ISSUES IN THE GPD FORMULATION OF DVCS*

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Virtual Compton scattering (VCS) can be used to explore the structure of hadrons. In the domain of large energy and momentum transfers, VCS is complementary to deep-inelastic scattering. If the kinematics is collinear, generalized parton distributions (GPDs) are widely used to describe VCS. This formulation satisfies electromagnetic gauge invariance only approximately. We propose to analyze experimental data in terms of Coulomb form factors, which occur in a gauge invariant formulation of Compton scattering, do not depend on the kinematics and can be related to GPDs in the deeply virtual limit.

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1. Introduction

Light-cone dominated processes can be related to parton distributions. In deeply virtual Compton scattering (DVCS), the relevant quantities are denoted as Generalized Parton Distributions (GPDs) [1]. They occur in a factorized form of the scattering amplitude (see Fig. 1), which was shown to be valid for handbag-type diagrams in the deeply virtual regime, where the virtuality of the incoming photon Q^2 is much larger than the masses of the particles involved and the square of the momentum-transfer to the target hadron, Mandelstam t. Two questions may arise: Is this formulation valid in a realistic kinematics, where t and the masses are not negligible compared to Q^2 (*cf.* Ref. [2]), and does this formulation satisfy electromagnetic gauge invariance?

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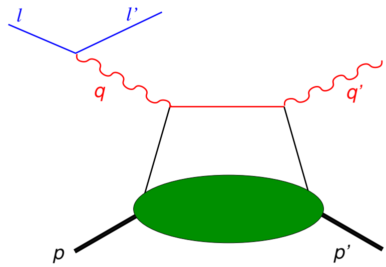

Fig. 1. DVCS amplitude.

In any kinematics, the VCS amplitude can be written as the contraction of a second-rank tensor and the polarization vectors of the photons

$$A\left(p', q', h'; p, q, h\right) = \epsilon^*\left(q'; h'\right)_\mu T^{\mu\nu} \epsilon(q; h)_\nu \,. \tag{1.1}$$

The most general form of the tensor depends on the momenta of the incident and outgoing particles and on their spins. The tensor $T^{\mu\nu}$ can be written as a linear combination of elementary tensors with scalar coefficients, the latter being denoted as the Compton form factors (CFFs). Below, we discuss the construction of the Compton tensor for a scalar hadron and show that if both the incident and the outgoing photons are virtual, there are five independent CFFs, while for a real photon in the final state only three CFFs occur.

2. Scalar target

From four-momentum conservation it follows that out of the external momenta occurring in the hadronic part of the amplitude, namely p, q, p', and q' one may choose three independent ones. We keep q and q', to be able to check the transversity of the tensor. For the remaining one, we choose the sum of the hadronic momenta, $\bar{P} := p' + p$. The most general second-rank tensor expressed in terms of this basis is then: $(k_1 = \bar{P},\ k_2 = q',\ k_3 = q)$

$$T^{\mu\nu} = \mathcal{H}_0\ g^{\mu\nu} + \sum_{i,j} \mathcal{H}_{ij}\ {k_i}^\mu {k_j}^\nu \,. \tag{2.1}$$

By contracting $T^{\mu\nu}$ with q'_μ and q_ν, which must give the result 0 for the physical tensor, one can determine the number of independent scalars $\mathcal{H}$. As there are ten $\mathcal{H}$s and the number of independent contractions is five, there are five independent scalars (CFFs) in the *effective* tensor. This number was mentioned before in Ref. [3] and numerous more recent papers. As these independent tensor structures can be chosen in an infinite number of ways, we look for a natural way to construct the effective tensor.

2.1. Tarrach's construction

Following Tarrach [3], we find it useful to construct the tensor $T^{\mu\nu}$ by applying a two-sided projector $\tilde{g}^{\mu\nu}$ to the most general second rank tensor expressed in terms of our basis

$$T^{\mu\nu} = \tilde{g}^{\mu m}\, t_{mn}\, \tilde{g}^{n\nu}\,, \qquad t_{mn} = t_0\, g_{mn} + \sum_{i,j} \mathcal{H}_{ij}\ k_{im} k_{jn}\,. \tag{2.2}$$

The two-sided projector $\tilde{g}$ is defined as follows

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q'^\nu}{q \cdot q'}\,. \tag{2.3}$$

This projector has the properties

$$q'_\mu\, \tilde{g}^{\mu\nu} = 0\,, \qquad \tilde{g}^{\mu\nu} q_\nu = 0\,. \tag{2.4}$$

Now, using the transverse momenta

$$\begin{aligned} \tilde{q}'^\mu_{\mathrm{L}} &= q'^\mu - \frac{q'^2}{q\cdot q'} q^\mu\,, \qquad & \tilde{q}^\nu_{\mathrm{R}} = q^\nu - \frac{q^2}{q\cdot q'} q'^\nu\,, \\ \tilde{P}^\mu_{\mathrm{L}} &= P^\mu - \frac{P\cdot q'^2}{q\cdot q'} q^\mu\,, \qquad & \tilde{P}^\nu_{\mathrm{R}} = P^\nu - \frac{q\cdot P}{q\cdot q'} q'^\nu\,, \end{aligned} \tag{2.5}$$

$T^{\mu\nu}$ can be written succinctly as follows

$$T^{\mu\nu} = \mathcal{H}_0\ \tilde{g}^{\mu\nu} + \mathcal{H}_1\ \tilde{P}^\mu_{\mathrm{L}} \tilde{P}^\nu_{\mathrm{R}} + \mathcal{H}_2\ \tilde{P}^\mu_{\mathrm{L}} \tilde{q}^\nu_{\mathrm{R}} + \mathcal{H}_3\ \tilde{q}'^\mu_{\mathrm{L}} \tilde{P}^\nu_{\mathrm{R}} + \mathcal{H}_4\ \tilde{q}'^\mu_{\mathrm{L}} \tilde{q}^\nu_{\mathrm{R}}\,. \tag{2.6}$$

This construction immediately gives us the number of CFFs, namely five, in the case that both q' and q are virtual. If either photon is real, the *effective* number of CFFs drops to three and for real Compton scattering only two CFFs remain. The reason is that, *e.g.* for a real photon with momentum q', the transverse momentum $\tilde{q}'_{\mathrm{L}}$ is identical to q' and thus is annihilated when the physical amplitude is obtained by contracting $T^{\mu\nu}$ with $\epsilon(q')_\mu$. An advantage of working with the resulting *effective* tensor is that the number of CFFs appearing is the same as the number of independent physical matrix elements.

3. Spin-1/2 case

Let us write again the physical amplitude as the contraction of a tensor

$$A\left(p', q', s', h'; p, q, s, h\right) = \epsilon^*\left(q'; h'\right)_\mu T^{\mu\nu}\left(p', s'; p, s\right) \epsilon(q; h)_\nu\,. \tag{3.1}$$

The case of a spin-1/2 target is more complicated because the tensor $T^{\mu\nu}$ is a spin matrix element and thus will, in general, contain besides structures expressible in terms of momenta only, Dirac operators like γ^μ and $\sigma^{\mu\nu}$. Following Tarrach, one finds ten basis tensors which coincide with the ones found in the scalar case, ten more which are obtained by multiplying these with $\bar{\not{Q}} = \gamma \cdot (q' + q)$. In addition, one finds six that contain one of the basis momenta and one γ matrix, six more, where these are commuted with $\bar{\not{Q}}$. Finally, one may add $\sigma^{\mu\nu}$ and the commutator of $\sigma^{\mu\nu}$ and $\bar{\not{Q}}$. The 34 structures found this way are overcomplete, but using two linear relations between them, Tarrach succeeded in reducing the basis to 32 elementary tensors, which constitute a complete basis for a complex second-rank tensor in four-dimensional space.

Upon using the same construction as before, namely contracting the elementary structures with $\tilde{g}$ from the left and from the right, one finds that the spin-1/2 VCS tensor has 18 CFFs. If either of the photons is real, this number reduces to 12; if both are real, the number is 8 for real Compton scattering. These numbers do not depend on the kinematics but for the values of q'^2 and q^2. In particular, the limit $Q^2 = -q^2 \to \infty$ does not change these numbers.

3.1. Tree level tensor: scalar and spinor cases

The tree-level amplitudes are given by the diagrams in Fig. 2. The seagull occurs only for scalar targets.

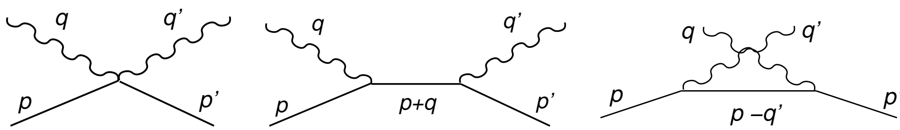

Fig. 2. Seagull, s-, and u-channel amplitudes at tree level.

In the case of a scalar target, they can be expressed in terms of two CFFs

$$\mathcal{H}_0 = -2\,, \qquad \mathcal{H}_1 = \frac{1}{s - M^2} + \frac{1}{u - M^2}\,, \tag{3.2}$$

where s and u are the usual Mandelstam variables. Clearly, these two CFFs will occur for any kinematics independent of the values of q'^2 and q^2. If dynamics beyond tree-level is considered, not only will the other three CFFs occur (one only if either the incoming or outgoing photon is real), but $\mathcal{H}_0$ and $\mathcal{H}_1$ will change too. This consideration has some bearing on the twist expansion used in the GPD formulation of DVCS. For a spin-1/2 target, at lowest twist only four GPDs are considered, while at higher twist the number of GPDs is increased.

4. Simple models

In order to understand what the condition of electromagnetic gauge invariance means for the minimal number of dynamical corrections to the tree-level amplitudes, we consider a simple model: A charged particle of mass M interacts with a neutral one of mass μ. The lowest-order in g amplitudes for Compton scattering of the charged particle are just the ones we have seen before. In the case of the spin-1/2 target, they are the s- and u-channel amplitudes, while for the scalar target also the seagull occurs. The corresponding diagrams for the tensors are given in Figs. 3–5.

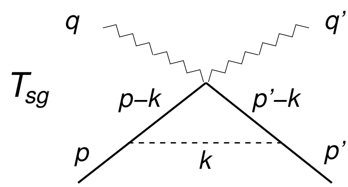

Fig. 3. One-loop correction to the seagull amplitude $T_{sg}^{\mu\nu}$.

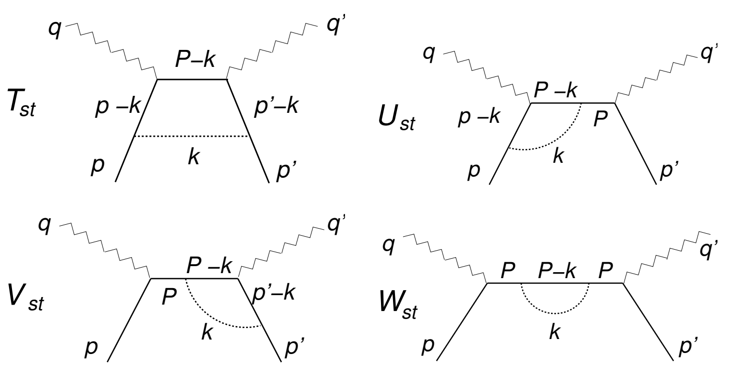

Fig. 4. One-loop corrections to the tensors $T_{st}^{\mu\nu}$, $U_{st}^{\mu\nu}$, $V_{st}^{\mu\nu}$, and $W_{st}^{\mu\nu}$.

The diagrams with the self-energy corrections to the propagators connecting the photon vertices diverges and must be regulated by subtraction of the counter terms appearing in the mass renormalization as well as the photon vertex corrections. Then from a straight forward calculation of the contractions with q'_μ one finds that they cancel out, as well as the contractions with q_ν. However, the handbag diagrams $T_{sg}^{\mu\nu}$, $T_{st}^{\mu\nu}$, and $T_{ut}^{\mu\nu}$ do not, which means that in order to satisfy electromagnetic gauge invariance other contributions from the photon vertex corrections must be included to the Compton tensor.

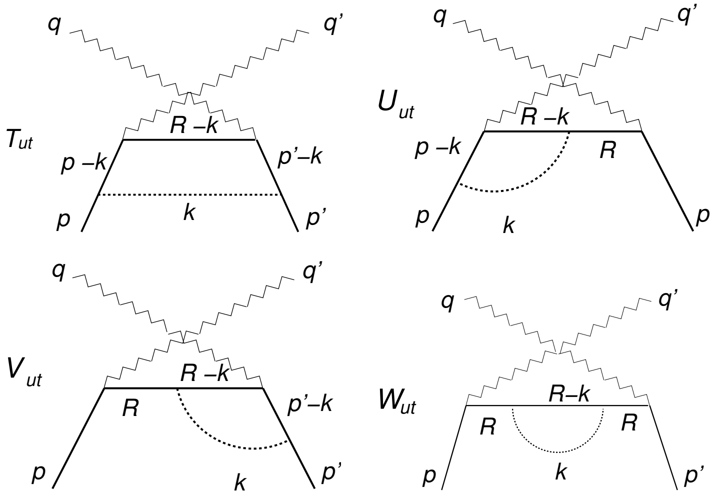

Fig. 5. One-loop corrections to the tensors $T^{\mu\nu}_{ut}$, $U^{\mu\nu}_{ut}$, $V^{\mu\nu}_{ut}$, and $W^{\mu\nu}_{ut}$.

For a spin-1/2 target, a similar analysis shows [2] that also in this case the handbag diagrams by themselves do not satisfy electromagnetic gauge invariance. The handbag diagrams considered in the literature in lowest twist violate this symmetry, indicating that other contributions must restore it.

5. Conclusion

Since the twist expansion does not guarantee electromagnetic gauge invariance order by order, we suggest that in the analysis of experimental data a form of the amplitude shall be used that satisfies electromagnetic gauge invariance and is fully covariant. This means that for any realistic experimental setup the data are to be analyzed in terms of the CFFs. The determination of GPDs will then be a further step, that may involve making approximations in the kinematics, *i.e.*, the forms of the basis tensors, as well as the dynamics, relevant for the relation of the CFFs to the GPDs.

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