EMERGENT ADS/QCD*

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We elaborate on the correspondence between a non-conformal 4d quantum field theory over the Minkowski space — *e.g.* quantum chromodynamics or an extension of the standard model that breaks the electroweak symmetry dynamically — and a 5d description over an AdS spacetime. Among other things, we are tracing contributions that break the conformal symmetry in the 4d theory to the warping of the 5d geometry, which resembles the soft-wall model.

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Quark-interchange dominance refers to the dominance of the exchange of quarks over the exchange of gluons in, for example, large-angle hadron–hadron scattering [1]. Among other things, it explains why at low energies the differential cross-section in proton–proton scattering scales with the Mandelstam variables like $\frac{d\sigma}{dt} \propto s^{-2}u^{-4}t^{-4}$ [2] and not like $\frac{d\sigma}{dt} \propto t^{-8}$ [3] as would be expected from gluon exchange.

The simplest object that describes the dominant diagrams without dynamic glue (see, *e.g.* Fig. 1 or the corresponding crossed diagram) is the generating functional

$$Z = \int \left[d\bar{\psi}\right] \left[d\psi\right] e^{i\int d^4x \bar{\psi}(i\not{D}-m)\psi} \,. \tag{1}$$

Here, for the sake of simplicity, we have limited ourselves to one fermionic flavour with mass m and to a vectorial source V^μ, which is contained in the 'covariant derivative' $D^\mu = \partial^\mu - iV^\mu$. The inclusion of further sources and/or flavours is straightforward and does not influence the following discussion fundamentally. Using scalar quarks has also no major impact, but allows

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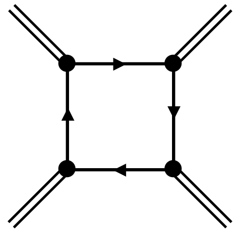

Fig. 1. A dominant diagram. Double lines are for hadrons, single for quarks.

us to avoid one more integration or finite-dimensional trace. Hence, we continue with the generating functional

$$\ln z = -\tfrac{1}{2}\mathrm{Tr}\ln\left(D^2+m^2\right) \tag{2}$$

and comment on the fermionic result at the end. Next, we express the logarithm with the help of a proper time integral,

$$\ln z = \frac{1}{2}\,\mathrm{Tr}\int_{\varepsilon}^{\infty}\frac{dT}{T}e^{-T\left(-D^2+m^2\right)}\,, \tag{3}$$

where we have continued to Euclidean space and introduced a proper-time regularisation $T \geq \varepsilon > 0$. (Subtracting the source-free part would permit us to take the limit $\varepsilon \to 0$.)

Next, we carry out the functional trace in the basis spanned by the eigenfunctions of the position operator and by converting into a one-dimensional path-integral representation,

$$\int d^4x\langle x|e^{TD^2}|x\rangle = \int[dp]\int_{\mathrm{P}}[dx]e^{\int_0^T d\tau[ip\cdot\dot{x}+D^2(x,p)]}\,. \tag{4}$$

Because of the trace the path-integral runs over periodic paths $x(0)=x(T)$. Subsequently, we integrate out the functional integral over the momentum p and obtain the effective action in the world-line formalism [4]

$$\ln z = \underbrace{\int_0^{\infty}\frac{dT}{T^3}e^{-m^2T}\int d^4x_0}_{\text{warped (soft}-\text{wall) AdS measure}}\;\underbrace{\frac{\mathcal{N}}{(4\pi)^2}\int_{\mathrm{P}}[dy]e^{-\int_{\varepsilon}^{T}d\tau\left(\frac{\dot{y}^2}{4}+i\dot{y}\cdot\dot{V}\right)}}_{=\mathcal{L}}\,. \tag{5}$$

The generating functional now appears in the form of an effective Lagrangian $\mathcal{L}$ integrated over a soft-wall–warped [5] AdS measure, which motivates the present comparison of (1) to a soft-wall AdS/QCD description. (If we assume m to be the constituent and not the current quark mass, the warp parameter m^2 is even of the phenomenologically preferred order of magnitude.) T takes the role of the fifth coordinate in the (unwarped) parametrisation $d^2s = \frac{dT^2}{T^2} + \frac{dx \cdot dx}{T}$. The integration over the variable x_0^μ has been split off, $x^\mu = x_0^\mu + y^\mu$, such that $\int_0^T d\tau y^\mu = 0$ and $\dot{x}^\mu = \dot{y}^\mu$. This is important for two reasons; it will make manifest energy-momentum conservation and will allow for the inversion of the corresponding propagator when integrating out the variable y, as x_0 constitutes a zero mode. $\mathcal{N}$ cancels the normalisation of the source-free $[dy]$ integration, which leaves an extra factor of $(4\pi T)^{-2}$. The above proper-time regularisation is directly analogous to the AdS/QCD UV regularisation, where the UV brane is not put directly at $T = 0$ but $T = \varepsilon > 0$. Furthermore, the effective Lagrangian $\mathcal{L}$ is locally invariant under chiral rotations of the source V, which is another feature of AdS/QCD models, where the chiral symmetry of the 4d theory becomes the gauge symmetry of the 5d description. This can be made manifest by rewriting the interaction part of the effective Lagrangian $\mathcal{L}$ in form of a Wilson loop,

$$\mathcal{L} = \frac{\mathcal{N}}{(4\pi)^2} \int_{\mathrm{P}} [dy] e^{-\int_0^T d\tau \frac{\dot{y}^2}{4}} e^{-i \oint dy \cdot V} . \tag{6}$$

A correspondence between an *a priori* 4d computation and deformed AdS/QCD models has also been found in light-front holography [6]. There the fifth-dimensional coordinate T is identified with $\zeta^2 = x(1-x)\vec{b}_\perp^2$, where x stands for the light-front momentum fraction x of one of the mesonic constituent and $\vec{b}_\perp$ for the transverse separation of the constituents. ζ is the invariant separation between the constituents. Different deformations of AdS space, which in these AdS/QCD models account for the breaking of conformal invariance by confinement correspond to different interquark potentials of the 4d formulation. In particular, the soft-wall warping corresponds to a harmonic oscillator potential.

In the present study, due to the periodicity condition of the path-integration, T is also a measure for the separation between the constituents, as it limits the size of the closed orbit; at this point there is, however, no one-to-one correspondence to ζ^2.

For the sake of concreteness let us take a look at certain correlators. To this end, we integrate out the field y from Eq. (6) and obtain

$$\mathcal{L} \subset \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \prod_{j=1}^{n} \left(\int \frac{d^4 q_j}{(2\pi)^4} \int_0^T \mathrm{d}\tau_j \right)^n \frac{e^{-i x_0 \cdot (\sum_{i=1}^n q_i)}}{(4\pi)^2} l_n \,, \tag{7}$$

$$l_n = e^{\frac{1}{2} \sum_{i,j=1}^n \left(G_{ij} q_i \cdot q_j + 2\mathrm{i} \dot{G}_{ij} \tilde{V}_i \cdot q_j + \ddot{G}_{ij} \tilde{V}_i \cdot \tilde{V}_j \right)} , \tag{8}$$

which is the master formula of Bern and Kosower [7]. Here $G_{ij} = G(\tau_i, \tau_j)$ stands for the world-line propagator, which satisfies the equation of motion

$$\frac{1}{2}\frac{d^2}{d\tau^2}G\left(\tau,\tau'\right) = \delta\left(\tau-\tau'\right) - \frac{1}{T}\,. \tag{9}$$

(The additional inhomogeneity $\frac{1}{T}$ on the left-hand side is required to have net-zero charge in our compact space (interval) to have a well-defined Poisson problem.) $\dot{G}_{ij}$ and $\ddot{G}_{ij}$ stand for the first and second derivatives, respectively, of the propagator with respect to its first argument. $\tilde{V}_j = \tilde{V}(q_j)$ stands for the Fourier transform of the source. The d^4x_0 integration will impose energy-momentum conservation, as it only concerns the Fourier phase in Eq. (7). "$\subset$" indicates that not all terms of the right-hand side contribute to the effective Lagrangian $\mathcal{L}$ but only those that at order n are linear in all n $\tilde{V}_i$. Then the two-point function reads

$$\int d^4x_0\mathcal{L}_2 \;=\; -\frac{1}{32\pi^2}\int\frac{d^4q}{(2\pi)^4}T^2\int_0^1 d\hat{\tau}s_2\,, \tag{10}$$

$$s_2 \;=\; e^{-G_{12}q^2}\tilde{V}^*_\mu(q)\tilde{V}_\nu(q)\dot{G}^2_{12}\left(-q^\mu q^\nu + q^2\eta^{\mu\nu}\right)\,. \tag{11}$$

Now, let us compare this expression to the result from a soft-wall AdS/QCD model. There the 4d sources are extended to 5d fields, $V(x)\to\mathcal{V}(x,T)$. The two-point function is encoded in the quadratic action

$$S_2^{5D} = -\frac{1}{4}\int d^4x\frac{dT}{T^3}e^{-m^2T}g^{\mu\kappa}g^{\nu\lambda}\mathcal{V}_{\mu\nu}\mathcal{V}_{\kappa\lambda}\,, \tag{12}$$

which, according to the holographic dictionary, does not feature an explicit fifth-dimensional mass term for this vector field with fermionic constituents. It must be evaluated on the classical solution, which leaves a surface term at small values of T, which me must choose as ε rather then 0 for the sake of regularisation. After a 4d Fourier transformation, at the saddle point of this action the transverse modes obey the equation of motion

$$\left(4\partial_T^2 + q^2/T - m^4\right)e^{-m^2T/2}\breve{\tilde{\mathcal{V}}}_\nu(q,T) = 0\,. \tag{13}$$

One boundary condition for this second-order equation is given by $\breve{\tilde{\mathcal{V}}}_\lambda(q,\varepsilon) = \tilde{V}_\lambda(q)\tilde{v}(q,\varepsilon)$ with $\tilde{v}(q,\varepsilon)=1$. As a consequence, the action evaluated on the classical solution reads

$$\breve{S}_2^{5D} = \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}e^{-m^2\varepsilon}\left(\frac{q^\nu q^\lambda}{q^2} - \eta^{\nu\lambda}\right)\tilde{V}^*_\nu(q)\tilde{V}_\lambda(q)\left[\partial_T\tilde{v}(q,\varepsilon)\right]\,. \tag{14}$$

The standard prescription is to select the normalisable solution in lieu of the second boundary condition. In that case

$$\tilde{v}(q,T) = \Gamma\left(1-\tfrac{q^2}{4m^2}\right) TU\left(1-\tfrac{q^2}{4m^2}, 2, m^2T\right) \tag{15}$$

and to leading order in small ε (the position of the ultraviolet brane)

$$\partial_T\left[\Gamma\left(1-\tfrac{q^2}{4m^2}\right) TU\left(1-\tfrac{q^2}{4m^2}, 2, m^2T\right)\right] \approx -\frac{q^2}{4}\ln\left(m^2T\right) . \tag{16}$$

The purely 4d result in Eq. (10) features the same leading small-ε behaviour after identifying the two regularisation parameters, proper time and UV-brane position. The two results differ for subleading terms.

The 4d result can be reproduced exactly by adopting

$$\partial_T\tilde{v}(q,\varepsilon) \propto q^2\int_0^1 d\hat{\tau}(1-2\hat{\tau})^2\int_0^\infty \frac{dT}{T}e^{-T\left[m^2-q^2\left(\hat{\tau}-\hat{\tau}^2\right)\right]} \tag{17}$$

as second boundary condition instead of the usually invoked normalisability of the AdS/QCD solution. Choosing the normalisable solution instead, we get

$$\partial_T\tilde{v}(q,\varepsilon) = -\frac{q^2}{4}\left[\ln\left(\varepsilon m^2\right)+\gamma+\psi\left(1-\tfrac{q^2}{4m^2}\right)+\gamma\right] . \tag{18}$$

The finite part can be expressed as

$$\begin{aligned}\gamma+\psi\left(1-\tfrac{q^2}{4m^2}\right) &= m^2\int_0^\infty dT\frac{e^{-m^2T}-e^{-\left(m^2-\frac{q^2}{4}\right)T}}{1-e^{-m^2T}} \\ &= m^2\sum_{n=1}^\infty\int_0^\infty dTe^{-nm^2T}2\left[e^{q^2G(0)}-e^{q^2G\left(\frac{1}{2}\right)}\right] . \end{aligned}\tag{19}$$

On the second line, we have reexpressed the fraction by a geometric series, reflecting the presence of a tower of states with a constant spacing between their squared masses, known for the soft-wall model. Furthermore, we have exploited that the world-line propagator that solves Eq. (9), *i.e.*

$$G(\hat{\tau}) = T\left(\hat{\tau}-\hat{\tau}^2\right) , \tag{20}$$

where $\hat{\tau}=\tau/T$, takes the minimum value $G(0)=0$ and the maximum value $G(\frac{1}{2})=\frac{T}{4}$. Finally, the expression in square brackets can be rewritten as

$$e^{q^2G(0)}-e^{q^2G\left(\frac{1}{2}\right)} = \int_0^1 d\hat{\tau}\dot{G}e^{q^2G} = 2\int_{G(0)}^{G\left(\frac{1}{2}\right)} dg\dot{G}\left[\hat{\tau}(g)\right]e^{q^2g} , \tag{21}$$

which is close to what we arrive at in the above purely 4d computation, but with one important difference: In the 4d set-up there appears another factor of the first derivative of the world-line propagator,

$$\int_0^1 d\hat{\tau}\dot{G}^2 e^{q^2 G} = 2\int_{G(0)}^{G(\frac{1}{2})} dg\dot{G}\left[\hat{\tau}(g)\right] e^{q^2 g}\,. \tag{22}$$

An analogous mismatch exists if we return to the original fermions instead of the scalars, where, up to a numerical factor, $\dot{G}^2$ is replaced by G. This means that the 5d set-up corresponds to a different 4d setting in which the conformal symmetry is not (only) broken by a mass term, but also an interquark potential. (The determination of the latter is left for later work.) This is also in line with the observation that light-cone holography contains such a potential as one of its ingredients.

In conclusion, in a kinematic setting, where contributions from dynamical gluons are subdominant, we elaborated on the correspondence between a 4d and a 5d description. Both descriptions share the same dominant UV behaviour. The 5d description could be made to coincide with the 4d for selected boundary conditions, albeit not for the canonical choice. The correspondence becomes quantitatively close when adopting constituent instead of current quark masses. Adjusting the 4d to the 5d approach the 4d must be extended to incorporate an infinite tower of states with evenly spaced squared masses. This, however, is not enough for matching the two descriptions, but the introduction of an interquark potential is necessary as known from light-front holography.

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