LIGHT-FRONT QUARK MODEL UPDATE ON MASS SPECTRUM CALCULATIONS FOR GROUND STATE PSEUDOSCALAR AND VECTOR MESONS*

Ho-Meoyng Choi

Department of Physics, Teachers College, Kyungpook National University
Daegu, Korea 702-701

Chueng-Ryong Ji, Ziyue Li

Department of Physics, North Carolina State University
Raleigh, North Carolina 27695-8202, USA

(Received January 7, 2013)

We present an update on our light-front quark model constrained by the variational principle for the QCD-motivated effective Hamiltonian. By smearing out the Dirac delta function in the hyperfine interaction and taking a larger harmonic oscillator basis in our trial wave function, we improved our model predictions for both mass spectra and decay constants of ground state pseudoscalar and vector mesons.

DOI:10.5506/APhysPolBSupp.6.281
PACS numbers: 12.39.Ki, 13.40.Gp, 13.40.Hq, 14.40.–n

1. Introduction

Effective degrees of freedom to describe a strongly interacting system of hadrons have been one of the key issues in understanding the non-perturbative nature of QCD in the low energy regime. Within an impressive array of effective theories available nowadays, the constituent quark model has been quite useful in providing a good physical picture of hadrons just like the atomic model for the system of atoms. In addition, a proper way of dealing with the relativistic aspect of a hadron system is also quite essential and the formulation of light-front dynamics (LFD) [1] provides a natural framework to include such relativistic effects in describing hadrons. Taking advantage of the constituent quark picture and the formulation of LFD, we have developed a light-front quark model (LFQM) [2–4] based on a simple QCD-motivated effective Hamiltonian for a description of mesons. The key

* Presented at the Light Cone 2012 Conference, Kraków, Poland, July 8–13, 2012.

idea of our LFQM was to treat the radial wave function as a trial function for the variational principle to the QCD-motivated effective Hamiltonian with the well-known linear plus Coulomb interaction. Both the masses and the hadronic properties of ground state pseudoscalar and vector mesons in our LFQM were fairly well reproduced by taking just a $1S$-state harmonic oscillator (HO) wave function as a trial function. Computing the meson mass spectra [4], however, we have treated the hyperfine interaction as a perturbation rather than including it in the variation procedure to avoid the negative infinity from the Dirac delta function contained in the hyperfine interaction.

The main purpose of this work is to update our previous model [2–4] by including the hyperfine interaction also in our variational method to get the optimal model parameters, and examine if it improves our numerical results compared to the ones obtained by the perturbative treatment of the hyperfine interaction. To achieve this goal, we smeared out the Dirac delta function by a Gaussian distribution in order to resolve the infinity problem when variational principle is applied. Moreover, we improved our trial wave function by taking a larger HO basis to analyze this effect of improving trial wave functions on our numerical results [5].

The paper is organized as follows: in Sec. 2, we describe our QCD-motivated effective Hamiltonian with the smeared-out hyperfine interaction. The optimum values of model parameters are also presented in this section. In Sec. 3, we present our numerical results of the mass spectra and the decay constants of mesons and compare them with the experimental data. Summary and conclusion follow in Sec. 4.

2. Model description

In our LFQM for mesons, we approximate the system as effectively dressed valence quarks governed by the following QCD-motivated effective Hamiltonian in which motion of the quarks inside a meson is relativistic [2–4]: $H = (m_q^2 + \vec{k}^2)^{1/2} + (m_{\bar{q}}^2 + \vec{k}^2)^{1/2} + V$, where

$$V = V_{\rm conf} + V_{\rm coul} + V_{\rm hyp} = a + br - \frac{4\alpha_{\rm s}}{3r} + \frac{2}{3}\frac{\boldsymbol{S}_q \cdot \boldsymbol{S}_{\bar{q}}}{m_q m_{\bar{q}}}\nabla^2 V_{\rm coul}\,. \tag{1}$$

The LF Hamiltonian can be taken as the above Hamiltonian in the rest frame of the meson, *i.e.* the center of mass frame of the two-body system. The LF wave function of the ground state mesons is given by $\Psi(x_i, \boldsymbol{k}_{\perp i}, \lambda_i) = \mathsf{R}^{JJ_z}_{\lambda_q\lambda_{\bar{q}}}(x_i, \boldsymbol{k}_{\perp i})\phi(x_i, \boldsymbol{k}_{\perp i})$, where ϕ is the radial wave function and R is the interaction-independent spin-orbit wave function. The wave function is represented by the Lorentz invariant internal variables $x_i = p_i^+/P^+$, $\boldsymbol{k}_{\perp i} = \boldsymbol{p}_{\perp i} - x_i\boldsymbol{P}_\perp$ and helicity λ_i, where P^μ is the momentum of the meson and p_i^μ

is the momenta of constituent quarks. Since the spin-orbit wave functions satisfy $\sum_{\lambda_q\lambda_{\bar{q}}} \mathsf{R}^{JJ_z\dagger}_{\lambda_q\lambda_{\bar{q}}}\mathsf{R}^{JJ_z}_{\lambda_q\lambda_{\bar{q}}} = 1$, we will focus on the radial wave functions in the calculation of meson mass spectra.

We use the same trial wave function ϕ for both pseudoscalar and vector mesons, but we try two different forms: one simply takes the $1S$-state HO wave function $\phi_{\mathrm{A}} = \phi_{1S}$ and the other one is expanded with the two lowest order HO wave functions $\phi_{\mathrm{B}} = \sum_{i=1}^{2} c_i\phi_{iS}$, where both wave functions are proportional to $\exp(-\vec{k}^2/2\beta^2)$ with the Gaussian parameters β depending on the quark flavors. The Jacobi factor of the variable transformation between the LF internal variables and the three momentum here should be taken into account for the LF wave functions used in the calculations of the decay constants presented in the next section. We then evaluate the expectation value of the Hamiltonian with $\phi_{\mathrm{A(B)}}$, *i.e.* $\langle\phi_{\mathrm{A(B)}}|H|\phi_{\mathrm{A(B)}}\rangle$ which depends on β. In our previous works [2–4], which we call "CJ model", we first evaluate the expectation value of the central Hamiltonian $T + V_{\mathrm{conf}} + V_{\mathrm{coul}}$ with the trial function ϕ_{A}, where T is the kinetic energy. Once the model parameters are fixed by minimizing the expectation value $\langle\phi_{\mathrm{A}}|(T + V_{\mathrm{conf}} + V_{\mathrm{coul}})|\phi_{\mathrm{A}}\rangle$, then the mass eigenvalue of each meson is obtained as $M_{q\bar{q}} = \langle\phi_{\mathrm{A}}|H|\phi_{\mathrm{A}}\rangle$. The hyperfine interaction V_{hyp} in CJ model was treated as perturbation and was left out in the variational process that optimizes the model parameters.

But now we want to include the hyperfine interaction in our parameterization process. To avoid the negative infinity, we use a Gaussian smearing function to weaken the singularity of $\delta^3(\boldsymbol{r})$ in hyperfine interaction, *viz.* [6, 7], $\delta^3(\boldsymbol{r}) \to \frac{\sigma^3}{\pi^{3/2}}e^{-\sigma^2\boldsymbol{r}^2}$. Once the delta function is smeared out, a true minimum for the mass occurs at a finite value of β. The analytic formulae of mass eigenvalues for our modified Hamiltonian with the smeared-out hyperfine interaction, *i.e.* $M^{\mathrm{A(B)}}_{q\bar{q}} = \langle\phi_{\mathrm{A(B)}}|H|\phi_{\mathrm{A(B)}}\rangle$, are found in [5]. We then apply the variational principle, *i.e.* $\partial M^{\mathrm{A(B)}}_{q\bar{q}}/\partial\beta = 0$, to find the optimal model parameters in order to get a best fit for the mass spectra of ground state pseudoscalar and vector mesons (see [5] for more detailed description). Our optimized potential parameters are obtained as $a = -0.5575$ (-0.6664) GeV, $b = 0.18$ (0.18) GeV^2, $\alpha_{\mathrm{s}} = 0.5174$ (0.5348) for $\phi_{\mathrm{A(B)}}$. Our optimal constituent quark masses [GeV] and the smearing parameter σ [GeV] obtained from $\phi_{\mathrm{A(B)}}$ are $m_q = 0.220(0.221)$, $m_s = 0.432(0.456)$, $m_c = 1.77(1.77)$, and $m_b = 5.2(5.2)$, and $\sigma = 0.405(0.423)$. For the best fit of the mass spectra, we find that $|c_1|^2 = 0.7$ and $|c_2|^2 = 0.3$ for ϕ_{B}. Since we included the hyperfine interaction with smearing function entirely in our variational process, we now obtained the two different sets of β values, one for pseudoscalar and the other for vector mesons, respectively. The optimal Gaussian parameters $\beta_{q\bar{q}}$ for pseudoscalar and vector mesons obtained by the two trial functions ϕ_{A} and ϕ_{B} are listed in Table I.

TABLE I

The Gaussian parameter β [GeV] for ground state pseudoscalar (upper panel) vector (lower panel) mesons obtained by $\phi_{\rm A}$ and $\phi_{\rm B}$. $q = u$ and d.

Model	β_{qq}	β_{qs}	β_{qc}	β_{cs}	β_{cc}	β_{qb}	β_{bs}	β_{bc}	β_{bb}
$\phi_{\rm A}$	0.6376	0.5513	0.5810	0.5994	0.7916	0.6686	0.7132	1.0577	1.6455
$\phi_{\rm B}$	0.4520	0.3799	0.3960	0.4078	0.5286	0.4461	0.4757	0.6891	1.0549
$\phi_{\rm A}$	0.3480	0.3952	0.5283	0.5727	0.7849	0.6436	0.7010	1.0554	1.6450
$\phi_{\rm B}$	0.2416	0.2742	0.3579	0.3892	0.5233	0.4278	0.4671	0.6871	1.0544

3. Numerical results

We show in Fig. 1 our prediction of the meson mass spectra obtained from the variational principle to the modified Hamiltonian with the smeared-out hyperfine interaction using $\phi_{\rm A}$ and $\phi_{\rm B}$ and compare them with the experimental data [8]. We also include the results obtained from the CJ model with the linear confining potential [3, 4]. We should note that the masses

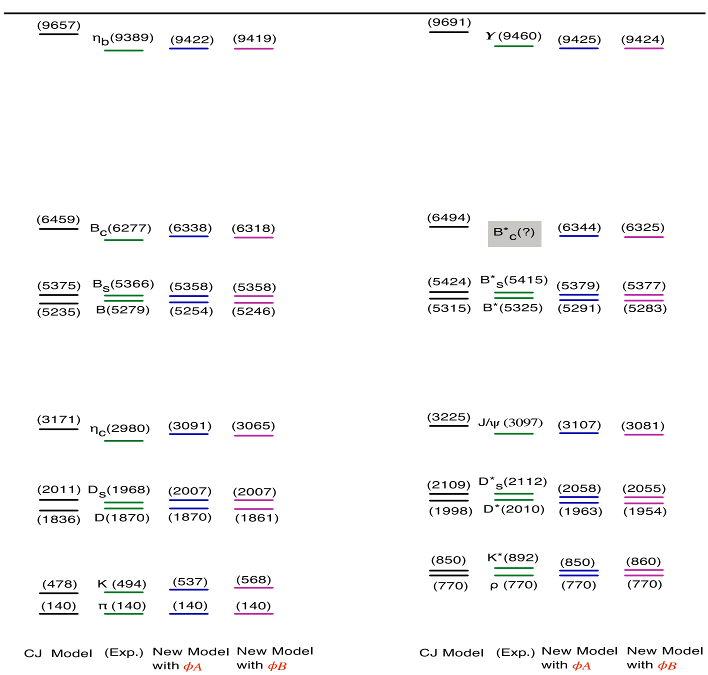

Fig. 1. Fit of the ground state meson masses [MeV] for $\phi_{\rm A(B)}$ compared with the fit from our previous calculations using CJ model [4] as well as the experimental values [8].

of π and ρ mesons are used as inputs in our calculation. As one can see, the single $1S$ state HO wave function $\phi_{\rm A}$ already generates a good enough fitting for the spectrum, and a more complicated trial wave function $\phi_{\rm B}$ does not change the $1S$ results too much. Except for the mass of K, our predictions for the masses of $1S$-state pseudoscalar and vector mesons are within 4% error. Especially, our modified Hamiltonian clearly improves the predictions of heavy-light and heavy quarkonia systems such as $(\eta_c, J/\psi, B_c, \eta_b, \Upsilon)$ compared to the CJ model adopting the contact hyperfine interaction.

In Table II, we list our predictions for the decay constants of light mesons (π, K, ρ, K^*) obtained by using $\phi_{\rm A(B)}$ and compare them with CJ model and the experimental data [8]. As one can see, $\phi_{\rm A}$ generates decay constants that are quite high for light mesons indicating that just $1S$-state HO wave function alone cannot be a good trial wave function for the entire Hamiltonian including the smeared hyperfine interaction. However, the results from $\phi_{\rm B}$ are much closer to the experimental data than those from $\phi_{\rm A}$. In Table III, we list our predictions for the charmed- and bottomed-meson decay constants together with CJ model and the available experimental data. We should note that our results of the ratios

TABLE II

Decay constants for light mesons (in unit of MeV) obtained from our updated LFQM.

Model	f_π	f_ρ	f_K	f_{K^*}
$\phi_{\rm A}$	155	234	190	261
$\phi_{\rm B}$	139	211	176	234
CJ	130	246	161	256
Exp. [8]	130.4 ± 0.2	221 ± 1	156.1 ± 0.8	217 ± 7

$f_{D_s}/f_D = 1.13[1.14]$ and $f_{\eta_c}/f_{J/\Psi} = 0.88[0.91]$ obtained from $\phi_{\rm A}[\phi_{\rm B}]$ are quite comparable with the available experimental data, $f_{D_s}/f_D = 1.25 \pm 0.06$ [8] and $f_{\eta_c}/f_{J/\Psi} = 0.81 \pm 0.19$ [9], respectively.

4. Conclusion

In this work, we updated our LFQM by smearing out the Dirac delta function in the hyperfine interaction and including the smeared hyperfine interaction in our calculation based on the variational principle rather than using the perturbation method to handle the delta function in the contact hyperfine interaction. Using the two trial wave functions, *i.e.* the $1S$ state HO wave function $\phi_{\rm A}$ and the mixed wave function $\phi_{\rm B}$ of $1S$ and $2S$ HO states, we calculated both the mass spectra of the ground state pseudoscalar and vector mesons and the decay constants of the corresponding mesons. We

TABLE III

Charmed meson decay constants (in unit of MeV) obtained from our updated LFQM.

Model	f_D	f_{D^*}	f_{D_s}	$f_{D_s^*}$	f_{η_c}	$f_{J/\psi}$
$\phi_{\rm A}$	244	279	276	322	406	460
$\phi_{\rm B}$	218	241	249	282	354	390
CJ	197	239	232	273	326	360
Exp.	206.7 ± 8.9 [8]	—	257.5 ± 6.1 [8]	—	335 ± 75 [9]	416 ± 6 [8]
	f_B	f_{B^*}	f_{B_s}	$f_{B_s^*}$	f_{η_b}	f_Υ
$\phi_{\rm A}$	229	243	267	288	805	871
$\phi_{\rm B}$	195	202	229	242	654	692
CJ	171	185	205	220	507	529
Exp.	229^{+36+34}_{-31-37} [10]	—	—	—	—	715 ± 5 [8]

found that our predictions of the meson mass spectra are in good agreement with the data both for $\phi_{\rm A}$ and $\phi_{\rm B}$. However, $\phi_{\rm B}$ turns out to be much better in the calculation of decay constants than $\phi_{\rm A}$. Since our modified Hamiltonian together with $\phi_{\rm B}$ provides very good results as discussed in this work, it may be also desirable to investigate further and see if we can improve our previous calculations of other wave function related observables such as form factors. We shall explore them in our future work.

This work is supported by the U.S. Department of Energy (No. DE-FG02-03ER41260). The work of H.-M. Choi was supported in part by the Korea Research Foundation Grant funded by the Korean Government (KRF-2010-0009019) and that of Z. Li was supported in part by the JSA/Jefferson Lab fellowship.

REFERENCES

[1] S.J. Brodsky, H.-C. Pauli, S.S. Pinsky, *Phys. Rep.* **301**, 299 (1998).
[2] H.-M. Choi, C.-R. Ji, *Phys. Rev.* **D59**, 074015 (1999).
[3] H.-M. Choi, C.-R. Ji, *Phys. Lett.* **B460**, 461 (1999).
[4] H.-M. Choi, C.-R. Ji, *Phys. Rev.* **D80**, 054016 (2009).
[5] H.-M. Choi, C.-R. Ji, Z. Li, `arXiv:1206.3351v1 [hep-ph]`.
[6] S. Capstick, N. Isgur, *Phys. Rev.* **D34**, 2809 (1986).
[7] S. Godfrey, N. Isgur, *Phys. Rev.* **D32**, 189 (1985).
[8] J. Beringer *et al.* [Particle Data Group], *Phys. Rev.* **D86**, 010001 (2012).
[9] K.W. Edwards *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **86**, 30 (2001).
[10] K. Ikado *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **97**, 251802 (2006).