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PION–PHOTON TRANSITION DISTRIBUTION AMPLITUDES IN NON-LOCAL CHIRAL QUARK MODEL*

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We study the pion–photon transition distribution amplitudes (TDAs) within semibosonized Nambu–Jona-Lasinio model with momentum dependent constituent quark mass. In order to satisfy the Ward–Takahashi identities, we use the non-local currents. We analyse the axial and vector channels and find that our TDAs satisfy polynomiality and the normalization requirement due to the axial anomaly. We calculate the related form factors (for $\pi^{\pm} \to e^{\pm}\nu\gamma$ decay) and find that the value of the axial form factor at zero momentum transfer is shifted towards the experimental value due to the non-locality of the model. We also analyse the pion–photon transition form factor for $\pi^0 \to \gamma^*\gamma$ and compare it with data.

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1. Introduction

One of the most important tools in high energy physics are factorization theorems, *i.e.* a systematic way of separating perturbatively calculable part from well-defined non-perturbative matrix elements. The most tested is the collinear factorization, where the corresponding non-perturbative matrix elements sandwich non-local quark and gluon operators on the light-cone. Besides the most common application in inclusive and semi-inclusive processes, there are certain theorems allowing for collinear factorization also in exclusive processes [1]. The most prominent is perhaps deeply virtual Compton scattering (DVCS) with corresponding non-perturbative part encoded in so-called generalized parton distributions (GPDs) (for a review see *e.g.* [2]).

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In a DVCS process $\gamma^* H \to H\gamma$ (Fig. 1), the highly virtual photon scatters off the hadron H and the final state hadron flies approximately in the forward direction in the CM frame of the incoming particles. Thus, the momentum transfer to the hadronic matrix element is small, what actually allows for the factorization to occur. There are certain generalizations, *e.g.* to hard pseudoscalar meson electroproduction [3], which uses GPDs and distribution amplitudes (DAs) as non-perturbative objects. One can, however, still generalize the collinear factorization by introducing so-called transition distribution amplitudes (TDAs): whereas GPDs are defined via matrix elements non-diagonal in hadron momenta, TDAs are also non-diagonal in hadronic states. Thus, one can basically consider baryon–meson [4] and hadron–photon [5] TDAs. In the right part of Fig. 1, we show an example of a process with baryon–photon TDA. Note, that it is the same process as in the left figure, however in a different kinematic regime — in so-called 'backward kinematics', *i.e.* when the final state photon flies approximately in the same direction as the initial state hadron.

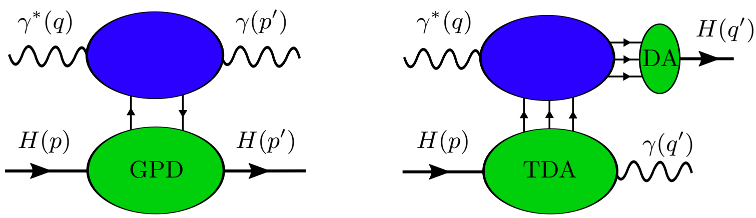

Fig. 1. Factorization of the process $\gamma^* H \to H\gamma$ in two different kinematic regimes: left corresponds to the 'forward kinematics', while the right to the 'backward kinematics'. The dark grey (blue) blobs denote pertubatively calculable parts.

So far, little is known experimentally about GPDs, notably TDAs (see Ref. [6] for a review). The situation is somewhat better on the theory side, especially if processes with simple hadronic states are considered. In particular, probably the most graceful states are pions being the Goldstone bosons of spontaneously broken chiral symmetry and in the same time quark–antiquark bound states. Their low energy behaviour can be described by various effective models, one of which shall be discussed in Sec. 2. Next, in Sec. 3, we will define pion–photon TDAs and give our main results.

2. Non-local chiral quark model from instanton vacuum

At low energy scales (of the order of a few hundred MeV) the relevant degrees of freedom are pions and constituent quarks. The characteristic feature of the last is that they posses relatively large 'constituent' mass $M_0 \approx$ 350 MeV. This mass is generated dynamically due to the spontaneous chiral symmetry breaking and is, in general, not a constant, *i.e.* for a momentum of a quark k, we have $M(k) = M_0 F^2(k)$, with $F(0) = 1$ and $F(k \to \infty) = 0$. The function $F(k)$ was derived within instanton theory of QCD vacuum [7] and turned out to have highly non-trivial form given in Euclidean space. The interaction part of the effective action for quarks and pions reads

$$S_{\rm int} = M_0 \int \frac{d^4k\, d^4l}{(2\pi)^8} \bar{\psi}(k) F(k)\, U^{\gamma_5}(k-l) F(l)\, \psi(l)\,, \tag{1}$$

with chiral field $U^{\gamma_5}(x) = \exp(\frac{i}{F_\pi}\tau^a \cdot \pi^a(x)\gamma_5)$, where π^a is the triplet of pion fields, τ^a are Pauli matrices and $F_\pi \approx 93\,\mathrm{MeV}$ is pion decay constant. Here, we consider the chiral limit, *i.e.* the current quark mass is set to zero.

Instead of using the original complicated formula for $F(k)$, we use the following simple ansatz [8],

$$F(k) = \left(\frac{-\Lambda_n^2}{k^2 - \Lambda_n^2 + i\epsilon}\right)^n \tag{2}$$

which can be used in Euclidean as well as in the Minkowski space, provided a special prescription for dealing with multiple poles is used. Our formula possesses an important feature, namely it depends not only on the 'cut-off' parameter Λ_n, but also on the exponent n which dictates the shape of the regulator (thus, the influence of a fall-off behaviour can be studied). The parameter Λ_n is adjusted for given n in such a way that the experimental value of the pion decay constant is recovered. Using the Birse–Bowler formula [9], we obtain in our model [10]

$$\begin{aligned} F_\pi^2 \;=\; -\frac{N_c\, M_0^2}{4\pi^2} \sum_{i,j=1}^{4n+1} f_i f_j \eta_i^{2n} \big(&(1 + 2n\,(1+2n))\, \eta_j^{2n+1} + (1{+}4n\,(1{+}3n))\, \eta_j^{2n} \\ &+2n\,(1+6n)\, \eta_j^{2n-1} + 4n^2 \eta_j^{2n-2}\big) \left(\frac{\epsilon_{ij}}{\eta_i - \eta_j} \ln\frac{1+\eta_i}{1+\eta_j} + \frac{\delta_{ij}}{1+\eta_i}\right) \end{aligned} \tag{3}$$

with ϵ_{ij} being Levi–Civita symbol and δ_{ij} being Kronecker delta. The complex numbers η_i are the roots of $G(z) = z^{4n+1} + z^{4n} - (M_0/\Lambda_n)^2$. Fixing $F_\pi = 93\,\mathrm{MeV}$, we get *e.g.* for $n = 1$ the value $\Lambda_1 = 836\,\mathrm{MeV}$.

Although the momentum dependent quark mass seems to be very physical regulator, it generates a problem. Namely, the standard vector and axial currents are not conserved. This generates a set of problems, including the normalization of certain distributions. In order to avoid them, the currents have to be modified, *i.e.* new additional non-local pieces have to be added. In our work, we have used the modification proposed in Ref. [11].

The model just presented has been used to calculate pion DA [8], photon DAs [10, 12] up to twist-4, chiral condensates [13], pion generalized DAs [14] and pion–photon TDAs [15, 16]. All the DAs were calculated analytically up to the solution of the equation $G(z) = 0$.

3. Results

There are two TDAs of interest: the vector one (VTDA) which we denote V and the axial TDA (ATDA), A. They are defined as [5]

$$\int \frac{d\lambda}{2\pi} e^{2i\lambda X P^+} \left\langle \gamma(p',\varepsilon) \right| \bar{d}(-\lambda n)\gamma^\mu u(\lambda n) \left| \pi^+(p) \right\rangle = \frac{-e}{4\sqrt{2}F_\pi P^+}\, \epsilon^{\mu\nu\alpha\beta} \varepsilon^*_\nu p_\alpha p'_\beta\, V\left(X,\xi,t\right)\,, \tag{4}$$

$$\int \frac{d\lambda}{2\pi} e^{2i\lambda X P^+} \left\langle \gamma(p',\varepsilon) \right| \bar{d}(-\lambda n)\gamma^\mu \gamma_5 u(\lambda n) \left| \pi^+(p) \right\rangle = \frac{ie}{4\sqrt{2}F_\pi P^+}\, p'^\mu p\cdot\varepsilon^*\, A\left(X,\xi,t\right) + \ldots\,, \tag{5}$$

where u, d are up, down quark fields, e is the electric charge, ε is the photon polarization vector. The light-like vector $n = (1, 0, 0, -1)$ defines a 'plus' component of any vector v, *i.e.* $v^+ = v \cdot n$. Moreover, we have defined $P^+ = \left(p^+ + p'^+\right)/2$, the momentum transfer $t = \left(p' - p\right)^2$ and so-called skewedness $\xi = \left(p'^+ - p^+\right)/2P^+$. The dots on the r.h.s. in the definition of ATDA denote the structure proportional to pion DA.

Using the model described in the previous section, we have evaluated the above matrix elements to one loop accuracy. In order to recover the correct normalization for VTDA required by the axial anomaly, $\int_{-1}^{1} dX\, V\left(X,\xi,0\right) = 1/2\pi^2$, we had to use the non-local currents not only for quark-photon vertex, but also for the bilocal vertices. In the same time, the normalization of ATDA is substantially decreased (Fig. 2). It has observable consequences: VTDA and ATDA are related via sum rules to the form factors $F_{\rm V}$, $F_{\rm A}$ for the process $\pi^\pm \to e^\pm \nu\gamma$ (Table I). At zero momentum transfers the experimental values are [17] $F_{\rm V}^{\rm exp}\left(0\right) = 0.0254 \pm 0.0017$, $F_{\rm A}^{\rm exp}\left(0\right) = 0.0119 \pm 0.0001$. Thus the ratio is approximately one-half. In the local models (*i.e.* where the quark mass does not depend on the momentum), this ratio is always one [18, 19]. Thus the conclusion is that the non-local interactions are important at low energies, if the physical results are to be recovered (see also [20]).

There is also another interesting measurable quantity, namely pion–photon transition form factor $F_{\pi\gamma}$, directly related to $F_{\rm V}$. We plot the results in Fig. 3 together with the data [21–24]. Recently, there have been debates concerning its high-Q^2 behaviour, see *e.g.* [25, 26]. Here we, however, concentrate mainly on low-Q^2 regime, where our model should apply.

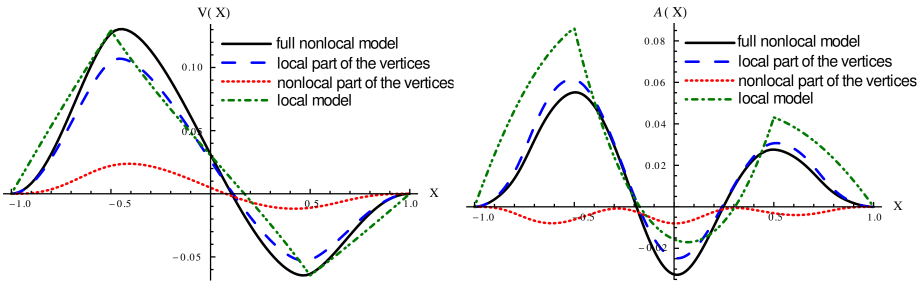

Fig. 2. Some of the model results (black solid) for VTDA (left) and ATDA (right) for $t = -0.1\,\mathrm{GeV}^2$ and $\xi = 0.5$. For VTDA the correct normalization (equal to the one of the local quark model (dash-dotted/green) is recovered thanks to the non-local part of the vertices (dotted/red). For ATDA this contribution is negative and substantially decreases the normalization.

TABLE I

The results for axial form factor at zero momentum transfer.

M_0 [MeV]	n	$F_{\mathrm{A}}(0)$	$F_{\mathrm{A}}(0)/F_{\mathrm{V}}(0)$
225	1	0.0217	0.80
350	1	0.0168	0.62
350	5	0.0163	0.60
400	1	0.0161	0.60
400	5	0.0152	0.56
Local models		0.0272	1.00

It is, of course, interesting to look also at the results for large Q^2 (right of Fig. 3). We observe that in that regime our curves saturate at a value dependent on the model parameters, thus the model is consistent with Belle data, rather than BaBar.

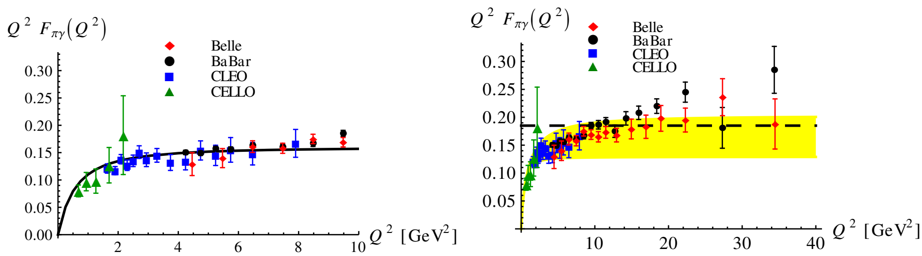

Fig. 3. The pion–photon transitions form factor. Left: low-Q^2 regime for $M_0 = 300\,\mathrm{MeV}$ and $n = 1$ (solid). Right: behaviour for high-Q^2 (shaded/yellow). The dashed line is the asymptotic QCD prediction.

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REFERENCES

[1] G.P. Lepage, S.J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980).
[2] A. Belitsky, A. Radyushkin, *Phys. Rep.* **418**, 1 (2005).
[3] J.C. Collins, L. Frankfurt, M. Strikman, *Phys. Rev.* **D56**, 2982 (1997).
[4] L. Frankfurt, P. Pobylitsa, M.V. Polyakov, M. Strikman, *Phys. Rev.* **D60**, 014010 (1999).
[5] B. Pire, L. Szymanowski, *Phys. Rev.* **D71**, 111501 (2005).
[6] B. Pire, K. Semenov-Tian-Shansky, L. Szymanowski, J. Wagner, `arXiv:1109.0179 [hep-ph]`.
[7] D. Diakonov, V.Y. Petrov, *Nucl. Phys.* **B272**, 457 (1986).
[8] M. Praszalowicz, A. Rostworowski, *Phys. Rev.* **D64**, 074003 (2001).
[9] R. Bowler, M. Birse, *Nucl. Phys.* **A582**, 655 (1995).
[10] P. Kotko, M. Praszalowicz, *Phys. Rev.* **D81**, 034019 (2010).
[11] B. Holdom, R. Lewis, *Phys. Rev.* **D51**, 6318 (1995).
[12] P. Kotko, *Acta Phys. Pol. B Proc. Suppl.* **2**, 509 (2009).
[13] M. Praszalowicz, A. Rostworowski, *Phys. Rev.* **D66**, 054002 (2002).
[14] M. Praszalowicz, A. Rostworowski, *Acta Phys. Pol. B* **34**, 2699 (2003).
[15] P. Kotko, M. Praszalowicz, *Phys. Rev.* **D80**, 074002 (2009).
[16] P. Kotko, M. Praszalowicz, *Acta Phys. Pol. B* **40**, 123 (2009).
[17] J. Beringer *et al.*, *Phys. Rev.* **D86**, 010001 (2012).
[18] A. Courtoy, S. Noguera, *Phys. Rev.* **D76**, 094026 (2007).
[19] W. Broniowski, E.R. Arriola, *Phys. Lett.* **B649**, 49 (2007).
[20] D. Gomez Dumm, S. Noguera, N. Scoccola, *Phys. Lett.* **B698**, 236 (2011).
[21] H. Behrend *et al.*, *Z. Phys.* **C49**, 401 (1991).
[22] J. Gronberg *et al.*, *Phys. Rev.* **D57**, 33 (1998).
[23] B. Aubert *et al.*, *Phys. Rev.* **D80**, 052002 (2009).
[24] S. Uehara *et al.*, *Phys. Rev.* **D86**, 092007 (2012) [`arXiv:1205.3249 [hep-ex]`].
[25] N. Stefanis, A.P. Bakulev, S. Mikhailov, A. Pimikov, *Nucl. Phys. Proc. Suppl.* **225–227**, 146 (2012).
[26] A. Bakulev, S. Mikhailov, A. Pimikov, N. Stefanis, *Phys.Rev.* **D86**, 031501 (2012).