QUARK CONFINEMENT IN THE HEAVY QUARK LIMIT*

Carina Popovici

Institut für Theoretische Physik, Universität Giessen
Heinrich-Buff-Ring 16, 35392 Giessen, Germany

(Received September 13, 2012)

In this presentation, we investigate the heavy quark sector of Coulomb gauge QCD using a leading-order heavy quark mass expansion of the QCD action adapted from M. Neubert, *Phys. Rep.* **245**, 259 (1994). In the limit where the Yang–Mills sector is truncated to only include dressed two-point functions, we show that the rainbow-ladder approximation to the gap and Bethe–Salpeter equations is exact, and provide a direct connection between the physical string tension and the temporal gluon propagator. Furthermore, we derive an exact solution for the four-point quark Green's function and show that a natural separation between the physical and unphysical poles arises.

DOI:10.5506/APhysPolBSupp.5.1179
PACS numbers: 11.10.St, 12.38.Aw

1. Introduction

The Gribov–Zwanziger scenario is explicitly realized in Coulomb gauge, as the confining force is carried by the temporal part of the gluon propagator while its spatial component is suppressed in the infrared [1]. In this context, it is pertinent to investigate the connection between the physical (nonperturbative) string tension of a quark–antiquark pair and the underlying Yang–Mills interaction.

For the results presented in this paper, we use an expansion in the heavy quark mass and a truncation excluding pure Yang–Mills vertices (while the propagators are kept dressed non-perturbatively). In the work described here [2, 3], we retain only the first order term in the mass expansion. We consider the homogeneous Bethe–Salpeter equation for quark–antiquark systems and show that with our assumptions the rainbow-ladder approximation is exact. In the second part of this presentation, we consider the four-point

* Presented at the Workshop "Excited QCD 2012", Peniche, Portugal, May 6–12, 2012.

quark–antiquark Green's functions. We present exact analytical solutions, which exhibit an explicit separation between physical and unphysical poles. Furthermore, we show that the physical poles of the Bethe–Salpeter equation are also singularities of the Green's function. The results presented here could be applied for further studies of phenomenological meson and baryon models (as in the numerical analysis of [4]).

A complete description is beyond the scope of this proceeding, for a more detailed and rigorous description we refer to Ref. [2, 3].

2. Quark propagator in the heavy mass limit

To work in the large mass limit, we first decompose the quark field according to the heavy quark transformation

$$\begin{aligned} q_\alpha(x) &= e^{-imx_0}\left[h(x)+H(x)\right]_\alpha , \\ h_\alpha(x) &= e^{imx_0}\left[\frac{\mathbf{1}+\gamma^0}{2}q(x)\right]_\alpha , \qquad H_\alpha(x) = e^{imx_0}\left[\frac{\mathbf{1}-\gamma^0}{2}q(x)\right]_\alpha , \end{aligned} \tag{1}$$

where the h field represents "light degrees of freedom", while the H field represents "heavy degrees of freedom" which are integrated out. At leading order in the $1/m$ expansion, the quark contribution to the QCD action reduces to

$$\mathcal{S}_q = \int d^4x\, \bar{h}\left(\imath\partial_0 + gT^aA_0^a\right)h + \mathcal{O}(1/m)\,. \tag{2}$$

One notes that due to the form of the projectors in the decomposition (1), there are no Dirac gamma matrices in the above expression, reflecting the fact that in the infinite mass limit the spin degree of freedom decouple. Further, the quark field only couples to the temporal part of the gluon, and since there is only a time derivative at this order in the mass expansion, there is no dependence on the quark three-momentum $\vec{k}$.

The Dyson–Schwinger equation for the heavy quark propagator reads

$$\begin{aligned} \left[W_{\bar{q}q}(k)\right]^{-1} &= \left[W_{\bar{q}q}^{(0)}(k)\right]^{-1} \\ &\quad + gT^a\int đ\omega\, W_{\bar{q}q}(\omega)\Gamma^b_{\bar{q}qA_0}(\omega,-k,k-\omega)W^{ab}_{A_0}(k-\omega)\,. \end{aligned} \tag{3}$$

We now insert the leading order expressions for the ingredients in this equation. The bare heavy quark propagator can be derived directly from the action Eq. (2) and is given by

$$W_{\bar{q}q}^{(0)}(k_0) = \frac{-i}{k_0 - m + \imath\epsilon} + \mathcal{O}(1/m)\,. \tag{4}$$

Further, we use the fact that the gluon dressing is largely independent of the energy [5], *i.e.* $W^{ab}_{A_0}(k) = \delta^{ab} W_{A_0}(\vec{k})$ (this assumption is supported by lattice findings [6]). The functional dependence on the momentum (which we will only use at then end of this presentation) is suggested by lattice results displaying a $1/\vec{k}^2$ divergence in the infrared. Within our approximation, the quark-gluon vertex can be rewritten with the help the Slavnov–Taylor identity in terms of two point functions as follows

$$\Gamma^b_{\bar{q}qA_0}(k_1,k_2,k_3) = \frac{ig}{k^0_3} T^b \left\{ [W_{\bar{q}q}(k_1)]^{-1} - [W_{\bar{q}q}(-k_2)]^{-1} \right\} + \mathcal{O}(1/m) \,. \quad (5)$$

From the equations (3), (5) we obtain the exact (in the leading order in the mass expansion and using our truncation) solution for the quark propagator

$$[W_{\bar{q}q}(k)]^{-1} = i \left[k_0 - m - \frac{1}{2} g^2 C_F \int đ\,\vec{\omega}\, W_{A_0}(\vec{\omega}) \right] , \qquad C_F = \frac{N_c^2 - 1}{2N_c} \,. \quad (6)$$

Inserting this expression back in the Slavnov–Taylor equation yields for the quark-gluon vertex

$$\Gamma^d_{\bar{q}qA_0}(k_1,k_2,k_3) = gT^d + \mathcal{O}\left(1/m\right) , \quad (7)$$

which means that the temporal quark-gluon vertex remains bare nonperturbatively, and this implies that the gap equation reduces to the rainbow truncation.

3. Homogeneous Bethe–Salpeter equation

We now use the results of the previous section for the full homogeneous Bethe–Salpeter equation for quark–antiquark bound states (depicted in Fig. 1)

$$\Gamma(p) = - \int đ\,k\, K(p,k) W_{\bar{q}q}(k_+) \Gamma(k) W_{\bar{q}q}(k_-) \,. \quad (8)$$

Our truncation corresponds for the Bethe–Salpeter kernel K to the ladder approximation, $K(p,k) = -g^2 W_{A_0}(\vec{p} - \vec{k}) \left[T^a \Gamma(k) T^a\right]$ (see [2] for an explicit derivation). Writing

$$\left[T^a \Gamma(\vec{r}) T^a\right]_{\alpha\beta} = C_M \Gamma_{\alpha\beta}(\vec{r}) \,, \quad (9)$$

where C_M is a color factor assigned to the Bethe–Salpeter vertex Γ, yet to be identified, we find for the total energy of the $\bar{q}q$ pair

$$P_0 = g^2 \int đ\,\vec{\omega}\, W_{A_0}(\vec{\omega}) \left[C_F - e^{\imath \vec{\omega} \cdot \vec{r}} C_M \right] . \quad (10)$$

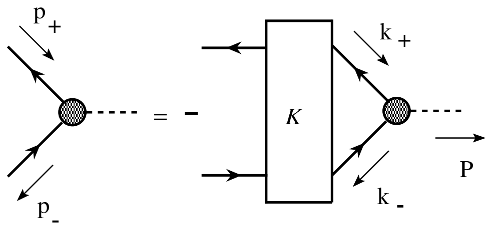

Fig. 1. Homogeneous Bethe–Salpeter equation for quark–antiquark bound states. Internal propagators are fully dressed, solid lines represent the quark propagator and filled blobs represent the Bethe–Salpeter vertex function Γ. The box represents the Bethe–Salpeter kernel K.

Assuming that the temporal gluon propagator is more infrared divergent than $1/|\vec{\omega}|$, we find that C_M has to be equal to C_F in order to ensure the convergence of the integral in Eq. (8). This implies that the Bethe–Salpeter equation can only have a finite solution for color singlet states, *i.e.*, $\Gamma_{\alpha\gamma}(\vec{x}) = \delta_{\alpha\gamma}\Gamma(\vec{x})$. Further, if we assume the infrared behavior suggested by the lattice, $W_{A_0}(\vec{\omega}) = X/|\vec{\omega}|^4$, where X is some combination of constants, we obtain

$$P_0 = \sigma|\vec{r}| \tag{11}$$

with $\sigma = g^2 C_F X/(8\pi)$, in accordance with the results from [7]. This result shows that there is a direct connection between the physical string tension σ and the nonperturbative temporal gluon propagator.

4. Four-point quark–antiquark Green's functions

In this section, we apply the results of the preceding sections to the calculation of the four point Green's function. The diagrammatic representation of its Dyson–Schwinger equation is given in Fig. 2.

Here, we will consider the flavor non-singlet case in the s-channel (that is, we consider the quark and the antiquark as two distinct flavors with the same mass) so that the terms (a), (c), (i) do not contribute. In our approximation, diagram (b) vanishes upon performing the energy integral, and the two-gluon two-quark vertex of term (d) can be shown to vanish using its Slavnov–Taylor identity [3]. We then proceed by neglecting the terms (e), (f) and (g), which contain the four-point Green's function, and solving the truncated equation (diagrammatically depicted Fig. 3). One can then show that the diagrams (e), (f) and (g) cancel and hence our assumption is consistent (see [3] for a complete derivation). A formal simplification can be

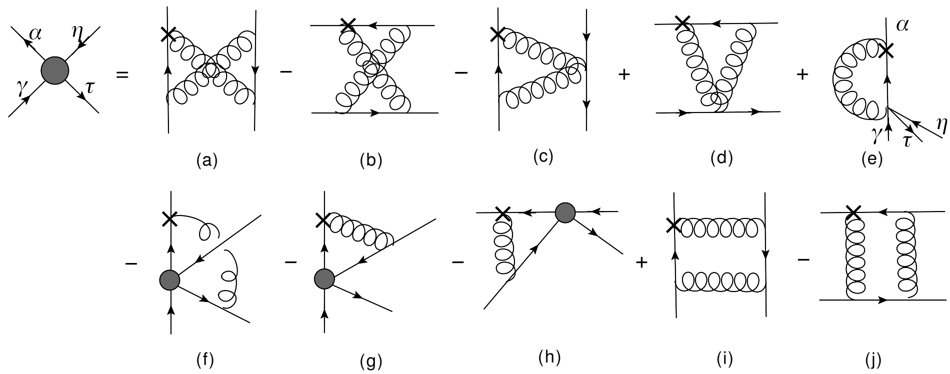

Fig. 2. Diagrammatic representation of the Dyson–Schwinger equation for the 1PI 4-point quark–antiquark Green's function. Blobs represent dressed 1PI 4-point vertex, solid lines represent the quark propagator, springs denote gluon propagator and cross denotes the tree level quark-gluon vertex.

achieved by deriving the amputated 4-point Green's function, *i.e.* cutting the quark legs. The final result for this function reads

$$\begin{aligned} G^{(4)}_{\alpha\gamma;\tau\eta}(x) \;=\; & \frac{g^2}{2}\frac{g_1(x)}{P_0 - g^2\int d\!\!{}^-\vec{\omega}\, W_{A_0,\vec{\omega}}\left[C_F + \frac{e^{i\vec{\omega}\cdot\vec{x}}}{2N}\right] + i\epsilon} \\ & \times\left[\delta_{\alpha\gamma}\delta_{\tau\eta}\frac{g_2(x)}{P_0 - g^2 C_F\int d\!\!{}^-\vec{\omega}\, W_{A_0,\vec{\omega}}\left[1 - e^{i\vec{\omega}\cdot\vec{x}}\right] + i\epsilon} - \delta_{\alpha\eta}\delta_{\tau\gamma}\frac{1}{N}\right], \end{aligned} \tag{12}$$

where $g_1(x)$ and $g_2(x)$ are functions of the separation $x = |\vec{x}|$ associated with the momentum $\vec{p}_1 + \vec{p}_4$ (notice the momentum rooting in Fig. 3). Again, assuming an energy dependence $W_{A_0}(\vec{\omega}) \simeq 1/\vec{\omega}^4$, we recognize in the second fraction our expression Eq. (10) for the total energy of the quark pair. Thus, we have provided an explicit analytical dependence of the four-point Green's function on the $\bar{q}q$ bound state energy, which in turn results from

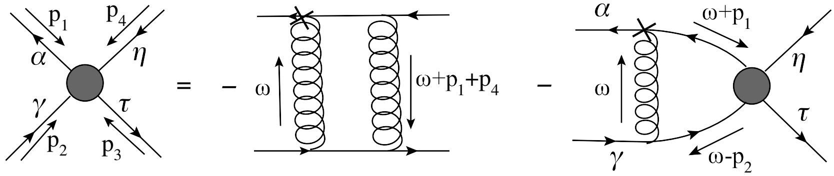

Fig. 3. Truncated Dyson–Schwinger equation for the 1PI 4-point Green's function in the s-channel. Same conventions as in Fig. 2 apply.

the homogeneous Bethe–Salpeter equation. The first fraction displays an unphysical pole, which does not appear in the homogeneous Bethe–Salpeter equation and can be interpreted as a part of the normalization.

5. Conclusions

In this presentation, we have used a leading-order heavy quark mass expansion and a truncation including only the (non-perturbative) temporal gluon propagator to discuss the Dyson–Schwinger and Bethe–Salpeter equations for quark–antiquark systems in Coulomb gauge.

We find that with these approximations, the rainbow approximation to the quark gap equation and the corresponding ladder approximation to the homogeneous Bethe–Salpeter equation are exact. We show that confining (finite energy) solutions exist only for color singlet meson states, and otherwise the system has infinite energy. Furthermore, we provide exact analytic solutions for the four-point quark–antiquark Green's function, and find that the physical and nonphysical poles disentangle, the former coinciding with the bound state solutions of the homogeneous Bethe–Salpeter equation.

The author thanks the Organizers, in particular Francesco Giacosa, for the kind invitation. Support by the Helmholtz International Center for FAIR within the LOEWE program of the State of Hesse, and the Helmholtz Young Investigator Group No. VH-NG-332 is acknowledged.

REFERENCES

[1] V.N. Gribov, *Nucl. Phys.* **B139**, 1 (1978); D. Zwanziger, *Nucl. Phys.* **B485**, 185 (1997); **B518**, 237 (1998).

[2] C. Popovici, P. Watson, H. Reinhardt, *Phys. Rev.* **D81**, 105011 (2010).

[3] C. Popovici, P. Watson, H. Reinhardt, *Phys. Rev.* **D83**, 125018 (2011).

[4] M. Blank, A. Krassnigg, *Comput. Phys. Commun.* **182**, 1391 (2011).

[5] P. Watson, H. Reinhardt, *Phys. Rev.* **D77**, 025030 (2008).

[6] M. Quandt, G. Burgio, S. Chimchinda, H. Reinhardt, *PoS* **CONFINEMENT8**, 066 (2008).

[7] S.L. Adler, A.C. Davis, *Nucl. Phys.* **B244**, 469 (1984).