ANOMALOUS QUARK–GLUON CHROMOMAGNETIC INTERACTION AND HELICITY AMPLITUDES OF HIGH ENERGY ρ-MESON ELECTROPRODUCTION*

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In this article, we present some results of our investigation of the influence of instanton induced anomalous quark chromomagnetic moment on spin properties of electroproduced ρ-meson. The full set of helicity amplitudes has been calculated. It is shown that the existence of a large anomalous chromomagnetic moment of quark gives a tiny contribution to spin-flip amplitudes, but it is important for non-spin-flip amplitudes.

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1. Introduction

It is well known that the perturbative high-energy QCD conserves helicity of quarks in s-channel. But the QCD vacuum is full of fluctuations with non-trivial topological structure — instantons (for the reviews [1, 2]). This topological fluctuations generate the celebrated multiquark t' Hooft interaction [3]. Moreover, it was shown that instantons generate a non-trivial spin-flip, s-channel helicity non-conserving, *quark–gluon interaction* [4]. This interaction can be described in terms of an anomalous chromomagnetic moment of quarks (ACMQ) complementary to the perturbative Dirac coupling. A magnitude of the ACMQ can be related to the instanton density in the QCD vacuum.

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The exclusive vector meson electroproduction is a good lab for testing the s-channel helicity properties of the quark–gluon coupling. Previously, it was found that, s-channel helicity non-conserving transitions from photons to vector mesons are possible even within the perturbative QCD, because a helicity of mesons can be different from a sum of the quark and anti-quark helicities (see [6]). The ACMQ would introduce an extra contribution to both the s-channel helicity conserving and non-conserving vector meson production amplitudes.

2. Anomalous chromomagnetic moment of quarks

In the most general case, the interaction vertex of massive quark with gluon can be written in the following form

$$V_\mu\left(k_1^2,k_2^2,\kappa^2\right)t^a=-g_s t^a\left[F_1\left(k_1^2,k_2^2,\kappa^2\right)\gamma_\mu+\frac{\sigma_{\mu\nu}\kappa_\nu}{2m}F_2\left(k_1^2,k_2^2,\kappa^2\right)\right], \quad (1)$$

where the first term is a conventional perturbative QCD quark–gluon vertex and the second term in our case comes from the non-perturbative sector of QCD. $k_{1,2}$ are the momenta of incoming and outgoing quarks, respectively, $\kappa=k_2-k_1$, m is the quark mass. We focus on effects of the novel color chromomagnetic vertex and keep $F_1(k_1^2,k_2^2,\kappa^2)=1$. The anomalous quark chromomagnetic moment is $\mu_a=F_2(0,0,0)$.

In an earlier paper [4], it was shown that instantons generate an ACMQ which is proportional to the instanton density. In terms of the average size of instantons ρ_c and the dynamical quark mass m in non-perturbative QCD vacuum, one finds [5]

$$\mu_a=-\frac{3\pi(m\rho_c)^2}{4\alpha_{\rm s}(\rho_c)} \quad (2)$$

which exhibits a strong sensitivity of the ACMQ to a dynamical mass of quarks. We emphasize an implicit assumptions that quarks are light, the above estimates of ACQM are valid for u,d,s, while for heavy quarks ACMQ vanishes. For the average instanton size $\rho_c^{-1}=0.6\,\mathrm{GeV}$, the resulting ACMQ for light quarks is numerically quite large. We use $\mu_a=-1$.

The form of F_2 explicitly comes from the instanton model

$$F_2\left(k_1^2,k_2^2,\kappa^2\right)=\mu_a\Phi_q(\mid k_1\mid\rho/2)\Phi_q(\mid k_2\mid\rho/2)F_g(\mid\kappa\mid\rho)\,, \quad (3)$$

where

$$\begin{aligned}\Phi_q(z) &= -z\frac{d}{dz}\left(I_0(z)K_0(z)-I_1(z)K_1(z)\right),\\ F_g(z) &= \frac{4}{z^2}-2K_2(z)\,.\end{aligned}$$

Here, $I_\nu(z)$, $K_\nu(z)$ are the modified Bessel functions and ρ is the instanton size.

3. Calculation of amplitudes

Hereafter, we follow the k_t-factorization analysis developed in [8–10]. In frame of BFKL considerations, a high energy diffractive production of vector meson is usually described by an exchange of a color-singlet two-gluon tower in the t-channel. The corresponding diagrams are presented in Fig. 1.

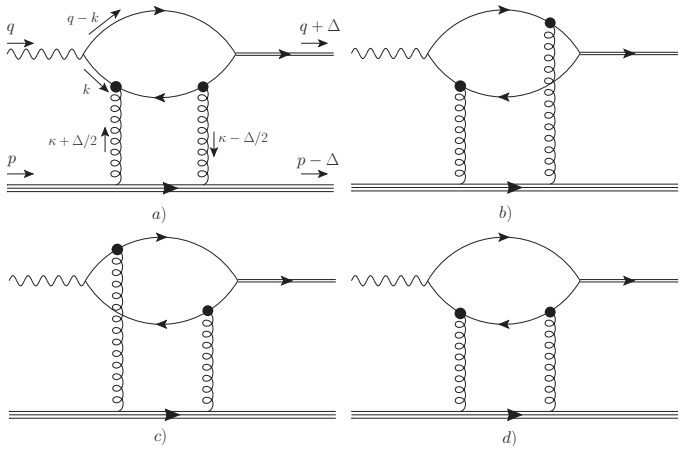


Fig. 1. The diagrams which contribute to high energy exclusive vector meson electroproduction off proton by two gluon exchange. Here, the blob stands for the generalized quark–gluon vertex Eq. (1).

The imaginary part of the amplitude takes the form

$$
\begin{aligned}
&A\left(x,Q^2,\vec{\Delta}\right)\\
&=-is\frac{c_{\rm V}\sqrt{4\pi\alpha_{\rm em}}}{4\pi^2}\int\frac{dz}{z(1-z)}\int d^2\vec{k}\psi\left(z,\vec{k}\right)\int\frac{d^2\vec{\kappa}}{\vec{\kappa}^4}\alpha_{\rm s}\mathcal{F}\left(x,\vec{\kappa},\vec{\Delta}\right)\\
&\times\left[\frac{1-z}{z}\frac{I^{(a)}}{\vec{k}_{1a}^2+m^2+z(1-z)Q^2}+\frac{I^{(b)}}{\vec{k}_{1b}^2+m^2+z(1-z)Q^2}\right.\\
&\left.+\frac{I^{(c)}}{\vec{k}_{1c}^2+m^2+z(1-z)Q^2}+\frac{z}{1-z}\frac{I^{(d)}}{\vec{k}_{1d}^2+m^2+z(1-z)Q^2}\right]. \qquad (4)
\end{aligned}
$$

Here, $\alpha_{\rm em}$ is the fine-structure constant, $c_V=1/\sqrt{2}$ is coming from the flavor part of the ρ-meson wave function, $\mathcal{F}(x,\vec{\kappa},\vec{\Delta})$ is the differential gluon density. $\psi(z,\vec{k})$ is the light-cone wave function of the ρ-meson, for which we use a simple parameterization

$$\psi=c\exp\left(-\frac{a^2}{2}\left(\vec{k}^2+\frac{1}{4}(2z-1)^2M^2\right)\right). \qquad (5)$$

Two constants a and c were fixed by the normalization of wave function and decay width $\Gamma(\rho \to e^+e^-)$. We find $a = 3.927\,\mathrm{GeV}^{-1}$, $c = 17.44$. M is the mass of $\bar{q}q$ pair on mass-shell

$$M^2 = \frac{\vec{k}^2 + m^2}{z(1-z)}\,, \tag{6}$$

where $m = 220\,\mathrm{MeV}$ is the quark mass. For the QCD running coupling, we use

$$\alpha_{\rm s}\left(q^2\right) = \frac{4\pi}{9\ln\left(\left(q^2+m_g^2\right)/\Lambda^2_{\rm QCD}\right)}\,, \tag{7}$$

where $\Lambda_{\rm QCD} = 0.28\,\mathrm{GeV}$ and the value $m_g = 0.88\,\mathrm{GeV}$ imposes the infrared freezing at $\alpha_{\rm s}(1/\rho_c^2) \approx \pi/6$ [2].

$I^{(a,b,c,d)}$ is the trace over quark line in corresponding (a), (b), (c), (d) diagram from Fig. 1 divided by $2s^2$. It involves three parts: perturbative, chromomagnetic and interference between them

$$I^{(i)} = I^{(i)}_{\rm p} + I^{(i)}_{\rm cm} + I^{(i)}_{\rm mix}\,. \tag{8}$$

Below we are using the following notation for the transverse momentum of quark

$$\begin{aligned}
\vec{k}_{1a} &= \vec{k} - (1-z)\vec{\Delta}\,,\\
\vec{k}_{1b} &= \vec{k} - (1-z)\vec{\Delta} + \vec{\kappa} + \tfrac{1}{2}\vec{\Delta}\,,\\
\vec{k}_{1c} &= \vec{k} - (1-z)\vec{\Delta} - \vec{\kappa} + \tfrac{1}{2}\vec{\Delta}\,,\\
\vec{k}_{1d} &= \vec{k} + z\vec{\Delta}\,.
\end{aligned} \tag{9}$$

Besides, $[\vec{a}\vec{b}] = a_x b_y - a_y b_x$.

Let us start from simplest case — LL transition. We denote $I(\lambda_\gamma\lambda_{\rm V})$, k_I^2 and k_{II}^2 are the average virtualities of off-shell quarks in photon and meson vertices.

$$\begin{aligned}
I^{(c)}_{\rm p}({\rm LL}) &= -4QMz^2(1-z)^2\,,\\
I^{(c)}_{\rm cm}({\rm LL}) &= -4QMz^2(1-z)^2\vec{\kappa}^2 F_2\left(k_I^2,\kappa^2\right) F_2\left(k_{II}^2,\kappa^2\right)\,,\\
I^{(a)}_{\rm mix}({\rm LL}) &= I^{(b)}_{\rm mix}({\rm LL}) = I^{(c)}_{\rm mix}({\rm LL}) = I^{(d)}_{\rm mix}({\rm LL}) = 0\,.
\end{aligned} \tag{10}$$

For TL transition

$$\begin{aligned}
I^{(c)}_{\rm p}({\rm TL}) &= 2z(1-z)(1-2z)M\left(\vec{e}\vec{k}_{1c}\right)\,,\\
I^{(c)}_{\rm cm}({\rm TL}) &= I^{(c)}_{\rm p}({\rm TL})\kappa^2 F_2\left(k_I^2,\kappa^2\right) F_2\left(k_{II}^2,\kappa^2\right)\,,\\
I^{(c)}_{\rm mix}({\rm TL}) &= 2z(1-z)Mm\left(\vec{\kappa}\vec{e}\right)\left(F_2\left(k_I^2,\kappa^2\right) - F_2\left(k_{II}^2,\kappa^2\right)\right)\,.
\end{aligned} \tag{11}$$

For LT transition

$$
\begin{aligned}
I_{\rm p}^{(c)}({\rm LT}) &= -2z(1-z)(1-2z)Q\left(\vec{k}\vec{V}^*\right), \\
I_{\rm cm}^{(c)}({\rm LT}) &= I_{\rm p}^{(c)}({\rm LT})\kappa^2 F_2\left(k_I^2,\kappa^2\right) F_2\left(k_{II}^2,\kappa^2\right), \\
I_{\rm mix}^{(c)}({\rm LT}) &= 2z(1-z)Qm\left(\vec{\kappa}\vec{V}^*\right)\left(F_2\left(k_I^2,\kappa^2\right)-F_2\left(k_{II}^2,\kappa^2\right)\right). \qquad (12)
\end{aligned}
$$

For previous formulas, we can use $I^{(b)} = I^{(c)} = -\frac{1-z}{z} I^{(a)} = -\frac{z}{1-z} I^{(d)}$ with corresponding $\vec{k}_{1i}$ to obtain other traces.

The TT transition is more complicated

$$
\begin{aligned}
I_{\rm p}^{(c)}({\rm TT}) &= \left[\left(\vec{e}\vec{V}^*\right)\left(m^2+\vec{k}\vec{k}_{1c}\right)+\left(\vec{V}^*\vec{k}\right)\left(\vec{e}\vec{k}_{1c}\right)(1-2z)^2-\left(\vec{e}\vec{k}\right)\left(\vec{V}^*\vec{k}_{1c}\right)\right], \\
I_{\rm cm}^{(a)}({\rm TT}) &= I_{\rm p}^{(a)}({\rm TT})\kappa^2 F_2\left(k_I^2,\kappa^2\right) F_2\left(k_{II}^2,\kappa^2\right), \\
I_{\rm cm}^{(d)}({\rm TT}) &= I_{\rm p}^{(d)}({\rm TT})\kappa^2 F_2\left(k_I^2,\kappa^2\right) F_2\left(k_{II}^2,\kappa^2\right), \\
I_{\rm cm}^{(b)}({\rm TT}) &= \left[\left((1-2z)^2\left(\vec{V}^*\vec{k}\right)\left(\vec{k}_{1b}\vec{e}\right)-\left(\vec{k}\vec{k}_{1b}\right)\left(\vec{e}\vec{V}^*\right)+\left(\vec{k}\vec{e}\right)\left(\vec{V}^*\vec{k}_{1b}\right)\right)\vec{\kappa}^2\right. \\
&\quad \left.+m^2\left(2\left(\vec{e}\vec{\kappa}\right)\left(\vec{V}^*\vec{\kappa}\right)-\left(\vec{e}\vec{V}^*\right)\vec{\kappa}^2\right)\right] F_2\left(k_I^2,\kappa^2\right) F_2\left(k_{II}^2,\kappa^2\right). \qquad (13)
\end{aligned}
$$

The replacement $\vec{k}_{1b} \to \vec{k}_{1c}$ in the last expression leads to the formula for $I_{\rm cm}^{(c)}$.

The interference of the pQCD and ACQM vertices gives

$$
\begin{aligned}
I_{\rm mix}^{(a)}({\rm TT}) &= \frac{zm}{1-z}\left[\left(\vec{V}^*\vec{k}\right)\left(\vec{\kappa}\vec{e}\right)(1-2z)-\left[\vec{e}\vec{\kappa}\right]\left[\vec{V}^*\vec{k}\right]-\left(\vec{e}\vec{k}_{1a}\right)\left(\vec{\kappa}\vec{V}\right)\right. \\
&\quad \left.\times(1-2z)-\left[\vec{e}\vec{k}_{1a}\right]\left[\vec{\kappa}\vec{V}^*\right]\right]\left(F_2\left(k_{II}^2,\kappa^2\right)-F_2\left(k_I^2,\kappa^2\right)\right), \qquad (14) \\
I_{\rm mix}^{(d)}({\rm TT}) &= \frac{(1-z)m}{z}\left[\left(\vec{V}^*\vec{k}\right)\left(\vec{\kappa}\vec{e}\right)(1-2z)+\left[\vec{e}\vec{\kappa}\right]\left[\vec{V}^*\vec{k}\right]-\left(\vec{e}\vec{k}_{1d}\right)\left(\vec{\kappa}\vec{V}^*\right)\right. \\
&\quad \left.\times(1-2z)+\left[\vec{e}\vec{k}_{1d}\right]\left[\vec{\kappa}\vec{V}^*\right]\right]\left(F_2\left(k_{II}^2,\kappa^2\right)-F_2\left(k_I^2,\kappa^2\right)\right), \qquad (15) \\
I_{\rm mix}^{(b)}({\rm TT}) &= m\left[\left(\vec{V}^*\vec{k}\right)\left(\vec{\kappa}\vec{e}\right)(1-2z)-\left[\vec{e}\vec{\kappa}\right]\left[\vec{V}^*\vec{k}\right]-\left(\vec{e}\vec{k}_{1b}\right)\left(\vec{\kappa}\vec{V}\right)(1-2z)\right. \\
&\quad \left.-\left[\vec{e}\vec{k}_{1b}\right]\left[\vec{\kappa}\vec{V}^*\right]\right] F_2\left(k_I^2,\kappa^2\right)-m\left[\left(\vec{V}^*\vec{k}\right)\left(\vec{\kappa}\vec{e}\right)(1-2z)+\left[\vec{e}\vec{\kappa}\right]\right. \\
&\quad \left.\times\left[\vec{V}^*\vec{k}\right]-\left(\vec{e}\vec{k}_{1b}\right)\left(\vec{\kappa}\vec{V}^*\right)(1-2z)+\left[\vec{e}\vec{k}_{1b}\right]\left[\vec{\kappa}\vec{V}^*\right]\right] F_2\left(k_{II}^2,\kappa^2\right). \\
& \qquad (16)
\end{aligned}
$$

The $I_{\rm mix}^{(c)}({\rm TT})$ can be obtained from $I_{\rm mix}^{(b)}({\rm TT})$ if we change overall sign, interchange k_I and k_{II}, and replace $k_{1b} \to k_{1c}$.

4. Some numerical results

In Fig. 2, results of different amplitude ratios are shown and one can see that contribution of ACMQ is negligible in the case of amplitudes with a single spin-flip. The ACMQ interaction gives the notable contribution only in $A_{DF}/A_{\rm TT}$ and decreases it. It is important that the ACMQ contribution to the A_{DF} is tiny too and the difference at the plot comes from ACMQ contribution to the $A_{\rm TT}$.

So, finally, we can conclude that even a spin-flip quark–gluon interaction is included, the s-channel helicity conservation works well in ρ-meson electroproduction. The next aim is the calculation of spin density matrix elements, and comparison results with experimental data [11, 12].

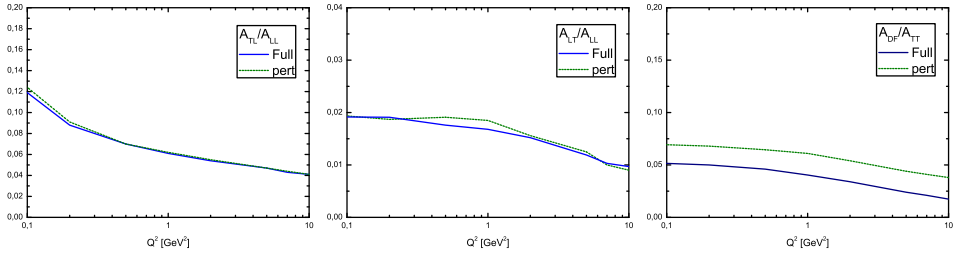


Fig. 2. The Q^2 dependence of the spin-flip to non-spin-flip amplitudes in the ρ-meson production. $W = 75\,\text{GeV}$, $t = 0.15\,\text{GeV}$. Solid/blue line is the result with AQCM and green/dashed line is only the perturbative contribution.

REFERENCES

[1] T. Schäfer, E.V. Shuryak, *Rev. Mod. Phys.* **70**, 323 (1998).
[2] D. Diakonov, *Prog. Part. Nucl. Phys.* **51**, 173 (2003).
[3] G. 't Hooft, *Phys. Rev.* **D14**, 3432 (1976).
[4] N.I. Kochelev, *Phys. Lett.* **B426**, 149 (1998).
[5] N. Kochelev, *Phys. Part. Nucl. Lett.* **7**, 326 (2010) [arXiv:0907.3555 [hep-ph]].
[6] I.P. Ivanov, N.N. Nikolaev, A.A. Savin, *Phys. Part. Nucl.* **37**, 1 (2006).
[7] J. Nemchik *et al.*, *Z. Phys.* **C75**, 71 (1997).
[8] E.V. Kuraev, N.N. Nikolaev, B.G. Zakharov, *JETP Lett.* **68**, 696 (1998) [arXiv:hep-ph/9809539].
[9] I.P. Ivanov, N.N. Nikolaev, *Phys. Atom. Nucl.* **64**, 753 (2001); *Phys. Rev.* **D65**, 054004 (2002).
[10] I.P. Ivanov, arXiv:hep-ph/0303053.
[11] A. Airapetian *et al.* [HERMES Collaboration], *Eur. Phys. J.* **C62**, 659 (2009) [arXiv:0901.0701 [hep-ex]].
[12] C. Adloff *et al.* [H1 Collaboration], *Eur. Phys. J.* **C13**, 371 (2000).