CFC
Faculdade de Ciências e Tecnologia da Universidade de Coimbra
3004-516 Coimbra, Portugal

(Received September 4, 2012)

Several relevant thermodynamic observables obtained within the (2+1) flavor and spin zero NJL and PNJL models with inclusion of the 't Hooft determinant and $8q$ interactions are compared with lattice-QCD (lQCD) results. In the case that a small ratio $R = \frac{\mu_B}{T_c} \sim 3$ at the critical end point (CEP) associated with the hadron gas to quark-gluon plasma transition is considered, combined with fits to the lQCD data of the trace anomaly, subtracted light quark condensate and continuum extrapolated data of the light quark chiral condensate, a reasonable description for the PNJL model is obtained with a strength $g_1 \sim 5 \ldots 6 \times 10^3$ GeV^{-8} of the $8q$ interactions. The dependence on the further model parameters is discussed.

DOI:10.5506/APhysPolBSupp.5.1171
PACS numbers: 11.10.Wx, 11.30.Rd, 11.30.Qc

In recent years, the role of effective chiral Lagrangians has grown as an important indicator of the order and universality class of phase transitions, as well as of the nature and location of the related CEP that may occur for the ground state of QCD in presence of external parameters, such as finite temperature T, baryonic chemical potential μ_B, magnetic field B [1]. In parallel, lQCD advances at zero and moderate chemical potential with masses approaching the physical values of the light quarks [2] and pion

* Presented by B. Hiller at the Workshop "Excited QCD 2012", Peniche, Portugal, May 6–12, 2012.

** Work supported by FCT, CERN/FP/116334/2010, QREN, UE/FEDER through COMPETE. Part of the EU Research Infrastructure Integrating Activity Study of Strongly Interacting Matter (HadronPhysics3) under the 7th Framework Programme of EU: Grant Agreement No. 283286.

mass [3], strongly indicate at a crossover transition from the hadronic to the quark-gluon phase at finite T and $\mu_B = 0$. Combining lQCD and chemical freeze-out data from relativistic heavy-ion collision facilities, the location of the CEP is presently conjectured to eventually occur at $R = \frac{\mu_B}{T_c} \sim 2$ and $\frac{T}{T_c} \sim 1$, [4, 5].

We consider the SU(3) flavor and spin-0 Nambu–Jona-Lasinio model (NJL) [6] with inclusion of the $U(1)_A$ breaking 't Hooft flavor determinant [7–9] and eight quark ($8q$) interactions [10, 11] (of which there exist two types, one of them violating the OZI rule, with strength g_1), and extend it to include the Polyakov loop (PNJL) [12–21]. The $8q$ have been firstly introduced to stabilize the effective potential of the model [10]. Their role turned out to be of significant importance in the behavior of model observables in presence of external parameters [17–19, 22–25]. Of particular interest is that the $8q$ coupling strengths g_1 can be varied in tune with the $4q$ interaction strength G without changing the vacuum condensates and low energy meson spectra, except for the σ-meson mass m_σ which decreases with increasing g_1. Fits to the low lying pseudoscalar and scalar meson spectra yield $m_\sigma \sim 560$ MeV for $g_1 = 6000$ GeV^{-8} and $m_\sigma \sim 690$ MeV for $g_1 = 1500$ GeV^{-8} [11]. In the μ, T plane (where $\mu = \frac{\mu_B}{3}$), the g_1, G interplay gives rise to a line of CEP, starting from the regime of large ratios $R \sim 20$ (NJL) and $R \sim 10$ (PNJL) in the case of weak $8q$ coupling g_1, to small ratios for strong g_1. In the first case, the chiral condensate is related with spontaneous symmetry breaking (SSB) driven by $4q$ interactions, in the second scenario SSB is induced by the $6q$ 't Hooft strength [11, 23, 25]. This continuous set of CEP is particular to the $8q$ extension of the model. However, a correlation between m_σ and the location of the CEP is also observed in the (2+1)-flavor quark-meson Lagrangian, where besides the 't Hooft term, a quartic mesonic contribution is present [26, 27], thus bearing a resemblance to the semi-bosonized version of the $8q$ NJL Lagrangian [11]. In order to restrict the g_1 values, one may: *(i)* calculate decays and scattering in the vacuum which are expected to narrow the choice, and *(ii)* compare with available lQCD data at finite T and moderate μ. In the present study, we try to explore the second option. For the PNJL case an extra uncertainty arises due to the parameters related with the choice of Polyakov potential $\mathcal{U}_\mathcal{P}$. In particular, the T_0 parameter of [14, 15] has a sizeable effect on the transition temperature.

First, we show in Fig. 1 the CEP lines in a (μ, T) *versus* g_1 diagram. The PNJL model (solid lines) enhances the effect of pushing R to small values as functions of g_1 in comparison with the NJL case (dashed lines). The crossing of the CEP(T) and CEP(μ) lines, (yielding $R = 3$), is reached for the PNJL at $g_1 \sim 6.4 \times 10^3$ GeV^{-8}, for the choice $T_0 = 190$ MeV, whereas it occurs for the NJL only at a much larger value, $g_1 \sim 8.4 \times 10^3$ GeV^{-8} (we remind that

with increasing g_1 the crossover becomes sharper and eventually gives rise to a first order transition at $\mu = 0$, which happens at $g_1 \sim 9 \times 10^3$ GeV^{-8} in the NJL case). Changing T_0, the CEP(T) is shifted up (down) with increasing (decreasing) T_0 (see caption of Fig. 1), while CEP(μ) remains sensibly unaltered.

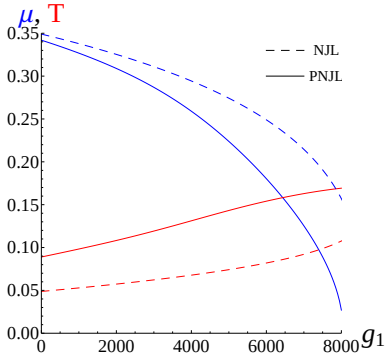

Fig. 1. Pairs (T, μ) of CEP as function of the $8q$ interaction strength g_1. Positive slope lines (red online) show T-dependence, negative slope lines (blue online) show μ dependence. All model parameters fixed as in [25], except for g_1, G. PNJL potential from [14]. Intersection ($R = 3$) of PNJL curves (solid lines) occur at ($\mu = T = 158; 167; 188$ MeV) with $g_1 = 6436; 6251; 6127$ GeV^{-8} for $T_0 = 190; 210; 270$ respectively (shown only for $T_0 = 190$ MeV). Intersection of NJL curves (dashed lines) at $\mu = T = 117$ with $g_1 = 8372$ GeV^{-8}.

In Fig. 2, the chiral condensates and dressed Polyakov loop for u, s quarks are shown for the NJL as function of T for $\mu = 0$ and with varying strength g_1 (see caption). For $g_1 10^{-3} = 1; 5; 6.5; 8$ GeV^{-8} the transition temperatures T_t defined at the corresponding inflection points of the curves are $T_\mathrm{t} = 192; 163; 147; 135$ MeV for the u-condensate, $T_\mathrm{t} = 197; 163; 150; 135$ MeV for the u-quark dressed Polyakov loop, $T_\mathrm{t} = 197; 160; 147; 135$ MeV at the first inflection point of s-quark condensate, $T_\mathrm{t} = 270; 240; 235; 225$ MeV at its 2nd inflection point, $T_\mathrm{t} = -; 166; 150; 135$ MeV at 1st inflection point of the dressed s-quark Polyakov loop and $T_\mathrm{t} = 270; 240; 235; 225$ MeV at its 2nd inflection point. The 1st set of inflection points in the case of the s-quark condensate and dressed Polyakov loop occur due to the gap equations that correlate the u and s variables, yielding similar T_t for the u and s observables. The 2nd inflection point occurs at temperatures T_t larger by ~ 80 MeV.

A similar pattern is observed for the PNJL model in Fig. 3, the second inflection points occur at roughly 55 MeV higher T_t values. Visually, these 2nd inflection points can barely be detected, the transition is very slow and smooth. This behavior can be traced back to the fact that for large T the

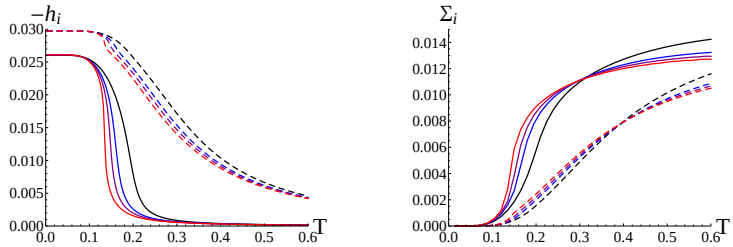

Fig. 2. The chiral condensates $= h_i/2$, $i = u, s$ and the dressed Polyakov loop Σ^i as functions of T for NJL; solid lines for light quarks, dashed for the strange quark. Up to down curves in left panel: $g_1 \times 10^{-3} = 1; 5; 6.5; 8$ GeV^{-8}, corresponding to black, blue, violet, red (color online). Same ordering in right panel, after crossing point.

s-quark constituent quark mass approaches asymptotically its current quark mass value, which is much larger than for the u-quark[1]. It is a disputable matter which temperature should be taken to characterize the transition for the s-quark in these observables. A calculation of the chiral and quark number susceptibilities associated with the s-quark in the NJL model display only one peak characterizing the transition temperature.

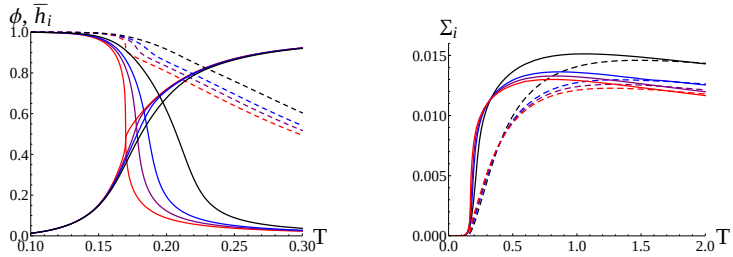

Fig. 3. The same as in Fig. 1 for the PNJL model. In left panel ϕ (curves growing with T) stand for the Polyakov loop and corresponding g_1 strengths revert order compared to the chiral condensate.

In Fig. 4, one sees however that in the PNJL case two peaks can occur again for the s-quark chiral susceptibility. Fig. 5 (left) shows the trace anomaly calculated for $g_1 = 6000$ GeV^{-8}, for various values of the parameter T_0 in comparison with lattice data. In Fig. 5 (right), the subtracted condensate Δ_{ls} is shown for several values for g_1, calculated with $T_0 = .19$ GeV, and compared to lQCD. In Fig. 6, the light chiral condensate is compared with lQCD data extrapolated to the continuum limit [2] for different values of g_1 and with $\mathcal{U}^1$ for the cases $T_0 = .15$ GeV (left) and $T_0 = .19$ GeV (right).

[1] We calculate the thermodynamic potential with the prescription of [21, 25], where we show that it leads to the correct large T asymptotic behavior for the quark masses (condensates), traced Polyakov loop and number of degrees of freedom.

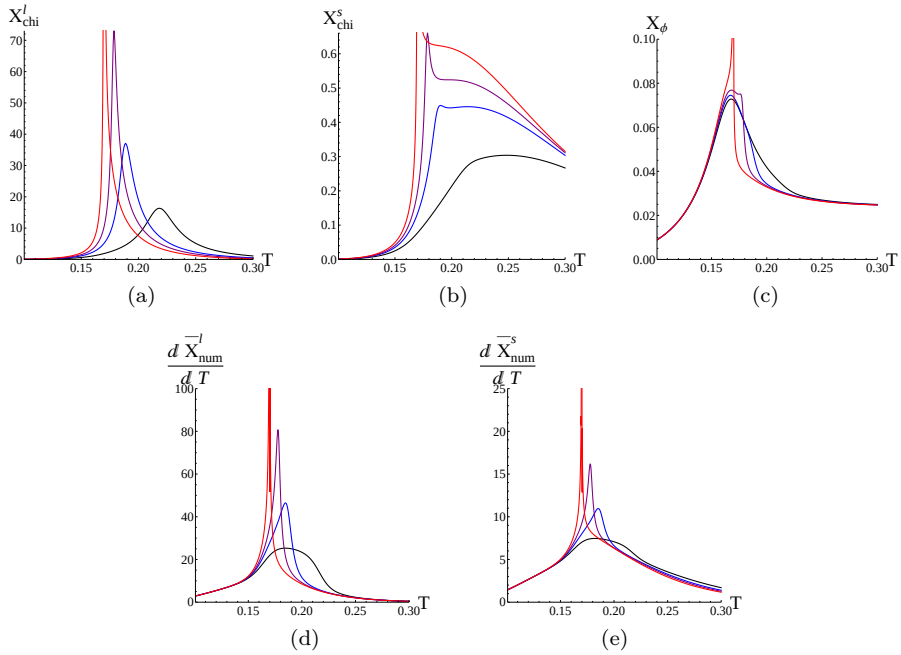

Fig. 4. Susceptibilities for PNJL, $\mathcal{U}_\mathcal{P}$ of [14] and $T_0 = 190$ MeV. (a) light quark chiral susceptibilites, (b) s quark chiral susceptibilities, (c) Polyakov loop susceptibility, (d) quark number (for u quark) and (e) quark number (for s-quark) susceptibilities. The peaks get more pronounced with increasing g_1. Color code as in Fig. 2.

From these comparisons we conclude *(i)* that the smaller the ratio $R = \frac{\mu_B}{T_c}$ related with the CEP location, the larger the $8q$ interaction strength g_1 must be chosen; a sizeable dependence on the T_0 parameter of the Polyakov potentials can induce shifts of the order of several tens of MeV in T_c (Fig. 1). For $R = 3$, we get g_1 of the order of 6000 GeV^{-8} and $T_c = 158$–188 MeV for the range $T_0 = 190$–270 MeV. *(ii)* Besides the $8q$ strength, the Polyakov loop plays also a substantial role in decreasing the ratio R. *(iii)* The observables calculated at $\mu = 0$ related with the light quarks, chiral condensates, traced Polyakov loop and dressed Polyakov loop (Fig. 3), chiral and quark number susceptibilities (Fig. 4 (a), (d)), as well as the s-quark number susceptibility (Fig. 4 (e)) and Polyakov loop susceptibility (Fig. 4 (c)) yield a crossover temperature $T_t \sim 179$ MeV for $g_1 = 6000$ GeV^{-8} and $T_0 = .19$ GeV. *(iv)* Some of the s-quark observables show two possible transition temperatures, Fig. 2, 3, 4 (b), the first close to the u-quark transition, the second about 50 MeV higher for the PNJL model. *(v)* The best fit to the trace anomaly is for $g_1 = 6000$ GeV^{-8} at $T_0 = .21$ GeV (Fig. 5 (a)) and for the

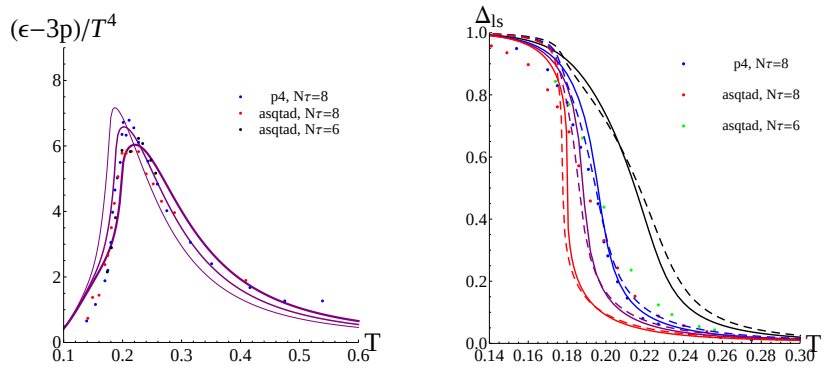

Fig. 5. Left: trace anomaly with $8q$ strength $g_1 = 6000$ GeV^{-8}, with $\mathcal{U}_\mathcal{P}$ from [14] and respective parameter $T_0 = .19, .21, .23$ GeV (which yield peak positions from left to right). The lQCD data is taken from [28]. Right: the lQCD data for Δ_{ls}, the subtracted chiral condensate value normalized to its zero T value, as defined in [28], compared to PNJL calculations for several g_1 strengths, color code as in Fig. 2. Solid lines: with $\mathcal{U}_\mathcal{P}$ from [14], dashed lines: with $\mathcal{U}_\mathcal{P}$ from [15], both at $T_0 = .19$ GeV.

observable Δ_{ls} we obtain a reasonable fit with $g_1 = 5 \ldots 6 \times 10^3$ GeV^{-8} and $T_0 = .19$ GeV (Fig. 5 (b)). *(vi)* The peak positions and heights of the continuum extrapolated light quark chiral susceptibility vary considerably (Fig. 6). This big spread allows to accommodate a large range of g_1 values, whose peak positions in turn depend also on the choice of the T_0 parameter. The value $g_1 \sim 5 \times 10^3$ GeV^{-8} is eventually the best choice if one takes the height of the peak also into consideration.

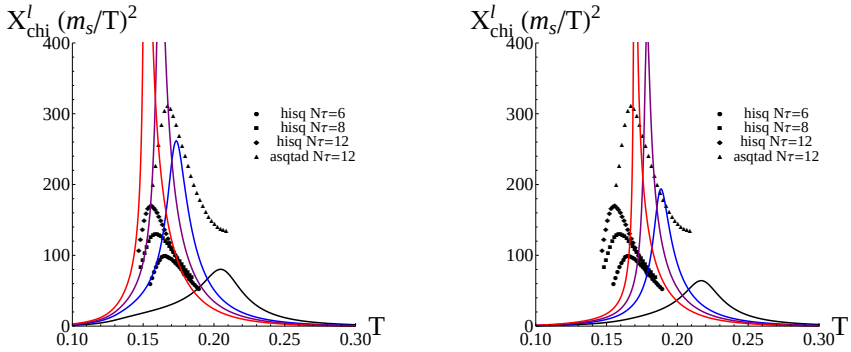

Fig. 6. The lQCD data for the light quark chiral susceptibility X^l_{chi} in the continuum limit taken from [2], in comparison with the PNJL model with $\mathcal{U}_\mathcal{P}$ [14] at $T_0 = .15$ GeV (left panel) and $T_0 = .19$ GeV (right panel) for different g_1 strengths (solid lines, narrower peaks correspond to increasing g_1).

REFERENCES

[1] J.M. Pawlowski, *AIP Conf. Proc.* **1343**, 75 (2011).

[2] A. Bazavov *et al.* [HotQCD Collaboration], *Phys. Rev.* **D85**, 054503 (2012).

[3] S. Borsanyi *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **5**, 1125 (2012), this issue.

[4] M. Stephanov, *Acta Phys. Pol. B* **35**, 2939 (2004).

[5] S. Gupta *et al.*, *Science* **332**, 1525 (2011).

[6] Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); **124**, 246 (1961).

[7] G. 't Hooft, *Phys. Rev.* **D14**, 3432 (1976); **D18**, 2199 (1978).

[8] V. Bernard, R.L. Jaffe, U.-G. Meissner, *Phys. Lett.* **B198**, 92 (1987).

[9] H. Reinhardt, R. Alkofer, *Phys. Lett.* **B207**, 482 (1988).

[10] A.A. Osipov, B. Hiller, J. da Providencia, *Phys. Lett.* **B634**, 48 (2006).

[11] A.A. Osipov, B. Hiller, A.H. Blin, J. da Providencia, *Ann. Phys.* **322**, 2021 (2007).

[12] K. Fukushima, *Phys. Lett.* **B591**, 277 (2004).

[13] E. Megías, E. Ruiz Arriola, L.L. Salcedo, *Phys. Rev.* **D74**, 065005 (2006).

[14] C. Ratti, M.A. Thaler, W. Weise, *Phys. Rev.* **D73**, 014019 (2006).

[15] S. Roessner, C. Ratti, W. Weise, *Phys. Rev.* **D75**, 034007 (2007).

[16] K. Fukushima, *Phys. Rev.* **D77**, 114028 (2008).

[17] Y. Sakai, K. Kashiwa, H. Kouno, M. Yahiro, *Phys. Rev.* **D77**, 051901 (2008); T. Sasaki, Y. Sakai, H. Kouno, M. Yahiro, *Phys. Rev.* **D82**, 116004 (2010).

[18] A. Bhattacharyya, P. Deb, S.K. Ghosh, R. Ray, *Phys. Rev.* **D82**, 014021 (2010); A. Bhattacharyya, P. Deb, A. Lahiri, R. Ray, *Phys. Rev.* **D83**, 014011 (2011).

[19] R. Gatto, M. Ruggieri, *Phys. Rev.* **D83**, 034016 (2011); **D82**, 054027 (2010).

[20] P. Costa *et al.*, *Phys. Rev.* **D79**, 116003 (2009).

[21] J. Moreira, B. Hiller, A.A. Osipov, A.H. Blin, *Int. J. Mod. Phys.* **A27**, 1250060 (2012).

[22] A.A. Osipov *et al.*, *Phys. Lett.* **B646**, 91 (2007).

[23] A.A. Osipov, B. Hiller, J. Moreira, A.H. Blin, *Phys. Lett.* **B659**, 270 (2008).

[24] A.A. Osipov, B. Hiller, A.H. Blin, J. da Providencia, *Phys. Lett.* **B650**, 262 (2007).

[25] B. Hiller, J. Moreira, A.A. Osipov, A.H. Blin, *Phys. Rev.* **D81**, 116005 (2010).

[26] B.-J. Schaefer, M. Wagner, J. Wambach, *Phys. Rev.* **D81**, 074013 (2010).

[27] S. Chatterjee, talk at this conference.

[28] A. Bazavov *et al.*, *Phys. Rev.* **D80**, 014504 (2009).