

THE MESON-EXCHANGE INDUCED LIGHT-BY-LIGHT CONTRIBUTION TO $(g - 2)_\mu$ WITHIN THE NONLOCAL CHIRAL QUARK MODEL*

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The current status of the muon anomalous magnetic moment (AMM) problem is briefly presented. The corrections to the muon AMM coming from the effects of hadronic light-by-light (LbL) scattering due to light quark–antiquark exchange in pseudoscalar and scalar channels are estimated within the nonlocal chiral quark model (N χ QM). Within this approach the full kinematic dependence on the photon and meson virtualities are taken into account. As a result, the meson exchange contributions to the muon AMM calculated within N χ QM are, in general, smaller than the contributions obtained within other models. The contribution from the scalar channel is positive and small, but stabilizes the combined with pseudoscalar channel result with respect to variation of the model parameters.

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1. Muon AMM: theory vs. experiment

The anomalous magnetic moment of the muon is known to an unprecedented accuracy. The latest result from the measurements of the Muon $(g - 2)$ Collaboration at Brookhaven is [1]

$$a_\mu^{\text{BNL}} = 11\,659\,208.0(6.3) \times 10^{-10}, \quad (1)$$

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which is a 0.54 ppm uncertainty over combined positive and negative muon measurements. Using e^+e^- annihilation and inclusive hadronic τ decay data, the standard model predicts [2]

$$a_\mu^{\text{SM}} = \begin{cases} 11\,659\,180.2(4.9) \times 10^{-10}, & [e^+e^-], \\ 11\,659\,189.4(5.4) \times 10^{-10}, & [\tau]. \end{cases} \quad (2)$$

The difference between the experimental determination of a_μ and the Standard Model using the e^+e^- or τ data for the calculation of the hadronic vacuum polarization (HVP) contribution is 3.6σ and 2.4σ , respectively.

The Standard Model prediction for a_μ consists of quantum electrodynamics, weak and hadronic contributions. The QED and weak contributions to a_μ have been calculated with great accuracy [5]

$$a_\mu^{\text{QED}} = 11\,658\,471.8951(0.0080) \times 10^{-10} \quad (3)$$

and [6]

$$a_\mu^{\text{EW}} = 15.4(0.2) \times 10^{-10}. \quad (4)$$

The theoretical errors in (2) are dominated by the uncertainties induced by the HVP and LbL effects. Thus, to confront usefully theory with the experiment requires a better determination of the hadronic contributions. In the last decade, a substantial improvement in the accuracy of the contribution from the HVP was reached. It uses, essentially, precise determination of the low energy spectrum of the total $e^+e^- \rightarrow$ hadrons and inclusive τ lepton decays cross-sections. The HVP contributions at order α^2 quoted in the most recent articles on the subject are given in Table I.

TABLE I

Phenomenological estimates and references for the leading order HVP contribution to the muon anomalous magnetic moment based on e^+e^- and τ data sets.

| | e^+e^- [2] | τ [2] | e^+e^- [3] | e^+e^- [4] |
|---|-----------------|-----------------|-------------------|-------------------|
| $a_\mu^{\text{HVP (1)}} \times 10^{10}$ | 692.3 ± 4.2 | 701.5 ± 4.7 | 681.23 ± 4.51 | 694.91 ± 4.27 |

The higher-order contributions at $O(\alpha^3)$ level to $a_\mu^{\text{HVP (2)}}$ were estimated in [4],

$$a_\mu^{\text{HVP (2)}} = -9.84(0.07) \times 10^{-10}, \quad (5)$$

by using analytical kernel functions and experimental data on the $e^+e^- \rightarrow$ hadrons cross-section. In addition, there exists the $O(\alpha^3)$ contribution to a_μ from the LbL diagram, a_μ^{hLbL} , that cannot be expressed as a convolution of

experimentally accessible observables and need to be estimated from theory. In some works [7], the value

$$a_{\mu}^{\text{hLbL}} = 10.5(2.6) \times 10^{-10} \quad (6)$$

is considered as an estimate of the hadronic LbL contribution to the muon AMM.

Assuming absence of New Physics effects, a phenomenological estimate of the total hadronic contributions to a_{μ}^{HVP} has to be compared with the value deduced from the $(g - 2)$ experiment (1) and known electroweak (4) and QED (3) corrections

$$a_{\mu}^{\text{BNL}} - a_{\mu}^{\text{QED}} - a_{\mu}^{\text{EW}} = 721.6(6.3) \times 10^{-10}. \quad (7)$$

Two new experiments on measurement of a_{μ} are proposed at Fermilab (E989) [8] and J-PARC [9] which plan to improve the experimental uncertainty by a factor of 4–5 with respect to the previous BNL experiment. In that respect, theoretical predictions of HVP and LbL contributions to the muon AMM should be at the same level or better than a precision of planned experiments. In the next part, we discuss the hadronic LbL contribution as it is calculated within the nonlocal chiral quark model (N χ QM) of low energy QCD and show that, within this framework, it might be possible realistically to determine this value to a sufficiently safe accuracy. We want to discuss how well this model (see, *e.g.*, [10, 11]) does in calculating a_{μ}^{LbL} .

2. LbL contribution to the muon AMM due to light quark–antiquark exchanges in pseudoscalar and scalar channels

The uncertainties of the SM value for a_{μ} are dominated by the uncertainties of the hadronic contributions, a_{μ}^{Strong} , since their evaluation involve quantum chromodynamics (QCD) at long-distances for which perturbation theory cannot be employed. Below, we discuss with some details theoretical status of hadronic LbL contribution to the muon AMM due to exchange by light mesons within N χ QM.

N χ QM is an effective model that has a numerous applications for description of low energy hadronic dynamics. We mention only those applications that are related to the problem of hadronic contributions to the muon AMM. The two-point VV correlator has been calculated in [11] and used for calculations of the HVP contribution to the muon AMM [12]. The three-point VAV correlator has been calculated in [13, 14] and used for calculations of the photon- Z -boson interference contribution to the muon AMM [15].

More recently, the LbL contribution due to exchange of pseudoscalar (P) and scalar (S) mesons (Fig. 1) was elaborated in [16–18]. The vertices containing the virtual meson M with momentum p and two photons with momenta $q_{1,2}$ and the polarization vectors $\epsilon_{1,2}$ can be written as [19]

$$\mathcal{A}\left(\gamma_{(q_1, \epsilon_1)}^* \gamma_{(q_2, \epsilon_2)}^* \rightarrow M_{(p)}^*\right) = e^2 \epsilon_1^\mu \epsilon_2^\nu \Delta_M^{\mu\nu}(p, q_1 q_2) \quad (8)$$

with

$$\Delta_P^{\mu\nu}(p, q_1, q_2) = -i\varepsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma F_P(p^2; q_1^2, q_2^2), \quad (9)$$

and

$$\Delta_S^{\mu\nu}(p, q_1, q_2) = A_S(p^2; q_1^2, q_2^2) P_A^{\mu\nu}(q_1, q_2) + B'_S(p^2; q_1^2, q_2^2) P_{B'}^{\mu\nu}(q_1, q_2), \quad (10)$$

where

$$\begin{aligned} P_A^{\mu\nu}(q_1, q_2) &= (g^{\mu\nu}(q_1 q_2) - q_1^\nu q_2^\mu), \\ P_{B'}^{\mu\nu}(q_1, q_2) &= \frac{(q_1^2 q_2^\mu - (q_1 q_2) q_1^\mu)(q_2^2 q_1^\nu - (q_1 q_2) q_2^\nu)}{(q_1 q_2)^2 - q_1^2 q_2^2}, \end{aligned}$$

and $p = q_1 + q_2$. The subject of model calculations¹ are the (P/S)VV vertice functions F_P, A_S, B'_S .

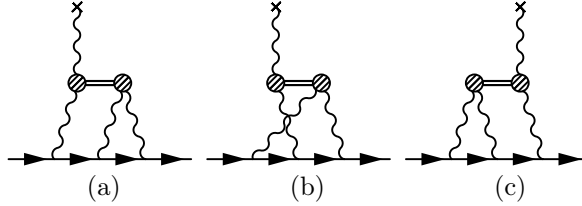


Fig. 1. Hadronic LbL scattering contribution due to quark–antiquark exchanges.

The expression for the LbL contribution to the muon AMM from the light meson exchanges can be written as

$$\begin{aligned} a_\mu^{\text{LbL}, M} &= -\frac{4\alpha^3}{3\pi^2} \int_0^\infty dQ_1^2 \int_0^\infty dQ_2^2 \int_{-1}^1 dt \sqrt{1-t^2} \frac{1}{Q_3^2} \\ &\quad \times \sum_M \left[\frac{\mathcal{N}_1^M(Q_1^2, Q_2^2, Q_3^2)}{Q_2^2 + M_M^2} + \frac{\mathcal{N}_2^M(Q_1^2, Q_3^2, Q_2^2)}{2(Q_3^2 + M_M^2)} \right], \end{aligned}$$

$$\mathcal{N}_{1,2}^P(Q_1^2, Q_2^2, Q_3^2) = F_P(Q_2^2; Q_2^2, 0) F_P(Q_2^2; Q_1^2, Q_3^2) T p_{1,2},$$

$$\begin{aligned} \mathcal{N}_{1,2}^S(Q_1^2, Q_2^2, Q_3^2) &= (A(Q_2^2; Q_2^2, 0) + \tfrac{1}{2} B'(Q_2^2; Q_2^2, 0)) \\ &\quad \times (A(Q_2^2; Q_1^2, Q_3^2) T s_{1,2}^{AA} + \tfrac{1}{2} B(Q_2^2; Q_1^2, Q_3^2) T s_{1,2}^{AB}), \end{aligned}$$

¹ For details see [18].

where $Q_3 = -(Q_1 + Q_2)$ and $B_S = B'_S/((q_1 q_2)^2 - q_1^2 q_2^2)$. The kinematic factors Tp_i and Ts_i can be found in [20] and [18], respectively.

3. The results of model calculations

The total contribution of pseudoscalar (π^0, η, η') exchanges is estimated as

$$a_\mu^{\text{LbL,P}} = (5.85 \pm 0.87) \times 10^{-10}, \quad (11)$$

and the combined value for the scalar (σ, a_0, f_0) and pseudoscalar contributions is [18]

$$a_\mu^{\text{LbL,P+S}} = (6.25 \pm 0.83) \times 10^{-10}. \quad (12)$$

We found that within the $N\chi\text{QM}$ the pseudoscalar meson contributions to the muon AMM are systematically lower than the results obtained in the other works. The full kinematic dependence² of the vertices on the pion virtuality diminishes the result by about 20–30% as compared to the case, where this dependence is neglected. For η and η' mesons the results are reduced by factor about 3 in comparison with the results obtained in other models, where the kinematic dependence was neglected (see Fig. 2 and discussion in [17, 18]). The scalar mesons contribution is small positive and partially compensates model dependence of the pseudoscalar contribution (Fig. 3).

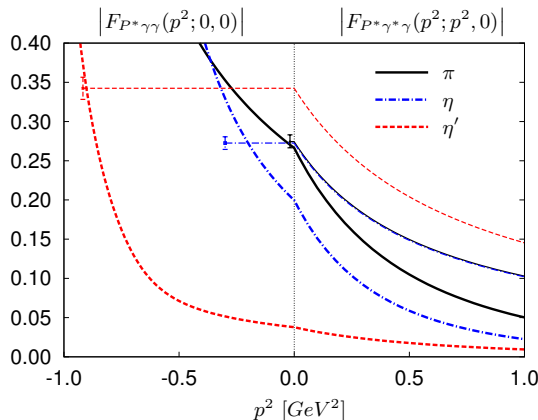


Fig. 2. Plots of the π^0, η and η' vertices $F_{P^*\gamma\gamma}(p^2; 0, 0)$ in the timelike region and $F_{P^*\gamma^*\gamma}(p^2; p^2, 0)$ in the space-like region in $N\chi\text{QM}$ model (thick lines) and VMD model (thin lines). The points with error bars correspond to the physical points of the meson decays into two photons. The VMD curves for π^0 and η are almost indistinguishable.

² This dependence also recently studied in [21].

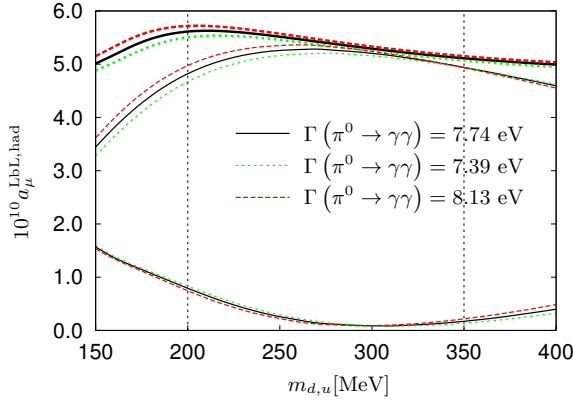


Fig. 3. LbL contribution to the muon AMM from the neutral pion and σ exchanges as a function of the dynamical quark mass. Bunch of three lower lines correspond to the σ contribution, the π^0 contribution is in the middle, and the upper lines are the combined contribution.

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REFERENCES

- [1] G.W. Bennett *et al.*, *Phys. Rev.* **D73**, 072003 (2006).
- [2] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, *Eur. Phys. J.* **C71**, 1515 (2011) [Erratum *ibid.* **C72**, 1874 (2012)] [arXiv:1010.4180 [hep-ph]].
- [3] M. Benayoun, P. David, L. DelBuono, F. Jegerlehner, arXiv:1210.7184 [hep-ph].
- [4] K. Hagiwara *et al.*, *J. Phys. G* **38**, 085003 (2011) [arXiv:1105.3149 [hep-ph]].
- [5] T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, *Phys. Rev. Lett.* **109**, 111808 (2012) [arXiv:1205.5370 [hep-ph]].
- [6] A. Czarnecki, W.J. Marciano, A. Vainshtein, *Phys. Rev.* **D67**, 073006 (2003) [Erratum *ibid.* **D73**, 119901 (2006)] [arXiv:hep-ph/0212229].
- [7] J. Prades, E. de Rafael, A. Vainshtein, Adv. Series on Direction in High Energy Physics, Vol 20, p. 303–318, World Scientific, 2009 [arXiv:0901.0306 [hep-ph]].
- [8] G. Venanzoni, *Frascati Phys. Ser.* **54**, 52 (2012) [arXiv:1203.1501 [hep-ex]].

- [9] N. Saito [J-PARC g-2/EDM Collaboration], *AIP Conf. Proc.* **1467**, 45 (2012).
- [10] I.V. Anikin, A.E. Dorokhov, L. Tomio, *Phys. Part. Nucl.* **31**, 509 (2000) [*Fiz. Elem. Chast. Atom. Yadra* **31**, 1023 (2000)].
- [11] A.E. Dorokhov, W. Broniowski, *Eur. Phys. J.* **C32**, 79 (2003) [arXiv:hep-ph/0305037].
- [12] A.E. Dorokhov, *Phys. Rev.* **D70**, 094011 (2004) [arXiv:hep-ph/0405153].
- [13] A.E. Dorokhov, *Eur. Phys. J.* **C42**, 309 (2005) [arXiv:hep-ph/0505007].
- [14] A.E. Dorokhov, *JETP Lett.* **82**, 1 (2005) [*Pisma Zh. Eksp. Teor. Fiz.* **82**, 3 (2005)] [arXiv:hep-ph/0505196].
- [15] A.E. Dorokhov, *Acta Phys. Pol. B* **36**, 3751 (2005) [arXiv:hep-ph/0510297].
- [16] A.E. Dorokhov, W. Broniowski, *Phys. Rev.* **D78**, 073011 (2008) [arXiv:0805.0760 [hep-ph]].
- [17] A.E. Dorokhov, A.E. Radzhabov, A.S. Zhevlakov, *Eur. Phys. J.* **C71**, 1702 (2011) [arXiv:1103.2042 [hep-ph]].
- [18] A.E. Dorokhov, A.E. Radzhabov, A.S. Zhevlakov, *Eur. Phys. J.* **C72**, 2227 (2012) [arXiv:1204.3729 [hep-ph]].
- [19] E. Bartos *et al.*, *Nucl. Phys.* **B632**, 330 (2002) [arXiv:hep-ph/0106084].
- [20] M. Knecht, A. Nyffeler, *Phys. Rev.* **D65**, 073034 (2002) [arXiv:hep-ph/0111058].
- [21] T. Goecke, C.S. Fischer, R. Williams, *Phys. Rev.* **D83**, 094006 (2011) [*Erratum ibid.* **D86**, 099901 (2012)] [arXiv:1012.3886 [hep-ph]].