NONEQUILIBRIUM EFFECTS IN DYNAMIC SYMMETRY BREAKING*

Marlene Nahrgang

SUBATECH, UMR 6457, Université de Nantes
Ecole des Mines de Nantes, IN2P3/CRNS
4 rue Alfred Kastler, 44307 Nantes cedex 3, France

Stefan Leupold

Department of Physics and Astronomy, Uppsala University
Box 516, 75120 Uppsala, Sweden

Marcus Bleicher

Frankfurt Institute for Advanced Studies (FIAS)
Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany

(Received September 20, 2012)

We study the evolution of the sigma field fluctuations in a scenario featuring a critical point and a first order phase transition using the model of nonequilibrium chiral fluid dynamics (NχFD).

DOI:10.5506/APhysPolBSupp.5.1191
PACS numbers: 25.75.q, 47.75.+f, 11.30.Qc, 24.60.Ky

1. Introduction

The QCD phase diagram is expected to exhibit a rich phase structure. At larger baryochemical potential, as achieved at the upcoming FAIR project [1] at GSI Darmstadt, a first order phase transition is expected from model studies [2–4]. Interesting observables could here be based on the growth of fluctuations due to the nonequilibrium effect of supercooling leading to nucleation and spinodal decomposition [5–8]. At zero baryochemical potential, the nature of the phase transition of QCD is well understood from lattice QCD calculations which show that it is an analytic crossover [9]. As a consequence, there must be a critical point which terminates the line of first order phase transitions.

* Presented at the Workshop "Excited QCD 2012", Peniche, Portugal, May 6–12, 2012.

The analytic investigations in this nonperturbative region of the phase transition though evolving cannot, at the present status, make deftnite statements on neither the existence nor the location of a critical point [10]. Additionally, the phase diagram of QCD can be studied experimentally in ultrarelativistic heavy-ion collisions. In this paper, we will concentrate on describing effects of the phase transitions in dynamic models of heavy-ion collisions. In thermodynamic systems, fluctuations and correlations of the order parameter diverge at the critical point. Coupling of the order parameter to measurable particles, like pions and protons, leads to an enhancement of event-by-event fluctuations in the net-charge or net-baryon number multiplicities [11, 12]. Scanning the phase diagram in heavy-ion collisions by varying the beam energy, one should see this enhancement in a nonmonotonic behavior. The key ingredient is the correlation length which becomes infinite in thermodynamic systems at a critical point. In a realistic evolution of a heavy-ion collision, however, the growth of the correlation length is limited by the size of the system and by the finite time, which the dynamic systems spends at a critical point. Relaxation times also become infinite at the critical point, a phenomenon called critical slowing down. Even if the system is in equilibrium above the critical point it is necessarily driven out of equilibrium by passing through the critical point. Assuming a phenomenological time evolution of the correlation length with parameters from the 3d Ising universality class, it was found that the correlation length does not grow beyond 2–3 fm [13].

The explicit propagation of fluctuations coupled to a dynamic model is a necessary step towards understanding the QCD phase diagram from heavy-ion collision experiments. In chiral fluid dynamic models [14, 15], the propagation of the fields in the chiral sector is coupled to a fluid dynamic propagation of the constituent quarks. The expansion and cooling of the fluid does thus drive the underlying model through the phase transition. In the following, we present the model of NχFD [16] with a focus o the evolution of the constituent quark masses.

2. Nonequilibrium chiral fluid dynamics

Both, the dynamics of the sigma field and of the fluid, are derived from the quark-meson model [16]. The parameters are chosen such that chiral symmetry is spontaneously broken in the vacuum, where $\langle\sigma\rangle = f_\pi = 93$ MeV. Due to explicit symmetry breaking $\langle\sigma\rangle$ does not vanish exactly in the chirally restored phase, but has a small finite value. The effective potential is given for $\mu_B = 0$, where the correct vacuum value for the constituent quark mass is obtained by a coupling of $g = 3.3$ between the sigma field and the quark fields. For a qualitative study, we fix the baryochemical potential at $\mu_B = 0$ and tune the phase transition by changing the quark-meson coupling constant. A constituent quark mass for $m_{q,\text{vac}} \simeq 307$ MeV in the vacuum is obtained for $g = 3.3$, where the transition is a crossover. For $g = 3.63$ one finds a corresponding critical point at $T_\text{c} \simeq 140$ MeV by the vanishing curvature of the effective potential at the minimum and a first order phase transition for $g = 5.5$ and $T_\text{c} \simeq 123$ MeV by the appearance of two degenerate minima. For these couplings the phenomenologically known value of the constituent quark mass comes out wrong. We, however, focus on the qualitative behaviour and leave the extension of the model to finite μ_B to future work. We furthermore concentrate on the dynamics of the order parameter and set the pion fields to their expectation value $\langle\vec{\pi}\rangle = 0$.

Within this approach, the Langevin equation for the sigma mean-field reads

$$\partial_\mu \partial^\mu \sigma + \frac{\delta V_\text{eff}}{\delta\sigma} + \eta\partial_t\sigma = \xi\,, \tag{1}$$

where the effective potential to fermionic one-loop level is given by

$$V_\text{eff}(\sigma, T) = U(\sigma) - 2d_q T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right) \tag{2}$$

with the quark energy $E^2 = p^2 + m_q^2$ and the classical potential U

$$U(\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q\sigma - U_0\,. \tag{3}$$

The quark mass $m_q = g\langle\sigma\rangle$ is generated dynamically at the phase transition.

The damping term is [16–19]

$$\eta = \begin{cases} g^2 \frac{d_q}{\pi}\left(1 - 2n_\text{F}\left(\frac{m_\sigma}{2}\right)\right)\frac{1}{m_\sigma^2}\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{3/2} & \text{for } m_\sigma > 2m_q = 2g\sigma_\text{eq}\,, \\ 2.2/\text{fm} & \text{for } 2m_q > m_\sigma > 2m_\pi\,, \\ 0 & \text{for } m_\sigma < 2m_\pi, 2m_q\,. \end{cases} \tag{4}$$

The stochastic field in the Langevin equation (1) has a vanishing expectation value

$$\langle\xi(t)\rangle_\xi = 0\,, \tag{5}$$

and the noise correlation is given by the dissipation-fluctuation theorem

$$\left\langle\xi(t)\xi\left(t'\right)\right\rangle_\xi = \frac{1}{V}\delta\left(t - t'\right) m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)\,. \tag{6}$$

The local pressure and energy density of the quarks are obtained from the thermodynamic relations

$$p(\sigma, T) = -V_\text{eff}(\sigma, T) + U(\sigma)\,, \qquad e(\sigma, T) = T\frac{\partial p(\sigma, T)}{\partial T} - p(\sigma, T)\,. \tag{7}$$

A source term S^ν in the relativistic fluid dynamic equations

$$\partial_\mu T^{\mu\nu} = S^\nu \tag{8}$$

allows for the energy dissipation from the system to the heat bath and thus assures energy-momentum conservation of the coupled system.

3. Evolution of the sigma field in nonequilibrium

In order to solve equations (1) and (8) numerically, we need to specify the initial conditions for the fluid dynamic fields and the sigma field. Here, we use averaged initial conditions from the hybrid approach to UrQMD [20]. We use the initial density profiles in the following way: We average over many central ($b < 2.75$ fm) UrQMD events which gives rather smooth distributions. Due to the special relations of the equation of state in (7), we rescale these distributions to temperatures which are well above the phase transition in the quark-meson model at $\mu_B = 0$. This way we achieve that large parts of the system are initially in the hot temperature, *i.e.* the chirally restored, phase. In Figs. 1 and 2, we compare the equilibrium expectation values of the sigma field to those obtained in the nonequilibrium evolution. In Fig. 1, we see a large discontinuity at the first order phase transition due to the two degenerate minima, while the transition is smooth at the critical point.

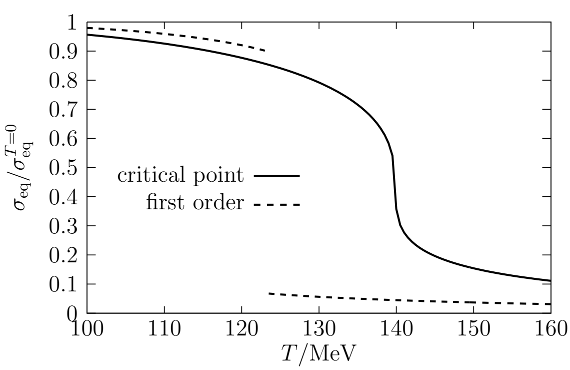

Fig. 1. The expectation value of the sigma field in thermodynamic equilibrium.

Due to the dynamic evolution of the locally equilibrated fluid and the nonequilibrium propagation of the fields, a similar correlation between temperature and the sigma field is not given directly in NχFD. For every time step, we extract the sigma field and the temperature averaged over the same central volume and confront these values in Fig. 2. Due to the interaction of the sigma field and the fluid, neither the evolution of the sigma field average

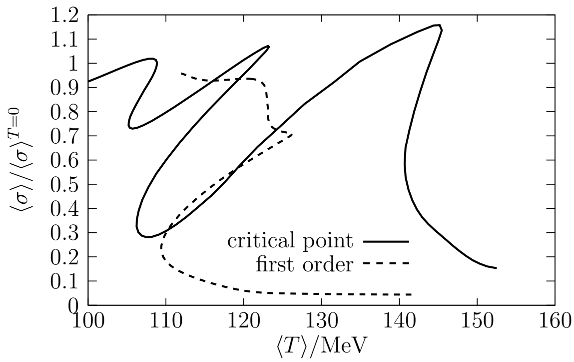

Fig. 2. The volume averaged sigma field confronted to the volume averaged temperature in a nonequilibrium dynamic system.

nor that of the temperature average is monotonic. Therefore, we observe a double-valued behavior in Fig. 2. The discontinuity of the first order phase transition is washed out and we can clearly observe a region of strong supercooling. Here the temperature is below the transition temperature but large parts are still in the chirally restored phase. Only after the systems starts to relax, which leads to reheating [21, 22], a steeper increase of $\langle \sigma \rangle$ at the transition temperature is seen. This is the remnant of the discontinuity in the thermodynamic system. In the vicinity of the critical point, both the sigma field and the temperature start to oscillate, which originates from the small sigma mass. Here the potential has a very small curvature and the damping vanishes locally, as given by equation (4). This facilitates the strong oscillations observed. The sigma field imprints its oscillations onto the temperature which amplifies the oscillations in Fig. 2.

4. Summary

We investigated the nonequilibrium effects on the evolution of the sigma field and the corresponding temperature of the system within NχFD applying initial conditions from UrQMD. The relaxation of the sigma field proceeds very differently in a critical point and a first order phase transition scenario. Plotting the volume averaged sigma field *versus* the averaged temperature in the same volume, we could observe the supercooling in the scenario with a first order phase transition and strong oscillations in the critical point scenario. It will be interesting to study the influence of this effect in coupling to the confinement–deconfinement phase transition. Work including the Polyakov-loop is in progress [23].

This work was supported by the Hessian Excellence Initiative LOEWE through the Helmholtz International Center for FAIR. We are grateful to the Center for Scientific Computing (CSC) at Frankfurt for providing the computing resources.

REFERENCES

[1] B. Friman *et al.* (eds.), *Lect. Notes Phys.* **814**, 1 (2011).
[2] O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, *Phys. Rev.* **C64**, 045202 (2001).
[3] C. Ratti, M.A. Thaler, W. Weise, *Phys. Rev.* **D73**, 014019 (2006).
[4] B.-J. Schaefer, J.M. Pawlowski, J. Wambach, *Phys. Rev.* **D76**, 074023 (2007).
[5] L.P. Csernai, J.I. Kapusta, *Phys. Rev.* **D46**, 1379 (1992).
[6] I.N. Mishustin, *Phys. Rev. Lett.* **82**, 4779 (1999).
[7] J. Randrup, *Phys. Rev.* **C82**, 034902 (2010).
[8] P. Chomaz, M. Colonna, J. Randrup, *Phys. Rep.* **389**, 263 (2004).
[9] Y. Aoki *et al.*, *Nature* **443**, 675 (2006).
[10] C.S. Fischer, J. Luecker, `arXiv:1206.5191 [hep-ph]`.
[11] M.A. Stephanov, K. Rajagopal, E.V. Shuryak, *Phys. Rev. Lett.* **81**, 4816 (1998).
[12] M.A. Stephanov, K. Rajagopal, E.V. Shuryak, *Phys. Rev.* **D60**, 114028 (1999).
[13] B. Berdnikov, K. Rajagopal, *Phys. Rev.* **D61**, 105017 (2000).
[14] I.N. Mishustin, O. Scavenius, *Phys. Rev. Lett.* **83**, 3134 (1999).
[15] K. Paech, H. Stoecker, A. Dumitru, *Phys. Rev.* **C68**, 044907 (2003).
[16] M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, *Phys. Rev.* **C84**, 024912 (2011).
[17] C. Greiner, B. Muller, *Phys. Rev.* **D55**, 1026 (1997).
[18] T.S. Biro, C. Greiner, *Phys. Rev. Lett.* **79**, 3138 (1997).
[19] D.H. Rischke, *Phys. Rev.* **C58**, 2331 (1998).
[20] H. Petersen *et al.*, *Phys. Rev.* **C78**, 044901 (2008).
[21] M. Nahrgang, S. Leupold, M. Bleicher, *Phys. Lett.* **B711**, 109 (2012).
[22] M. Nahrgang *et al.*, `arXiv:1105.1962 [nucl-th]`.
[23] C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, work in progress.