# OPTIMIZING POLAR ANGLE ASYMMETRY OBSERVABLES AT COLLIDERS*

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Angular asymmetries are simple, intuitive, model-independent observables used to identify spins of new elementary particles. In the case of Drell–Yan-like boson resonances, we generalize the well-known center-edge angular asymmetry to optimize spin identification when only a limited sample of events is available. By choosing simple weight functions $W(\theta)$ in integrals over the polar angle $\theta$, such as $W = \cos^n \theta$, we can improve spin discrimination significantly in production and decays of spin-0, spin-1, and spin-2 bosons. The power $n$ can be tuned in particular cases, but $n = 2$ ($n = 1$) works well for any forward–backward symmetric (non-symmetric) decay to massless particles.

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New particles, such as Higgs, $Z'$, excited gravitons, sparticles, *etc.*, may well be discovered in collider data from $p\bar{p}$ (Tevatron), $pp$ (LHC), or $e^+e^-$ (ILC) scattering. The spin of a new virtual particle is often determined in the experiment from the analysis of angular distributions in the heavy particle's decay. This analysis follows a standard quantum-mechanical expansion over spherical harmonics that depends on the spin of the massive particle state. In this contribution, we describe an analysis procedure for decay angular distributions that is more efficient in using limited event statistics than the commonly adopted method. One way to determine the spin is to exploit the "center-edge" asymmetry that typically exists between forward and transverse decay products from a boson [1, 2]. We show how to modify the usual "central-edge" asymmetry definition to increase its discriminating power for the spin. We keep the discussion transparent by working at the parton level, but consider a simple analytic model for typical experimental acceptance. We checked that our main conclusions remain valid in a more

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realistic calculation, by comparing our method to a fully differential calculation using ResBos [3] that includes proton PDFs, NLO corrections, and NNLL resummations in QCD.

For decay of a boson into two back-to-back massless particles, we define a general asymmetry by an integral

$$A = \int_{-1}^{1} W(z)\, P(z)\, dz\,. \tag{1}$$

Here $z = \cos\theta$, $\theta$ is the decay polar angle in the boson center-of-mass frame, $P(z) = \frac{1}{\sigma}\frac{d\sigma}{dz}$ is the normalized production probability density, and $W(z)$ is a weight function. One popular choice for $W(z)$ has been the center-edge step function [1, 2, 4–8]

$$W_{\rm CE}(z) = \ +1 \ \ (|z| > z^*); \quad -1 \ \ (|z| < z^*)\,, \tag{2}$$

with $z^* \sim 0.5$. We will propose an alternative definition for $W(z)$ that is more optimal than $W_{\rm CE}(z)$.

To discriminate between two possible boson spins $a$ and $b$ in a measurement, we introduce respective probability densities $P_a(z)$ and $P_b(z)$ and their asymmetries $A_a$ and $A_b$, and construct the ratio

$$R_{ab} = \frac{A_b - A_a}{\delta A_a + \delta A_b}\,. \tag{3}$$

The expected statistical uncertainty is estimated by

$$\delta A = \sqrt{\int P(z)\, W(z)^2\, dz - \left(\int P(z)\, W(z)\, dz\right)^2}\,. \tag{4}$$

$R_{ab}$ is independent of additive and multiplicative constants in $W(z)$. A larger $R_{ab}$ value indicates that $A$ with the chosen weight is more sensitive to spin. We wish to choose $W(z)$ to maximize $R_{ab}$, but we do not want to fine-tune $W(z)$ for each particular process/model/experiment. We will show for typical cross-sections of interest that the simple choice $W(z) = z^n$ works well. Although $n$ may be tuned, $n = 2$ ($n = 1$) is largely sufficient for distributions that are forward–backward (non)symmetric under $z \to -z$.

A choice of $W(z)$ that amplifies in the range of $z$, where the difference $\Delta P(z) = P_b - P_a$ is large in magnitude can increase the numerator in Eq. (3). (Note that $\Delta P$ is normalized to zero, so it must change sign.) However, amplifying this range of $z$ will unbalance the cancellation of statistical fluctuations across the whole $-1 \le z \le 1$ range and increase the denominator in Eq. (3), which must happen since $\Delta P$ is normalized. On the other hand, a uniform choice, such as $W_{\rm CE}$ in Eq. (2), gives equal weight to the whole

$z$ range, reducing the denominator through cancellation of fluctuations but not optimizing the numerator. It appears there is a minimization problem to solve.

Let us consider a concrete example. We compare the lowest order QCD densities $P_0$ and $P_1$ for spin-0 and spin-1 bosons produced by massless partons and decaying to massless fermion pairs

$$P_0 = \tfrac{1}{2}\,, \qquad P_1 = \tfrac{3}{8}\left(1 + z^2 + 2\frac{c-d}{c+d}z\right)\,.$$

The term $\frac{c-d}{c+d}$ arises for general boson–fermion chiral couplings, *e.g.*, it is non-zero in a parity-violating decay. Consider first the symmetric case $c = d$, so that $P_1 = (3/8)\cdot(1+z^2)$. We try a class of weight functions

$$W(z) = |z|^n\,, \tag{5}$$

with $n$ selected to emphasize contributions from the $z$ intervals with large $\Delta P(z)$. Figure 1 confirms the existence of an optimal weight that maximizes $R_{01}$. Recalling that confidence limits are determined by $R\sqrt{N}$ in the case of $N$ sample events, $W_{\rm CE}$ would require about $1/3$ more events to achieve the same level of significance as the optimal weight — see Table I. A similar conclusion holds for parity violating processes ($c \neq d$), with the results for the maximal symmetry violation $d = 0$ shown in Table I. (In this case, we modified the center-edge weights as $W(|z|) \to \mathrm{sign}(z)\,W(|z|)$.) Provided there is a significant monotonic variation of $\Delta P$ from the center to the edge of the range of $z$, the optimum weight will be close to quadratic for $c = d$ and linear for $d = 0$.

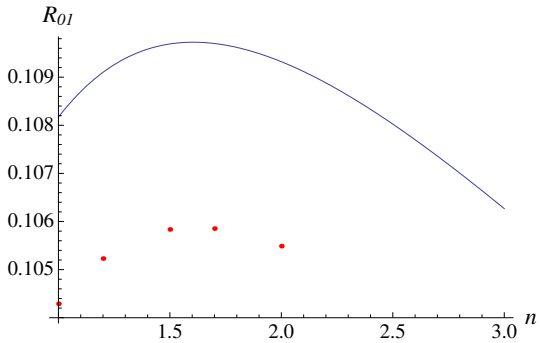



Fig. 1. The solid line is the statistical significance ratio $R_{01}$ for the particular case of spin-0 and spin-1 bosons decaying to forward–backward symmetric fermions. $R_{01}$ is plotted as a function of $n$ in the weight functions $W = |z|^n$. The points are obtained for discrete choices of $n$ after modulating the leading-order densities $P_{a,b}$ by a function (7) simulating experimental acceptance, for $\alpha = 6$ and $\beta = 0.2$.

TABLE I

The $R_{ab}$ ratio for a boson $B$ decaying to difermions $f\bar{f}$ or diphotons $\gamma\gamma$, computed for various spins $a$ and $b$, using $W_{\mathrm{CE}}$ and $W = |z|^n$ weights with the optimal $n$ value. Also shown is the percentage increase in the number of events that would be needed when using $W_{\mathrm{CE}}$ to achieve the same statistical significance as with the optimal $W = |z|^n$.

| $B \to f\bar{f}$ | | $R_{ab}$ with $W_{\mathrm{CE}}$ | Optimal $n$, $R_{ab}(n)$ | % more events with $W_{\mathrm{CE}}$ |
|---|---|---|---|---|
| $a$ | $b$ | | | |
| 0 | 1 | 0.095 | 1.6, 0.1097 | 34% |
| 0 | 2* | 0.106 | 4, 0.155 | 113% |
| 1 | 2* | 0.205 | 3, 0.264 | 66% |
| 0 | 1** | 0.45 | 0.8, 0.52 | 33% |
| $B \to \gamma\gamma$ | | | | |
| 0 | 2* | 0.239 | 1.3, 0.272 | 30% |

*($\epsilon_q = 0.1$) and **($d = 0$).

These conclusions stay valid when the above lowest-order numerical estimates for $R_{ab}$ are modified by higher-order QCD corrections to $P(z)$, smearing by parton distributions in the case of hadronic collisions, and detector acceptance constraints. For example, the angular dependence of $P(z)$ can be modified by the limits $y_{\min} < y < y_{\max}(z)$ on the boson rapidity. We can explore the impact of these corrections by multiplicatively modulating the lowest-order parton-level $z$-dependence of the differential cross section by a function

$$m(z) = \int_{M-\Delta M}^{M+\Delta M} dM \int_{-\infty}^{\infty} dy\ \Theta(y_{\max} - y) \times \Theta(y - y_{\min}) K(y, M) \tag{6}$$

with a boson mass $M$, $\Delta M$ bin size around the resonance peak, step functions $\Theta(y)$ indicating acceptance constraints imposed on the rapidity, and a function $K(y, M)$ containing all details of QCD corrections, parton distributions, boson propagator, *etc.* For a typical $z$ distribution, $m(z)$ can be modeled by

$$m(z) \propto (1 - z^{\alpha})^{\beta}, \tag{7}$$

where $\alpha$ and $\beta$ depend on details of the calculation. Figure 2 compares representative angular distributions for fermionic decays of massive bosons obtained with ResBos for a particular parameter set, which realistically includes the effect of PDFs, QCD NLO and NLL corrections, and acceptance cuts, against calculations using the modulation model (7). We see that the $m(z)$ function with $\alpha$ and $\beta$ fitted to the ResBos points captures the general features of the ResBos predictions.

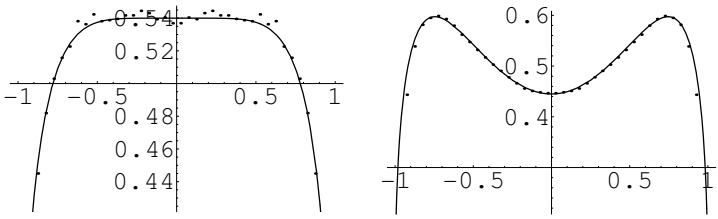



Fig. 2. Realistic $P(z)$ for spin-0 (left graph) and spin-1 (right graphs) bosons decaying to massless fermions. The data points were generated using ResBos [3] for boson mass $M = 1$ TeV produced in $pp$ collisions at $\sqrt{s} = 14$ TeV. This includes NLO and NLL QCD corrections, CTEQ 6.6 PDFs [9], and a $p_{\mathrm{T}} > 25$ GeV cut on the fermion's transverse momentum. The solid lines represent a fit using a modulated $P(z)$ with $m(z)$ from Eq. (7).

For our purposes of comparing the statistical performance of different polar weights, the precise values of the powers in (7) are not found to be crucial. The effect of this modulation on the polar asymmetry is also shown in Fig. 1. Although the statistical significance is reduced generally, as might be expected because the acceptance reduces the center-edge asymmetry, the conclusions about the optimum weight are essentially unchanged.

To demonstrate limitations of the above conclusions in relation to the overall amount of center-edge symmetry, we may look at the decay distribution of a spin-2 boson to fermions

$$P_2 = \tfrac{5}{8}\left(1 - 3z^2 + 4z^4\right)\epsilon_q + \tfrac{5}{8}\left(1 - z^4\right)\epsilon_g\,. \tag{8}$$

This depends, in addition, on the fractions of Drell–Yan events $\epsilon_q$ and $\epsilon_g$ produced via $q\bar{q}$ and gluon–gluon fusion respectively. Under the constraint $\epsilon_q + \epsilon_g = 1$ at a $pp$ collider, $\epsilon_q$ typically varies from $\epsilon_q \sim 0$ at very low boson masses $\ll 1$ TeV to $\epsilon_q \sim \frac{1}{2}$ at 4 TeV. We find that the statistical significance of an optimized power law $W = |z|^n$ is generally much better than $W_{\mathrm{CE}}$ across this range (see for example Table I). However, the advantage reduces as $\epsilon_q$ grows. This is because at large $\epsilon_q$ there is no longer any center-edge asymmetry at all and center-edge type observables are generally less useful. The optimal power is, of course, dependent on $\epsilon_q$, but a simple universal quadratic choice $W = z^2$ works very nearly as well.

Another important decay mode for identifying boson resonances is to two prompt photons, which are relatively clean to identify. The corresponding tree level probability densities in this case are

$$P_0 = \tfrac{1}{2}\,, \qquad P_1 = 0\,, \qquad P_2 = \tfrac{5}{32}\left(1 + 6z^2 + z^4\right)\epsilon_g + \tfrac{5}{8}\left(1 - z^4\right)\epsilon_q\,. \tag{9}$$

The statistical significance of the optimized power law and $W_{\rm CE}$ choices for spin-2 *versus* spin-0 with acceptance modeled by (7) follows a pattern similar to the di-fermion final state (see Table I).

Aside from the simple powers, we have also investigated other possible angular weights, particularly for those that strongly enhance the regions where $\Delta P$ is large, and find they are not superior in statistical significance. Choosing a weight function that was orthogonal to one of the two raw probability distributions used in any comparison did not produce better results for the $R_{ab}$ ratio when the experimental acceptance was taken into account; part of the reason is that the latter destroys orthogonality. To conclude, the sensitivity of the $z = \cos\theta$ distribution to the spin of the decaying heavy boson is increased by generalizing the center-edge asymmetry via Eq. (1) with the weight function $W(z) = z^n$ and $n = 2$ for a forward–backward symmetric ($n = 1$ for a forward–backward non-symmetric) distribution. These findings are best noticed in a leading-order analysis of the decay angle dependence, but they are robust against radiative and acceptance corrections arising in the full NLO differential calculation.

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