SOME APPROACHES TO GPDS IN AdS/QCD*

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(Received November 30, 2012)

We discuss a way to get some generalized parton distributions (GPDs) of quarks using AdS/QCD models. The approach is based on a matching procedure of sum rules relating the electromagnetic form factors to GPDs and AdS modes.

DOI:10.5506/APhysPolBSupp.6.19
PACS numbers: 11.10.Kk, 12.38.Lg, 13.40.Gp, 14.20.Dh

1. Introduction

Light-Front Holography (LFH) [1–3] is a semiclassical approximation to QCD based on the AdS/CFT correspondence that provides a precise mapping of the string modes $\Phi(z)$ in the AdS fifth dimension z to the hadron Light-Front Wave Functions (LFWFs) in physical space-time. This approach has been successfully applied to the description of the mass spectrum of mesons and baryons (reproducing the Regge trajectories), the pion leptonic constant, the electromagnetic form factors of pion and nucleons, *etc.* [4–9]. The mapping that allows to relate AdS modes with LFWF was obtained by matching certain matrix elements (*e.g.* electromagnetic pion form factor) in two approaches — string theory in AdS and Light-Front QCD in the Minkowski space-time.

* Presented by A. Vega at the Light Cone 2012 Conference, Kraków, Poland, July 8–13, 2012.

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The same idea can be used to calculate nucleonic generalized parton distributions (GPDs) [10–13], which are objects that encode important information about the hadronic structure. This is a new example of using the Gauge/Gravity ideas to calculate hadronic properties when QCD cannot be used in a straightforward way.

Here we discuss how to obtain the quark GPDs of the nucleon considering holographical models using a matching procedure similar to the one used in Light-Front Holography (LFH) applications [10, 13].

2. GPDs in AdS/QCD

The sum rules relating the electromagnetic form factors and the GPDs read as [14, 15]

$$F_1^p(t) = \int_0^1 dx\ \left(\tfrac{2}{3} H_v^u(x,t) - \tfrac{1}{3} H_v^d(x,t)\right)\ ,$$

$$F_1^n(t) = \int_0^1 dx\ \left(\tfrac{2}{3} H_v^d(x,t) - \tfrac{1}{3} H_v^u(x,t)\right)\ ,$$

$$F_2^p(t) = \int_0^1 dx\ \left(\tfrac{2}{3} E_v^u(x,t) - \tfrac{1}{3} E_v^d(x,t)\right)\ ,$$

$$F_2^n(t) = \int_0^1 dx\ \left(\tfrac{2}{3} E_v^d(x,t) - \tfrac{1}{3} E_v^u(x,t)\right)\ .$$

We restrict our analysis to the contribution of the u and d quarks and antiquarks, while the presence of the heavier strange and charm quark constituents is not considered.

Abidin and Carlson [16] have calculated the nucleon form factors using a hard wall and a soft wall AdS/QCD model. The AdS metric is specified as

$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right),$$

where $\mu, \nu = 0, 1, 2, 3$; $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski metric tensor and z is the holographical coordinate running from zero to ∞.

The relevant terms in the AdS/QCD action which generate the nucleon form factors are [16]

$$S = \int d^4x\, dz\, \sqrt{g}\, e^{-\Phi(z)} \left(\bar{\Psi}\, e_A^M \Gamma^A\, V_M\, \Psi + \frac{i}{2}\, \eta_{\mathrm{S,V}}\, \bar{\Psi}\, e_A^M\, e_B^N \left[\Gamma^A, \Gamma^B\right] F_{MN}^{(\mathrm{S,V})} \Psi\right),$$

where the basic ingredients of the AdS/QCD model are defined as [16]: $g = |\det g_{MN}|$; $\varPsi$ and V_M are the 5D Dirac and vector fields dual to the nucleon and electromagnetic fields, respectively; $F_{MN} = \partial_M V_N - \partial_N V_M$; $\varGamma^A = (\gamma^\mu, -i\gamma^5)$; $e_A^M = z\delta_A^M$ is the inverse vielbein; and $\eta_{\rm S,V}$ are the couplings constrained by the anomalous magnetic moment of the nucleon: $\eta_p = (\eta_{\rm S} + \eta_{\rm V})/2 = \kappa\, k_p/(2m_N\sqrt{2})$ and $\eta_n = (\eta_{\rm S} - \eta_{\rm V})/2 = \kappa\, k_n/(2m_N\sqrt{2})$. Here the indices S, V denote isoscalar and isovector contributions to the electromagnetic form factors.

Finally, the results for the nucleon form factors in AdS/QCD are given by [16]

$$\begin{aligned} F_1^p\left(Q^2\right) &= C_1\left(Q^2\right) + \eta_p C_2\left(Q^2\right), \qquad & F_2^p\left(Q^2\right) = \eta_p C_3\left(Q^2\right), \\ F_1^n\left(Q^2\right) &= \eta_n C_2\left(Q^2\right), & F_2^n\left(Q^2\right) = \eta_n C_3\left(Q^2\right), \end{aligned}$$

where $Q^2 = -t$ and $C_i(Q^2)$ are the structure integrals

$$\begin{aligned} C_1\left(Q^2\right) &= \int dz e^{-\varPhi}\frac{V(Q,z)}{2z^3}\left(\psi_{\rm L}^2(z) + \psi_{\rm R}^2(z)\right), \\ C_2\left(Q^2\right) &= \int dz e^{-\varPhi}\frac{\partial_z V(Q,z)}{2z^2}\left(\psi_{\rm L}^2(z) - \psi_{\rm R}^2(z)\right), \\ C_3\left(Q^2\right) &= \int dz e^{-\varPhi}\frac{2m_N V(Q,z)}{2z^2}\psi_{\rm L}(z)\psi_{\rm R}(z). \end{aligned}$$

$\psi_{\rm L}(z)$ and $\psi_{\rm R}(z)$ are the Kaluza–Klein modes (normalizable wave functions), which are dual to left- and right-handed nucleon fields and $V(Q,z)$ is the bulk-to-boundary propagator of the vector field in the axial gauge.

According to [10], GPDs calculated in the soft wall case are

$$H_v^q\left(x,Q^2\right) = q(x)\,x^a, \qquad E_v^q\left(x,Q^2\right) = e^q(x)\,x^a,$$

where $a = Q^2/(4\kappa^2)$, $q(x)$ and $e^q(x)$ are given by

$$q(x) = \alpha^q\gamma_1(x) + \beta^q\gamma_2(x), \qquad e^q(x) = \beta^q\gamma_3(x),$$

and the flavor couplings α_q, β_q and functions $\gamma_i(x)$ are written as

$$\alpha^u = 2, \qquad \alpha^d = 1, \qquad \beta^u = 2\eta_p + \eta_n, \qquad \beta^d = \eta_p + 2\eta_n,$$

$$\begin{aligned} \gamma_1(x) &= \tfrac{1}{2}\left(5 - 8x + 3x^2\right), \qquad \gamma_2(x) = 1 - 10x + 21x^2 - 12x^3, \\ \gamma_3(x) &= \frac{6m_N\sqrt{2}}{\kappa}(1-x)^2. \end{aligned}$$

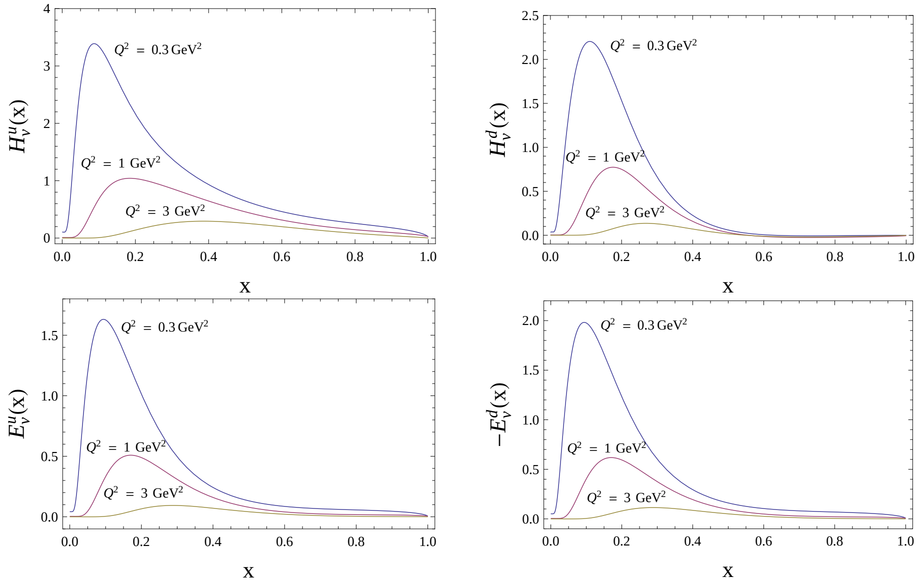

Fig. 1. The helicity-independent generalized parton distributions (GPDs) of quarks for the nucleon in the zero skewness calculated in the holographical hard wall model to different values of Q^2.

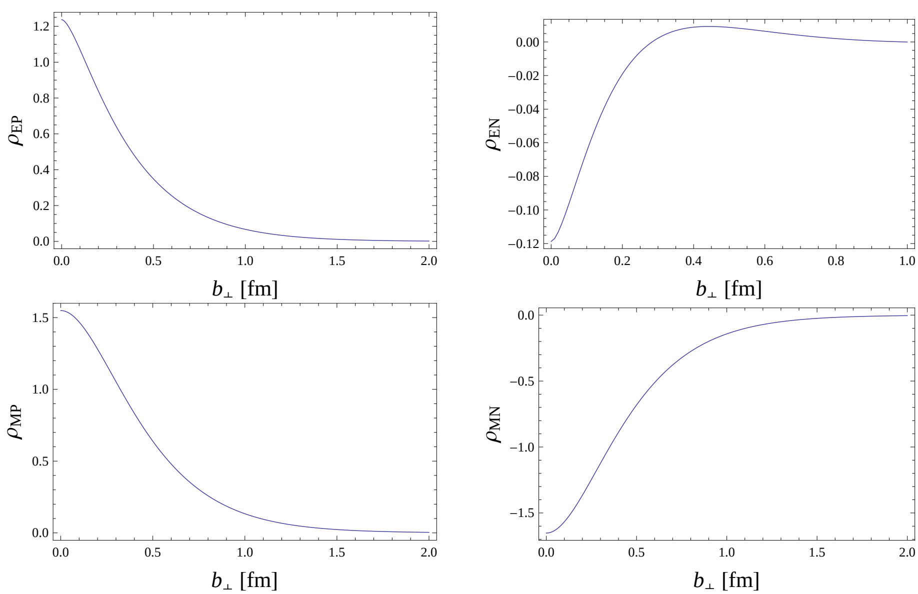

Fig. 2. Parton charge ($\rho_E(b)$) and magnetization ($\rho_M(b)$) densities in the transverse impact space, calculated in the soft wall model.

The parameters involved are the same as those used by Abidin and Carlson [16], *i.e.* $\kappa = 350$ MeV, $\eta_p = 0.224$, $\eta_n = -0.239$, which were fixed in order to reproduce the mass $m_N = 2\kappa\sqrt{2}$ and the anomalous magnetic moments of the nucleon. GPDs calculated in soft wall model are shown in Fig. 1.

Unfortunately, in the hard wall case it is not possible to get analytical results, although numerical calculations can be done without problems using Mathematica. Numerical results for some Q^2 values are shown in Fig. 2.

2.1. Nucleon GPDs in impact space

Another interesting aspect to consider is the nucleon GPDs in impact space. As shown by Burkardt [19], the GPDs in momentum space are related to impact parameter dependent parton distributions via Fourier transform. GPDs in impact space provide access to the distribution of partons in the transverse plane, which is quite important for understanding the nucleon structure. Here we consider just a couple of quantities in impact space.

Following Refs. [19] and [20, 21], we define the following set of nucleon quantities in impact space. The nucleon GPDs in impact space

$$q(x,b) = \int \frac{d^2k}{(2\pi)^2} H_q\left(x,k^2\right) e^{-ibk}\,, \qquad e^q(x,b) = \int \frac{d^2k}{(2\pi)^2} E_q\left(x,k^2\right) e^{-ibk}\,.$$

The parton charge ($\rho_E(b)$) and magnetization ($\rho_M(b)$) densities in transverse impact space are

$$\rho_E(b) = \sum_q e_q \int_0^1 dx q(x,b)\,, \qquad \rho_M(b) = \sum_q e_q \int_0^1 dx e^q(x,b)\,.$$

3. Conclusions

We discussed an alternative way to calculate the nucleon GPDs in both momentum and impact space using ideas of AdS/QCD. LFH and sum rules relating electromagnetic form factors to the GPDs functions $H_v^q(x,Q^2)$ and $E_v^q(x,Q^2)$ have been derived. The procedure used is similar to the one considered in some applications of LFH, where by comparing form factors it is possible to obtain mesonic light-front wave functions. In the present case, it is not necessary to reinterpretate the holographical coordinate z as in standard LFH, where z is the distance between constituent partons.

This work was supported by the FONDECYT (Chile) under Grants No. 3100028 and No. 1100287, by the DFG under Contract No. FA67/31-2 and No. GRK683. This research is also part of the European Community Research Infrastructure Integrating Activity "Study of Strongly Interacting Matter" (HadronPhysics2, Grant Agreement No. 227431), Russian President grant "Scientific Schools" No. 3400.2010.2, Federal Targeted Program "Scientific and Scientific-pedagogical Personnel of Innovative Russia" Contract No. 02.740.11.0238.

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