FORM FACTORS OF PSEUDOSCALAR MESONS IN A MODEL WITH UNITARITY AND REGGE EXCHANGES*

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We consider the transition and electromagnetic pseudoscalar form factors in a wide range of energy-momentum transfer, s. We employ dispersion relations to connect the time-like and space-like region. We find that hadronic resonances give sizable contributions in the currently available range of s. For the asymptotic contribution, we propose a model that operates with reggeized fermion exchanges.

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1. Introduction

Electromagnetic form factors of hadrons provide a clean tool to study their internal structure. In this paper, we consider the charged pion electromagnetic form factor $F_\pi(s)$, defined by the matrix element

$$\left\langle \pi^+\left(p'\right)\pi^-(p)|J_\mu|0\right\rangle = e\left(p'-p\right)_\mu F_\pi(s)\,, \tag{1}$$

and the transition form factors (TFF) between a neutral pseudoscalar meson $P=\pi^0,\eta,\eta'$ and a real photon, $F_{P\gamma}(s)$ determined by

$$\left\langle P\left(p'\right)\gamma(\lambda,p)|J_\mu|0\right\rangle = ie^2\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(\lambda)p'^\alpha p^\beta F_{P\gamma}(s)\,, \tag{2}$$

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where J_μ is the electromagnetic current and $s = (p' + p)^2$ is the four-momentum transfer squared. Current conservation implies $F_\pi(0) = 1$ and, in the chiral limit, axial anomaly determination of the $\pi^0 \to 2\gamma$ decay leads to the expectation $F_{\pi\gamma}(0) \approx 1/(4\pi^2 f_\pi) = 0.274\,\mathrm{GeV}^{-1}$ with $f_\pi = 92.4\,\mathrm{MeV}$ being the pion decay constant, which is in excellent agreement with the experimental value of $F^{\mathrm{exp}}_{\pi\gamma}(0) \approx 0.267\,\mathrm{GeV}^{-1}$. The measured values for the normalization of the η, η' transition form factors are $F^{\mathrm{exp}}_{\eta\gamma}(0) \approx 0.272\,\mathrm{GeV}^{-1}$ and $F^{\mathrm{exp}}_{\eta'\gamma}(0) \approx 0.341\,\mathrm{GeV}^{-1}$, respectively. All form factors were found to decrease with the virtuality of the photon, the fact that is interpreted as the evidence for a non-trivial internal structure of hadrons. At low virtualities, a successful description is based on the picture in which the photon couples to pions, the lightest hadronic degree of freedom. This picture includes hadronic resonances in the time-like region. At short distances quark/gluon interactions are asymptotically free, and the prediction of perturbative QCD (pQCD) is that at high $|s|$, these form factors are determined by hard scattering of the virtual photon with a small number of the QCD constituents [1–4]. A striking discrepancy with the pQCD prediction was observed in the pion transition form factor by BaBar [6]. For momentum transfers $10\,\mathrm{GeV}^2 \lesssim -s \lesssim 40\,\mathrm{GeV}^2$ the data suggest that the magnitude of $-sF_{\pi\gamma}(s)$ grows with $|s|$, while pQCD predicts $sF_{\pi\gamma}(s) \to 2f_\pi = \mathrm{const.}$ On the other hand, BaBar Collaboration also measured the η and η' transition form factors [7, 8] up to $-s \approx 40\,\mathrm{GeV}^2$ in the space-like region and at $s \approx 112\,\mathrm{GeV}^2$ in the time-like region, and these two form factors seem to be consistent with the pQCD expectations. Recently, Belle released pion TFF data [9] which are compatible with pQCD, while not contradicting the BaBar data within the experimental uncertainties.

2. Form factors from dispersion relations

2.1. Resonance contribution

We begin with the discussion of the transition form factor $F_{P\gamma}(s)$. Its discontinuity across the unitary cut at $s \geq 4m_\pi^2$ is given by

$$\mathrm{Im}F_{P\gamma} = \mathrm{Im}_{2\pi}F_{P\gamma} + \mathrm{Im}_{3\pi}F_{P\gamma} + \sum{}'\,\mathrm{Im}_X F_{P\gamma}\,, \tag{3}$$

where we explicitly include the discontinuity due to the lowest $2\pi, 3\pi$ intermediate states, and $\sum' = \sum_{X\neq 2\pi,3\pi}$. Provided that $\mathrm{Im}\,F_{P\gamma}$ vanishes at $s \to \infty$, its real part can be reconstructed for any s from the unsubtracted dispersion relation

$$F_{P\gamma}(s) = \frac{1}{\pi}\int\limits_{4m_\pi^2}^{\infty} ds'\,\frac{\mathrm{Im}F_{P\gamma}(s')}{s' - s}\,. \tag{4}$$

The two lowest mass intermediate states, $X = 2\pi$, 3π are dominated by the $\rho(770)$ and $\omega(782)$ resonances, respectively, and almost saturate the cut in the hadronic range $4m_\pi^2 < s \lesssim 1\,\mathrm{GeV}^2$. While the naïve vector dominance model (VDM) with these two resonances taken in the limit of zero width overestimates the anomaly, accounting for rescattering effects in the p-wave of the 2π channel (below referred to as unitarized VDM) allows one to improve the description [5, 10]. Since the TFF obeys the unsubtracted dispersion relation, it imposes a constraint on the multiparticle contribution,

$$F_{P\gamma}(0) = \frac{1}{\pi}\int\limits_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\left[\mathrm{Im}_{2\pi}F_{P\gamma}(s') + \mathrm{Im}_{3\pi}F_{P\gamma}(s') + \sum{}'\,\mathrm{Im}_X F_{P\gamma}(s')\right] \quad (5)$$

with the $2\pi, 3\pi$ terms saturating the anomaly at the level of 98% for all three TFF. We will discuss the multiparticle contribution in the next subsection.

In the case of the pion electromagnetic form factor, the discontinuity across the unitary cut reads

$$\mathrm{Im}F_\pi = \mathrm{Im}_{2\pi}F_\pi + \mathrm{Im}_{K\bar{K}}F_\pi + \sum{}'\,\mathrm{Im}_X F_\pi\,. \quad (6)$$

The two channels $X = 2\pi$ and $X = K\bar{K}$ are phenomenologically most significant in the hadronic domain, and are written explicitly, while we denote $\sum' = \sum_{X\neq 2\pi, K\bar{K}}$. The form factor can be written in the form $F_{2\pi}(s) = N(s)/D(s)$ with $N(s)$ containing inelastic cuts, and $D(s)$ constructed from the $\pi\pi$ phase shift, for full detail see [5, 11, 12].

2.2. Multiparticle contribution

Above $s = 1\,\mathrm{GeV}^2$, one has to include multi-π, K *etc.* channels in the description of the imaginary part. Alternatively, one can employ the idea of the quark–hadron duality, similarly as in the derivation of the QCD sum rules [13], where one takes the lowest hadronic resonances explicitly into account, while approximating the multiparticle contributions residing at higher energies by the asymptotic contribution due to quarks and gluons.

In a high energy e^+e^- collision, a highly virtual time-like photon converts into a quark and an antiquark that travel in the opposite directions (in the rest frame of the virtual photon). These highly energetic quarks radiate gluons. If the leading mechanism is an emission of a small number of gluons, as expected in pQCD, with hard gluon emission being power-suppressed, multi-particle production would result from a fire-ball decay of jets producing a broadening of the peaks around the initial q or $\bar{q}$ jet in the rapidity distribution, as shown in Fig. 1. To obtain the mid-rapidity plato that is observed in experiments, one needs to radiate a large number of gluons and $q\bar{q}$-pairs that neutralize color between the opposite side jets. They form hadron showers that fill in the mid rapidity region in Fig. 1. To account for this mechanism we recall that in QCD it was shown that multi-particle production corresponds to ladder exchanges. In particular, for $q\bar{q} \to gg$ scattering these lead to an effective $q\bar{q} \to gg$ amplitude with a reggeized quark exchanged in the t-channel [14].

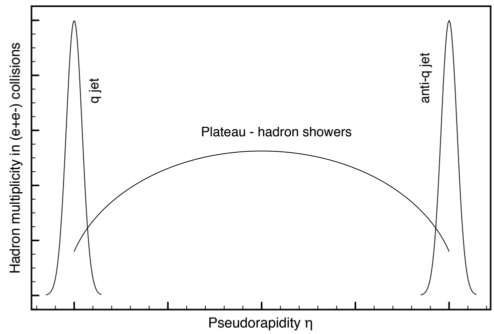

Fig. 1. Hadron multiplicity *vs.* pseudorapidity in e^+e^- collisions in arbitrary units.

Since the electromagnetic form factor F_X of a composite state decreases with energy-momentum transfer, asymptotically the highly virtual photon will couple to a single quark–antiquark pair for which $F_{q\bar{q}} = 1$. Thus, the mid-rapidity plateau contribution to the r.h.s. of Eq. (6) is given by the $X = q\bar{q}$, quark–antiquark intermediate state, as illustrated in Fig. 2, and analogously for the TFF. We assume a simple form for the reggeized amplitude $t^R_{q\bar{q}\to 2\pi}$ as function of large s at small momentum transfer t,

$$t^R_{q\bar{q}\to 2\pi}\left(s \geq \mu^2, t\right) = \beta_\pi e^{-b|t|} (s/s_0)^{\alpha_q(t)-1/2}\, \bar{q}\, \Gamma\, q \tag{7}$$

with Γ standing for the relevant Dirac structure, μ^2 an effective threshold for the asymptotic contribution, b the slope of the Regge residue, β_π the normalization to be determined from a fit, and $\alpha_q(t) = \alpha_q(0) + \alpha'_q t$ the quark Regge trajectory. To estimate these parameters phenomenologically, we square the above amplitude to obtain the imaginary part of the elastic $\pi\pi$ scattering amplitude and identify the meson Regge exchanges with the cut due to reggeized quark exchange. This leads to a simple relation $\alpha_\rho(0) = 2\alpha_q(0) - 1 \approx 0.5$, thus

$$\alpha_q(0) \approx 0.75 \tag{8}$$

and $b \approx 3\,\mathrm{GeV}^{-2}$. We fix α_q, b to the above values and use the normalization β_π ($\beta_{P\gamma}$ for TFF) and the threshold μ^2 as fit parameters. Once μ^2 is fixed (in the range $\mu^2 \sim 1$–$10\,\mathrm{GeV}^2$), $\beta_{P\gamma}$ is bounded by Eq. (5).

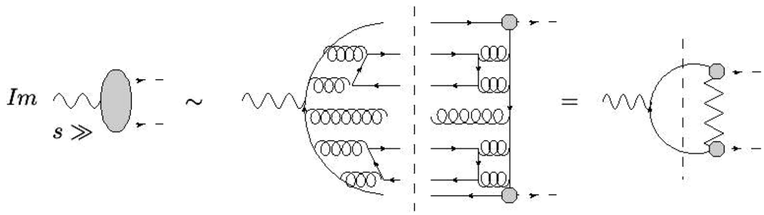

Fig. 2. Sum over multi-particle intermediate states contributions to $\mathrm{Im}F_\pi(s)$ at high energies $s \to \infty$ is given by a Regge limit of a fermion (spectator quark) exchange shown by a zigzag line.

3. Results

We display the results of the fit for all four form factors in Fig. 3.

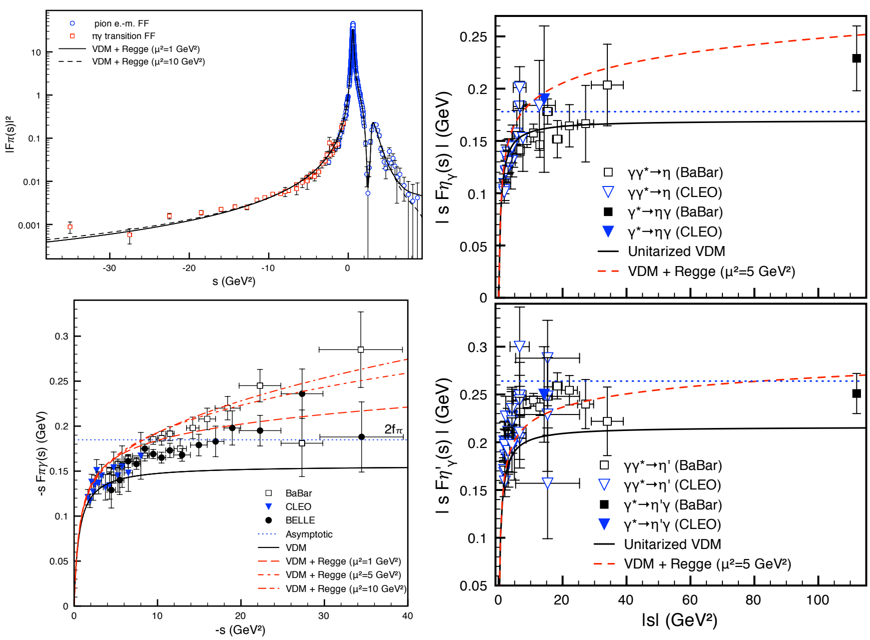

Fig. 3. Upper left panel: results with $\mu^2 = 1\,\mathrm{GeV}^2$ ($\mu^2 = 10\,\mathrm{GeV}^2$) shown by the solid (dashed) line are compared to the available experimental data on the pion electromagnetic form factor (open circles) and on pion TFF renormalized to $F_{\pi^0\gamma}(0) = 1$. Lower left panel: world data on pion TFF compared to pQCD asymptotics (thin dotted line), unitarized VDM (thick solid line), full model with $\mu^2 = 1, 5, 10\,\mathrm{GeV}^2$ are shown by long-dashed, dash-dotted, dash-double-dotted line, respectively. Upper right panel: data for the η TFF from [7, 8] in comparison with pQCD (thin dotted line), unitarized VDM (solid line), full calculation with $\mu^2 = 5\,\mathrm{GeV}^2$ (dashed line). Lower right panel: the same as above for the η' TFF.

We notice that the unitarized VDM (*i.e.*, dropping the asymptotic part) saturates the pQCD asymptotics for all three TFF at the level of 75–80%, thus even at highest s the asymptotic contribution contributes at most 50% of the full model result. This raises the question whether the inclusion of higher hadronic resonances would saturate the pQCD asymptotics. It is seen that we are able to describe all data over the whole available kinematical range. In order to fit the BaBar $F_{\pi^0\gamma}$ data alone, the model requires higher values of μ^2 (thus lower normalizations to reproduce the anomaly), while including Belle data allows to lower μ^2. For the η's TFF, we find that the asymptotic contribution is not much different in size from that for the pion TFF, and the flatter behavior is due to a larger relative size of the flat hadronic contribution. Finally, the authentic prediction of our model is that also for the pion electormagnetic form factor $|sF_\pi(s)|$ would grow asymptotically, should the rise of the BaBar data be confirmed. In fact, the upper left panel of Fig. 3 shows pion electromagnetic and transition form factors plotted on the same plot (pion TFF is renormalized to unity at $s = 0$) being compatible with each other over the available range of s.

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