QUARK ORBITAL ANGULAR MOMENTUM*

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For transversely polarized nucleons the distribution of quarks in the transverse plane is transversely shifted and that shift can be described in terms of Generalized Parton Distributions (GPDs). This observation provides a 'partonic' derivation of the Ji-relation for the quark angular momentum in terms of GPDs. Wigner distributions are used to show that the difference between the Jaffe–Manohar definition of quark orbital angular momentum and that of Ji is equal to the change of orbital angular momentum due to the final state interactions as the struck quark leaves the target in a DIS experiment.

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1. Distribution of quarks in the transverse plane

Fourier transforms of Generalized Parton Distributions (GPDs) describe the distribution of partons in the transverse plane [1]. In the case of transversely polarized quarks and/or nucleons, these show a significant deviation from axial symmetry. For example, in the case of unpolarized quarks in a nucleon polarized in the $+\hat{x}$ direction, this deformation is described by the $\perp$ gradient of the Fourier transform of the GPD E^q [2]

$$\begin{aligned} q_{q/p\uparrow}\left(x,\boldsymbol{b}_\perp\right) \;=\; & \int \frac{d^2\boldsymbol{x}_\perp}{(2\pi)^2} e^{-i\boldsymbol{b}_\perp\cdot\Delta_\perp} H^q\left(x,0,-\Delta_\perp^2\right) \\ & -\frac{1}{2M}\partial_y \int \frac{d^2\boldsymbol{x}_\perp}{(2\pi)^2} e^{-i\boldsymbol{b}_\perp\cdot\Delta_\perp} E^q\left(x,0,-\Delta_\perp^2\right) \end{aligned} \tag{1}$$

for quarks of flavor q. Since $E^q(x,0,t)$ also arises in the decomposition of the Pauli form factor $F_2^q = \int_{-1}^{1} dx E^q(x,0,t)$ for quarks with flavor q (here it is always understood that charge factors have been taken out) w.r.t. x, this

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allows to relate the $\perp$ flavor dipole moment to the contribution from quarks with flavor q to the nucleon anomalous magnetic moment (here it is always understood that charge factors have been taken out)

$$d^q \equiv \int d^2\boldsymbol{b}_\perp q_{+\hat{x}}\left(x,\boldsymbol{b}_\perp\right) b_y = \frac{1}{2M}F_2^q(0) = \frac{1}{2M}\kappa_{q/p}\,. \tag{2}$$

Here, $e_q\kappa_{q/p}$ is the contribution from flavor q to the anomalous magnetic moment of the proton. Neglecting the contribution from heavier quarks to the nucleon anomalous magnetic moment, one can use the proton and neutron anomalous magnetic moment to solve for the contributions from $q = u, d$, yielding $\kappa_{u/p} \approx 1.67$ and $\kappa_{d/p} \approx -2.03$. The resulting significant deformation ($|d_q| \sim 0.1\,\mathrm{fm}$) of impact parameter dependent PDFs in the transverse direction, which is in opposite directions for u and d quarks (Fig. 1), should have observable consequences in other experiments as well as will be discussed in the following.

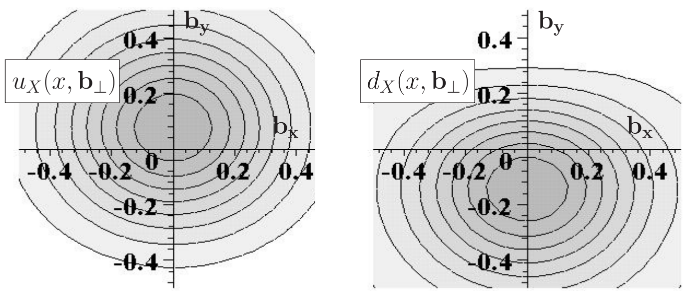

Fig. 1. Distribution of the j^+ density for u and d quarks in the $\perp$ plane ($x = 0.3$ is fixed) for a proton that is polarized in the x direction in the model from Ref. [2]. For other values of x the distortion looks similar.

2. Angular momentum sum rule

The transverse deformation also provides a parton interpretation for the 'Ji-relation' [3] between the 2nd moment of GPDs and the angular momentum carried by the quarks.

For a nucleon at rest, described using a spherically symmetric wave packet centered at the origin $\psi(\vec{r})$ the impact parameter dependent quark distribution is obtained as a convolution of the quark distribution relative to the center of momentum with the distribution of the nucleon described by the wave packet. Surprisingly, when the nucleon is transversely polarized, the resulting center of momentum for the entire nucleon is not at the origin but shifted sideways by half a Compton wavelength in a direction perpendicular to the spin polarization [4]

$$q_\psi\left(x,\boldsymbol{b}_\perp\right)=\int d^2r_\perp q_{q/P\uparrow}\left(x,b_\perp-r_\perp\right)\left(\left|\psi\left(\boldsymbol{r}_\perp\right)\right|^2-\frac{1}{2M}\frac{\partial}{\partial r_\perp}\left|\psi\left(\boldsymbol{r}_\perp\right)\right|^2\right). \tag{3}$$

This is a peculiar effect that arises due to the fact that the lower component of the Dirac wave function describing the nucleon wave packet has an orbital angular momentum that is positively correlated to the nucleon spin. However, even though this is a relativistic effect, it does not disappear in the limit of a large 'radius' R for the wavepacket, since in the evaluation of the center of momentum $\boldsymbol{r}_\perp \sim \mathcal{O}(R)$.

In order to relate the above distribution of quarks in a wave packet to the contribution from quarks of flavor q to the nucleon angular momentum, we use that for a nucleon polarized in the $+\hat{x}$ direction, rotational symmetry allows to replace $J_q^x = \int d^3r[yT_q^{0z} - zT_q^{0y}] \to 2\int d^3r\, yT_q^{0z}$, where $T_q^{\mu\nu}$ is the part of the energy-momentum tensor that involves q. The latter can be related to the light-cone momentum density $T^{++} = T^{00} + T^{0z} + T^{z0} + T^{zz}$ since only $T^{0z} + T^{z0}$ gives a nonzero contribution to $\int d^3r\, yT_q^{0z}$. One thus finds the Ji-relation [3] for the expectation value of J_q^x in terms of

$$\begin{aligned}\langle\psi|J_q^x|\psi\rangle &= \int d^3r\, y\langle\psi|T^{++}(\vec{r})|\psi\rangle = M\int dx\int d^2r_\perp q_\psi(x,\boldsymbol{r}_\perp)\\ &= \frac{1}{2}\int_0^1 dx\, x\left[H^q(x,0,0)+E^q(x,0,0)\right]\end{aligned} \tag{4}$$

moments of GPDs. Here we used the fact that $\int dzT^{++}(\vec{r})$ represents the distribution of the light-cone momentum fraction x in the transverse plane and the term involving the GPD H^q arises from the 'overall shift' (3) whereas the term involving E^q was caused by the 'intrinsic shift' (1).

In light-cone language, the angular momentum operator can be written as a sum of a 'good' and a 'bad' component. Fortunately, due to rotational invariance, the expectation value of the bad component equals that of the good component and we can restrict the calculation to the good component, which has a parton interpretation in terms of the transverse shift described by Eqs. (1) and (3).

3. Spin decompositions

The above derivation utilized the manifestly gauge invariant local energy-momentum tensor and thus the resulting quark orbital angular momentum (OAM) is also local. It enters the Ji-decomposition of the nucleon spin (5)

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left[\frac{1}{2}\Delta q + L_q\right] + J_g \tag{5}$$

in which two terms are experimentally accessible: Δq (through polarized DIS) and, through DVCS, the quark OAM

$$L_q = \int d^3x \, \langle P,S|\, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^z q(\vec{x}) \,|P,S\rangle \ . \tag{6}$$

An alternative decomposition has been suggested in Ref. [5]

$$\tfrac{1}{2} = \sum_q \tfrac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g \,, \tag{7}$$

where each term can be defined gauge invariantly, but except for Δq, definitions in terms of local operators only exist in light-cone gauge $A^+ = 0$. For example, in light-cone gauge the quark OAM is defined through

$$\mathcal{L}_q = \int d^3x \, \langle P,S|\, \bar{q}(\vec{x})\gamma^+ \left(\vec{x} \times i\vec{\partial}\right)^z q(\vec{x}) \,|P,S\rangle \ . \tag{8}$$

The most significant difference to (6) is the replacement $i\vec{D} \equiv i\vec{\partial} - g\vec{A} \longrightarrow i\vec{\partial}$. The quark spin contribution is the same as in (5), and ΔG is accessible though longitudinally polarized proton–proton scattering as well as through scaling violations in polarized DIS.

Model studies have shown that indeed $L_q \neq \mathcal{L}_q$ [6] and, therefore, it is not clear whether one should 'mix' the two decompositions (5) and (7) in order to accomplish a 'complete' decomposition of the nucleon spin.

4. Wigner distributions and quark OAM

Wigner distributions are defined through Fourier transforms of nonforward matrix elements of (nonlocal) quark correlation functions [7]

$$W\left(x, \vec{b}_\perp, \vec{k}_\perp\right) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{i\vec{b}_\perp\cdot\vec{q}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle \tag{9}$$

and provide a quasi-probabilistic description both in $\perp$ position $\boldsymbol{b}_\perp$ and momentum $\boldsymbol{k}_\perp$. Since we only consider $S_z = S'_z = +\frac{1}{2}$, the explicit dependence of $W(x, \vec{b}_\perp, \vec{k}_\perp)$ on S, S' will be suppressed to simplify notation.

Wigner distributions allow a unified description of GPDs $q(x, \boldsymbol{b}_\perp) = \int d^2k_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$, TMDs $f(x, \boldsymbol{k}_\perp) = \int d^2b_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$, and OAM [8]

$$L_z = \int dx \int d^2\boldsymbol{b}_\perp \int d^2\boldsymbol{k}_\perp W\left(x, \vec{b}_\perp, \vec{k}_\perp\right)(b_x k_y - b_y k_x) \,. \tag{10}$$

Since Eq. (9) involves non-local quark correlation functions, Wilson line gauge links connecting 0 with ξ must be inserted for a manifestly gauge invariant definition. This raises the immediate question regarding the choice of path for the Wilson line gauge link. One choice that may appear natural is a straight-line path from 0 to ξ. Such a choice leads to local definitions both for TMDs as well as OAM: For the OAM (10) with such a choice of path reduces to Ji's OAM (6). However, such a choice also leads to TMDs that have vanishing Single-Spin Asymmerties (SSAs) as a straight line gauge-link does not account for the final-state interactions (FSI) experienced by the struck quark in DIS. These FSI can be included into a definition of Wigner functions by using a straight Wilson line gauge links from the position of the quark field operators to $x^- = \infty$ (for fixed $\boldsymbol{x}_\perp$) and back with an additional connection at from $\mathbf{0}_\perp$ to $\boldsymbol{x}_\perp$ at $x^- = \infty$ (Fig. 2). In particular, in $A^+ = 0$ gage it is essential to include the segment at $x^- = \infty$. Indeed, in that gauge one finds for the average $\perp$ momentum

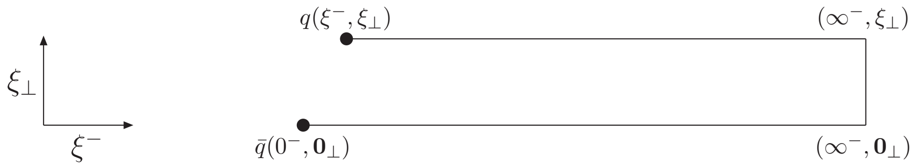

Fig. 2. Staple shaped path for gauge-link in the LC definition of Wigner functions.

$$\left\langle \vec{\mathcal{K}}_\perp \right\rangle \equiv \int d^2k_\perp \, \vec{k}_\perp \int d^2b_\perp W^q_{\rm LC}\left(x, \vec{b}_\perp, \vec{k}_\perp\right) \vec{k}_\perp = \langle P,S|\, \bar{q}(0)\gamma^+ i\vec{\mathcal{D}} q(0)\, |P,S\rangle \tag{11}$$

with $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \boldsymbol{x}_\perp)$, which is nonzero [9], but would vanish without the contribution from $\vec{A}(x^- = \infty, \boldsymbol{x}_\perp)$ [10]. In contradistinction, for the definition of the quark OAM, even for $A^+ = 0$ the piece at $x^- = \infty$ does not contribute [11]. This implies that $W^q_{\rm LC}$ provides a manifestly gauge invariant definition for $\mathcal{L}^q$ [12] also as

$$\begin{aligned} \mathcal{L}^q &= \int dx \int d^2\boldsymbol{b}_\perp \int d^2\boldsymbol{k}_\perp W^q_{\rm LC}\left(x, \vec{b}_\perp, \vec{k}_\perp\right) (b_x k_y - b_y k_x) \\ &= \int d^3b \, \langle P,S|\, \bar{q}\left(\vec{b}\right) \gamma^+ \left(\vec{b} \times i\vec{\mathcal{D}}\right)_z q\left(\vec{b}\right) |P,S\rangle \,. \end{aligned} \tag{12}$$

For this work the most important advance from using Wigner distributions to define quark OAM is that we are now in a position to compare the Jaffe–Manohar definition with that of Ji. Upon subtracting (6) from (12) and nothing that (in $A^+ = 0$ gauge)

$$A_\perp(x^- = \infty, \boldsymbol{x}_\perp) - A_\perp(0, \boldsymbol{x}_\perp) = \int_0^\infty dx^- G_{-\perp}(x^-, \boldsymbol{x}_\perp)\,, \tag{13}$$

where $\sqrt{2}G^{+y} = -E^y + B^x = -(\vec{E} - \hat{\vec{z}} \times \vec{B})$ is the $\hat{y}$ component of the color Lorentz force acting on a particle moving with the velocity of light in the $-\hat{z}$ direction. We thus find that the difference between the Jaffe–Manohar definition of OAM and that of Ji can be expressed in terms of the change in OAM as the quark leaves the target after being struck in DIS due to the FSI [13]

$$\mathcal{L}^q - L^q = \Delta L_{\rm FSI} = \int d^3x \int_{x^-}^{\infty} d\xi^- \left\langle P,S\right| \bar{q}(\vec{x})\gamma^+ T_z(\xi^-, \boldsymbol{x}_\perp) q(\vec{x}) \left|P,S\right\rangle \,, \quad (14)$$

where $T_z = g\left[\vec{x} \times \left(\vec{E} - \hat{\vec{z}} \times \vec{B}\right)\right]_z$ is the torque acting on the struck quark leaving the nucleon. This result also implies that the so-called 'potential angular momentum' [14] that arises when one tries to merge (5) with (7), may have a new interpretation in terms of the change in OAM of the quark as it leaves the target.

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