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RELATIONS BETWEEN ANOMALOUS AND EVEN-PARITY SECTORS IN ADS/QCD*

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We derive the $\mathcal{O}(p^6)$ Chiral Perturbation Theory Lagrangian in the massless quark limit for a class of gravity dual models of Quantum Chromodynamics with the chiral symmetry broken through boundary conditions. The odd $\mathcal{O}(p^6)$ couplings are related to the $\mathcal{O}(p^4)$ low-energy constants (LECs) in the even-parity sector. Some combinations of even $\mathcal{O}(p^6)$ couplings are found to be universal and independent of the peculiarities of the model. These relations turn out to be the manifestation at low energies of a broader relation between anomalous and even-parity amplitudes.

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1. Introduction

The relation between the anomalous and even-parity sectors of Quantum Chromodynamics (QCD) was studied in detail in Ref. [1] within the framework of holographic QCD [2–5]. This work was motivated by the analysis of Son and Yamamoto [6] of holographic models, where the chiral symmetry was broken through boundary conditions (b.c.). An interesting relation was found therein between the left–right correlator $\Pi_{\rm LR}(Q^2)$ and the transverse part $w_{\rm T}(Q^2)$ of the anomalous AVV Green's function [6] (studies of such a relation can be found in [7]). At low energies, this turned into a relation between the $\mathcal{O}(p^4)$ even-parity Chiral Perturbation Theory (χPT) coupling L_{10} and the $\mathcal{O}(p^6)$ odd-intrinsic-parity coupling C_{22}^W [8].

In [1], we derived the remaining $\mathcal{O}(p^6)$ odd-sector couplings — in the massless quark limit considered all along the analysis — and found analogous relations with the $\mathcal{O}(p^4)$ even low-energy constants (LECs). We focused on the odd couplings C_{22}^W and C_{23}^W [9], which can be directly related to

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the transition of a pion into two photons and two axial-vector currents, respectively. These amplitudes are found to be related to the vector form factor of the pion and the axial-vector form factor into three pions [1].

In the kind of models studied in this paper, with the chiral symmetry broken through b.c., the action is composed by the Yang–Mills (YM) and Chern–Simons (CS) terms, describing the even and anomalous QCD sectors, respectively [2–5]:

$$\begin{aligned} S &= S_{\rm YM} + S_{\rm CS}\,, \\ S_{\rm YM} &= -\int d^5x\, {\rm tr}\left[-f^2(z)\mathcal{F}_{z\mu}^2 + \frac{1}{2g^2(z)}\mathcal{F}_{\mu\nu}^2 \right]\,, \\ S_{\rm CS} &= -\frac{N_C}{24\pi^2}\int {\rm tr}\left[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5 \right]\,, \end{aligned} \tag{1}$$

with N_C the number of colors and the fifth coordinate z running from $-z_0$ to z_0 with $0 < z_0 \leq +\infty$. $\mathcal{A}(x,z) = \mathcal{A}_M dx^M$ is the 5D $U(N_f)$ gauge field and $\mathcal{F} = d\mathcal{A} - i\mathcal{A}\wedge\mathcal{A}$ is the field strength.

Chiral symmetry can be realized as a 5D gauge symmetry localized on the two boundaries at $z = \pm z_0$. The gauging of the chiral symmetry allows one to naturally introduce the corresponding right and left current sources, respectively $r_\mu(x)$ and $\ell_\mu(x)$, through the ultraviolet b.c. $\mathcal{A}_\mu(x,-z_0) = \ell_\mu(x)$ and $\mathcal{A}_\mu(x,z_0) = r_\mu(x)$ [1]. In the 5D gauge $\mathcal{A}_z = 0$, one has the field decomposition in on-shell states

$$\mathcal{A}_\mu(x,z) = i\mathit{\Gamma}_\mu(x) + \frac{u_\mu(x)}{2}\psi_0(z) + \sum_{n=1}^{\infty} v_\mu^n(x)\psi_{2n-1}(z) + \sum_{n=1}^{\infty} a_\mu^n(x)\psi_{2n}(z)\,, \tag{2}$$

where the commonly used tensors $u_\mu(x)$ and $\mathit{\Gamma}_\mu(x)$ from χPT contain the chiral Goldstones and $\ell_\mu(x)$ and $r_\mu(x)$ [9, 10]. The resonance wave-functions $\psi_n(z)$ are provided by the normalizable eigenfunctions of the equation of motion (EoM) for the transverse part of the gauge field and the pion wave-function $\psi_0(z)$ is the solution of the EoM at $q^2 = 0$ with b.c. $\psi_0(\pm z_0) = \pm 1$.

Once we have rewritten the 5D fields in terms of the chiral Goldstones and vector and axial-vector resonances, the derivation of the meson Lagrangian is straightforward. We substitute the $\mathcal{A}_\mu$ decomposition provided in Eq. (2) in the 5D action (1). This yields terms without resonance fields [3–5] which, at low energies, provide the Wess–Zumino–Witten (WZW) Lagrangian ($\mathcal{O}(p^4)$) [4, 5] and the $\mathcal{O}(p^2)$, and $\mathcal{O}(p^4)$ χPT action [11] with the corresponding LECs given in terms of the corresponding 5D integrals of pion wave-function [3].

2. Sum-rules and chiral couplings at $\mathcal{O}(p^6)$

The $\mathcal{O}(p^6)$ LECs are generated by the intermediate resonance exchanges. More precisely, we need just the one-resonance exchanges, given by the terms of the action with one-resonance field. The couplings $a_{\mathrm{V}v^n}$, $a_{\mathrm{A}a^n}$, $b_{v^n\pi\pi}$, $b_{a^n\pi^3}$ ... are defined by the corresponding 5D integrals of the wave-functions. At the level of the generating functional, in order to compute the diagrams with intermediate resonances, one must perform the functional integration over the heavy resonance configurations in the low-energy limit [1, 10].

Before proceeding to the actual computation, we will consider a series of resonance sum-rules which will be needed for the extraction of some LECs and the amplitudes. They are obtained through the EoMs of the 5D fields and the completeness condition for the wave-function solutions $\psi_m(z)$ [1]. We obtain, for instance

$$\sum_{n=1}^{\infty} a_{\mathrm{V}v^n} b_{v^n\pi\pi} m_{v^n}^2 = 2f_\pi^2\,, \qquad \sum_{n=1}^{\infty} a_{\mathrm{V}v^n} b_{v^n\pi\pi} = 4L_9\,, \tag{3}$$

$$\sum_{n=1}^{\infty} 3a_{\mathrm{A}a^n} c_{a^n} = 2\,, \qquad \sum_{n=1}^{\infty} 3a_{\mathrm{A}a^n} c_{a^n}/m_{a^n}^2 = 4(L_9 - 8L_1)/f_\pi^2\,, \tag{4}$$

respectively, related to the $\pi\pi$ vector form factor (VFF) and the $\pi\pi\pi$ axial-vector form factor (AFF) at high energies.

Through the integration of the heavy resonances in the generating functional, we obtain all the odd-parity sector LECs C_k^W in the massless quark case. By means of wave-function completeness relations and EoMs they can be reexpressed in terms of the $\mathcal{O}(p^4)$ χPT couplings of the even sector L_1 and L_9 and a constant Z [1]. In particular, we find

$$C_{22}^W = \frac{N_C}{32\pi^2 f_\pi^2} L_9\,, \qquad C_{23}^W = \frac{N_C}{96\pi^2 f_\pi^2}(L_9 - 8L_1)\,. \tag{5}$$

Taking into account the relation $L_9 = -L_{10}$ in this kind of holographic models [3], we recover the result $C_{22}^W = -\frac{N_C}{32\pi^2 f_\pi^2} L_{10}$ [8].

It is also possible to compute the $\mathcal{O}(p^6)$ even-sector LECs in the holographic model. In particular, we find some relations independent of the peculiarities of the model: the $\gamma\gamma\to\pi^0\pi^0$ amplitude is ruled by [1, 12]

$$\begin{aligned} 256\pi^4 f_\pi^2 (8C_{53} + 8C_{55} + C_{56} + C_{57} + 2C_{59}) &= N_C^2\,, \\ -128\pi^4 f_\pi^2 (C_{56} + C_{57} + 2C_{59}) &= N_C^2/6\,, \end{aligned} \tag{6}$$

and for $\gamma\gamma\to\pi^+\pi^-$ [1, 13]

$$\begin{aligned} 256\pi^4 f_\pi^2 (8C_{53} - 8C_{55} + C_{56} + C_{57} - 2C_{59} + 4C_{78} + 8C_{87} - 4C_{88}) &= 0\,, \\ -128\pi^4 f_\pi^2 (C_{56} + C_{57} - 2C_{59} - 4C_{78}) &= 0\,. \end{aligned} \tag{7}$$

3. Relations between anomalous and even-parity amplitudes

3.1. Green's function relation: LR versus AVV correlator

The relations between the odd-sector $\mathcal{O}(p^6)$ constants and the $\mathcal{O}(p^4)$ constants in the even sector indicate possible relations between hadronic amplitudes. One example is the Son–Yamamoto relation between the transverse structure function $w_{\rm T}(Q^2)$ of the AVV Green's function and the left–right correlator $\Pi_{\rm LR}(Q^2)$ [6]

$$w_T\left(Q^2\right) = \frac{N_C}{Q^2} + \frac{N_C}{f_\pi^2}\,\Pi_{\rm LR}\left(Q^2\right) \,, \tag{8}$$

where the Euclidean squared momentum transfer $Q^2 = -q^2$. Taking the $Q^2 \to 0$ limit on both sides, one gets the C_{22}^W LEC relation but with $L_9 = -L_{10}$ [8].

3.2. Form factor relation: $\gamma^* \to \pi\pi$ versus $\pi \to \gamma\gamma^*$

Other studies in two specific models showed that the $\pi^0 \to \gamma\gamma^*$ transition form-factor $\mathcal{F}_{\pi\gamma^*\gamma^*}(Q^2,0)$ was equal to the pion vector form-factor $\mathcal{F}_{\gamma^*\pi\pi}(Q^2)$ up to normalization [14, 15]. This relation was found to be universal in the class of models considered here [1]

$$\mathcal{F}_{\pi\gamma\gamma}\left(Q^2,0\right) = \frac{N_C}{12\pi^2 f_\pi}\,\mathcal{F}_{\gamma\pi\pi}\left(Q^2\right) \,. \tag{9}$$

In the low-energy limit $Q^2 \to 0$, we recover the C_{22}^W relation in (5).

The form-factors from different models may have completely different behaviours at $Q^2 \to \infty$ [16], as one can see in Fig. 1 compared to the experimental data. In order to reproduce the observed $1/Q^2$ behavior for the form factors, the models need to be asymptotically AdS in the UV, as this is the case of the "cosh" and hard wall models [2, 3]. One finds the large-Q^2 behavior of the form factors [14]

$$\mathcal{F}_{\pi\gamma^*\gamma^*}\left(Q^2,0\right) \quad \stackrel{Q^2\to\infty}{\longrightarrow} \quad \frac{N_C g_5^2 f_\pi}{12\pi^2 Q^2} \,. \tag{10}$$

If the 5D coupling, g_5 is fixed by the perturbative QCD logarithmic term of the axial-vector correlator at short distances, *i.e.*, $g_5^2 = 24\pi^2/N_C$ [2], one recovers the asymptotic behavior $Q^2\mathcal{F}_{\pi\gamma^*\gamma^*}(Q^2,0) \stackrel{Q^2\to\infty}{\longrightarrow} 2f_\pi$ [17].

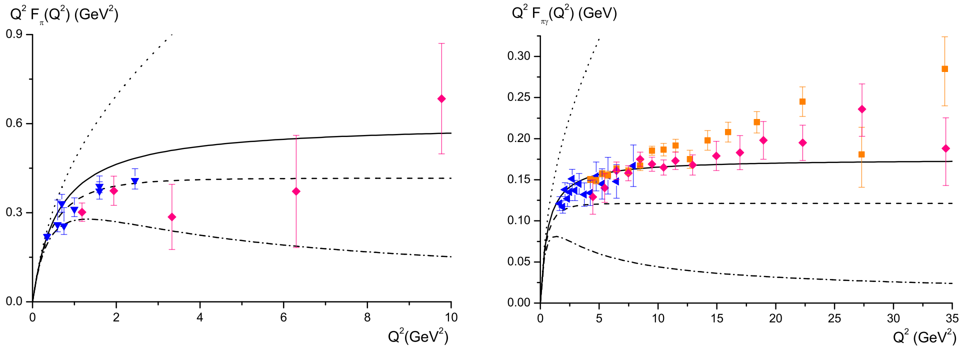

Fig. 1. Left: Vector form factor $\mathcal{F}_{\gamma^*\pi\pi}(Q^2)$ from the flat [2], "cosh" [2], hard wall [3] and Sakai–Sugimoto [4, 5] models, denoted by the dotted, solid, dashed and dash-dotted lines, respectively. The experimental data is taken from [18] (diamonds) and [19] (triangles). Right: Anomalous $\pi\gamma\gamma^*$ form factor: experimental data from CLEO [20] (triangles), BaBar [21] (squares) and Belle Collaboration [22] (diamonds).

3.3. Form factor relation: $A \to \pi\pi\pi$ versus $\pi \to AA$

A similar relation shows up between form factors involving the axial-vector source, related to the C_{23}^W expression in (5). This $\mathcal{O}(p^6)$ odd-parity coupling is related to the $\pi \to AA$ transition form-factor [1]

$$\begin{aligned}&\int \langle \pi^c(p)|T\left\{j_\mu^{5a}(x)j_\nu^{5b}(0)\right\}|0\rangle e^{-iq_1x}d^4x\\&= \frac{i\ N_C}{24\pi^2 f_\pi}\ d^{abc}\ \epsilon_{\mu\nu\alpha\beta}\ q_1^\alpha q_2^\beta\ \mathcal{F}_{\pi AA}\left(Q_1^2, Q_2^2\right)\end{aligned} \tag{11}$$

with $p = q_1+q_2$, $Q_1^2 = -q_1^2$, $Q_2^2 = -q_2^2$. In Ref. [1], we found that it is possible to relate this amplitude to the $A \to \pi\pi\pi$ form-factor in the massless quark limit [24]

$$\begin{aligned}&\langle \pi^a(p_1)\pi^b(p_2)\pi^c(p_3)|ij_\mu^{5d}|0\rangle = f^{bce}f^{ade}\ P_\mu^{\nu\perp}(q)\\&\times\left[\mathcal{F}_1(Q^2,s,t)(p_1-p_3)_\nu + \mathcal{F}_1\left(Q^2,t,s\right)(p_2-p_3)_\nu\right] + (a \leftrightarrow c)\ ,\end{aligned} \tag{12}$$

with $q = p_1+p_2+p_3$, $P_\alpha^{\mu\perp}(q) = \eta_\alpha^\mu - q_\alpha q^\mu/q^2$, $Q^2 = -q^2$, $s = (p_1+p_3)^2$, $t = (p_2+p_3)^2$, $u = (p_1+p_2)^2$ and the axial-vector current j_μ^{5a}.

In the particular kinematical regime $s = t = 0$, we find the relation between anomalous and even-parity sectors [1]

$$\frac{3f_\pi}{2}\,\mathcal{F}_1\left(Q^2,0,0\right) = \mathcal{F}_{\pi AA}\left(Q^2,0\right) = 1 - \sum_n \frac{3a_{Aa^n}c_{a^n}}{2}\,\frac{Q^2}{m_{a^n}^2+Q^2}\,. \tag{13}$$

The C_{23}^W relation in (5) is recovered by means of the low energy expansion of $\mathcal{F}_1(Q^2,0,0)$ and $\mathcal{F}_{\pi AA}(Q^2,0)$ in (13).

Based on these results, one may speculate that the relations between the $\mathcal{O}(p^6)$ odd and $\mathcal{O}(p^4)$ even-sector LECs represent the manifestation at low energies of further amplitude relations.

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