THE INFRARED FIXED POINT OF LANDAU GAUGE YANG–MILLS THEORY*

Axel Weber

Instituto de Física y Matemáticas
Universidad Michoacana de San Nicolás de Hidalgo
Edificio C-3, Ciudad Universitaria, A. Postal 2-82
58040 Morelia, Michoacán, Mexico

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Over the last decade, the infrared behavior of Yang–Mills theory in the Landau gauge has been scrutinized with the help of Dyson–Schwinger equations and lattice calculations. In this contribution, we describe a technically simple approach to the deep infrared regime via Callan–Symanzik renormalization group equations in an epsilon expansion. This approach recovers, in an analytical and systematically improvable way, all the solutions previously found as solutions of the Dyson–Schwinger equations and singles out the solution favored by lattice calculations as the infrared-stable fixed point (for space-time dimensions above two).

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After almost 40 years of Quantum Chromodynamics, it appears there, finally, is significant progress in achieving an analytical or semi-analytical description of the deep infrared (IR) regime, at least in the pure Yang–Mills (YM) sector, *i.e.*, in the absence of dynamical quarks. Maybe surprisingly, the methods employed, mainly Dyson–Schwinger (DS) equations [1–6] (and functional renormalization group equations [6, 7]), are standard methods in quantum field theory to go beyond perturbation theory. More recently, it has even been shown that perturbation theory is successful in describing the IR behavior of the propagators in the Landau gauge if a mass term for the gluons is taken into account [8, 9].

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In this contribution, we will employ renormalization group (RG) methods in order to reproduce the solutions of the DS equations and obtain deeper insight into the IR regime of YM theory. Among other things, we will find a natural explanation for the success of perturbation theory. The main results presented here have been derived in Ref. [10].

We start with the standard formulation of YM theory in the Landau gauge, with the Nakanishi–Lautrup field to implement the gauge fixing condition and ghost fields for the local expression of the Faddeev–Popov determinant leading to the action

$$S_{\mathrm{FP}} = \int d^D x \left(\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i B^a \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a D^{ab}_\mu c^b \right) \tag{1}$$

in D-dimensional Euclidean space-time. The existence of gauge copies in the Landau gauge [11], *i.e.*, the existence of different gauge equivalent fields that all satisfy the gauge condition $\partial_\mu A^a_\mu = 0$, forces one to restrict the functional integral over the gauge field to the first Gribov region[1]. It has been observed that this restriction of the functional integral breaks the BRST invariance of the theory [13, 14]. From an RG viewpoint it is natural to expect, and has indeed been confirmed in Ref. [14], that quantum corrections then generate a gluon mass term of the form

$$\int d^D x \, \tfrac{1}{2} A^a_\mu m^2 A^a_\mu \, . \tag{2}$$

For the following description of the IR regime of the theory, we add the latter term to the action (1). For small momenta $p^2 \ll m^2$, the mass term dominates over the other term in the action (1) that is quadratic in A^a_μ. In our IR analysis, we will hence effectively replace in the action (1) the term quadratic in A^a_μ by the mass term (2).

We will now perform Wilsonian RG transformations on this modified action, starting with the simple case of the "free" action $g = 0$ (without interaction terms). The only nontrivial step in the transformations is the rescaling of the fields. In order to keep the mass term (2) invariant, the gauge field has to transform as

$$A^a_\mu(x) \to s^{D/2} A^a_\mu(sx) \tag{3}$$

under a rescaling $x \to x/s$ of the space-time coordinates ($s > 1$). The scaling dimension of the gluon field is hence $D/2$ instead of the canonical

[1] Properly, the functional integral should be restricted to the fundamental modular region. Zwanziger has argued that this further restriction should have no effect on the correlation functions [12].

value $(D/2)-1$. The reason is that the scaling dimension of the gluon field in our case refers to the high-temperature fixed point rather than the usual critical fixed point (see, *e.g.*, Ref. [15]). The scaling dimension of the ghost field has the canonical value.

We now turn to the interacting theory in the vicinity of the fixed point of the free theory, *i.e.*, for small g. The scaling dimensions of gluon and ghost fields lead to the scaling

$$g_{\bar{c}Ac} \to s^{1-(D/2)} g_{\bar{c}Ac} \tag{4}$$

of the ghost–gluon coupling constant. Consequently, the ghost–gluon coupling is relevant for dimensions $D < 2$ and irrelevant for $D > 2$. The three- and four-gluon couplings are always irrelevant (for $D > 0$). The upper critical dimension of the theory is then 2. We will implement an epsilon expansion around $D = 2$ and neglect the three- and four-gluon couplings. The IR limit of the theory is then described by the action

$$S_{\mathrm{GD}} = \int d^D x \left(\tfrac{1}{2} A_\mu^a \, m^2 A_\mu^a + i B^a \partial_\mu A_\mu^a + \partial_\mu \bar{c}^a D_\mu^{ab} c^b \right) \tag{5}$$

in $D = 2 + \epsilon$ dimensions. Note that neglecting the three- and four-gluon couplings is equivalent to ghost dominance in the sense that to a given perturbative order only the diagrams with the biggest number of internal ghost propagators are retained. We will, therefore, refer to Eq. (5) as the ghost dominance approximation to the theory.

Standard renormalization of the theory defined by Eq. (5) leads to the beta function for the dependence of the dimensionless (with respect to the correct scaling dimensions of the fields) renormalized ghost–gluon coupling constant $\bar{g}_{\mathrm{R}}$ on the renormalization scale μ. To order ϵ, one obtains

$$\beta(\epsilon, \bar{g}_{\mathrm{R}}) = \mu^2 \frac{d}{d\mu^2} \, \bar{g}_{\mathrm{R}} = \frac{1}{2} \bar{g}_{\mathrm{R}} \left(\frac{\epsilon}{2} - \frac{1}{2} \frac{N \bar{g}_{\mathrm{R}}^2}{4\pi} \right) , \tag{6}$$

where N is the number of colors, see Ref. [10] for all details. Note that we are using the epsilon expansion *above* the critical dimension where the theory is, in the usual sense, perturbatively nonrenormalizable.

The beta function (6) has two fixed points: a trivial IR attractive one and a nontrivial IR unstable one at $N\bar{g}_{\mathrm{R}}^2/4\pi = \epsilon$ (for $\epsilon > 0$). For the IR unstable (and thus physically irrelevant) one, the solution of the Callan–Symanzik equations for the two-point functions yields *exactly* one of the two scaling solutions of the DS equations [4, 5], the one usually considered not physical. For the IR attractive (and hence physical) fixed point, we obtain the decoupling solution of the DS equations [16–18]. The fact that this fixed point is trivial implies the applicability of perturbation theory (with the inclusion of a gluon mass term) which has been demonstrated in Refs. [8, 9].

We can integrate Eq. (6) for the coupling constant to obtain information on the approach to this trivial fixed point. Solving the Callan–Symanzik equations for the renormalized propagators with the solution $\bar{g}_{\rm R}(\mu)$ obtained from Eq. (6) yields

$$\begin{aligned} \left\langle A^a_{{\rm R},\rho}(p) A^b_{{\rm R},\sigma}(-q)\right\rangle &= \frac{1}{m^2}\,\frac{1+(p^2/\Lambda^2)^{\epsilon/2}}{1+(\mu^2/\Lambda^2)^{\epsilon/2}}\left(\delta_{\rho\sigma}-\frac{p_\rho p_\sigma}{p^2}\right)\delta^{ab}(2\pi)^D\delta(p-q)\,, \\ \left\langle c^a_{\rm R}(p)\bar{c}^b_{\rm R}(-q)\right\rangle &= \frac{1}{p^2}\,\frac{1+(\mu^2/\Lambda^2)^{\epsilon/2}}{1+(p^2/\Lambda^2)^{\epsilon/2}}\,\delta^{ab}(2\pi)^D\delta(p-q)\,. \end{aligned} \tag{7}$$

Here, Λ is the characteristic scale where $N\bar{g}^2_{\rm R}(\Lambda)/4\pi = \epsilon/2$.

We can compare the IR behavior (7) directly with lattice calculations in Landau gauge in the limit of the lattice parameter $\beta \to 0$. In this limit, the gluonic part in Eq. (1) is completely absent from the action which precisely corresponds to the ghost dominance approximation (5) (remember that the gluon mass term originates from the BRST symmetry breaking due to the restriction to the Gribov region). Lattice calculations at $\beta = 0$ in $D = 3$ and 4 dimensions ($\epsilon = 1$ and 2) nicely confirm the qualitative behavior (7) [19].

For the full theory ($\beta > 0$), the linear rise with $|p|$ of the gluon propagator in three dimensions as predicted in Eq. (7) is also clearly seen in lattice simulations [20]. In four dimensions, however, it turns out that the p-dependence of the gluon propagator generated by the RG improvement in Eq. (7) is of the same order in p^2/m^2 as the contribution of the term quadratic in the gluon field in Eq. (1) that we have neglected in our IR analysis. We, therefore, have to reestablish the latter term (it does not receive quantum corrections to one-loop order) with the result that

$$\begin{aligned} &\left\langle A^a_{{\rm R},\rho}(p) A^b_{{\rm R},\sigma}(-q)\right\rangle \\ &= \left(p^2+\frac{m^2(\mu^2+\Lambda^2)}{p^2+\Lambda^2}\right)^{-1}\left(\delta_{\rho\sigma}-\frac{p_\rho p_\sigma}{p^2}\right)\delta^{ab}(2\pi)^D\delta(p-q)\,, \end{aligned} \tag{8}$$

for $\epsilon = 2$. The same form of the gluon propagator has been found in Ref. [14] (when the $\langle A^a_\mu A^a_\mu\rangle$-condensate is not taken into account) and is also qualitatively confirmed by lattice calculations in $D = 4$ dimensions [21–23]. Reintroducing the same term into the action in $D = 3$ dimensions also improves the concordance with the lattice calculations [20] for the gluon propagator.

From the beta function (6), we read off that the nontrivial (IR repulsive) fixed point is ultraviolet (UV) attractive. The fact that the corresponding scaling behavior arises in the UV regime of the ghost dominance approximation to the theory, or at lattice parameter $\beta = 0$, has lead to some confusion in the literature [19, 24, 25]. The limit $\beta \to 0$ has been associated with the IR regime of YM theory, and hence the appearance of scaling behavior was interpreted as evidence for the existence of the scaling solution in IR YM theory (see also Ref. [26]). Unfortunately, the values of the exponents of this scaling solution have not been firmly established. Our prediction for these values can be read off from the UV limit $p^2 \gg \Lambda^2$ in Eq. (7) (to the order of ϵ). It should be clear that the scaling behavior is exclusively related to the UV stable fixed point of the ghost dominance approximation and cannot appear in the full YM theory since the latter has a different UV stable fixed point, the well-known trivial fixed point associated with asymptotic freedom (while the ghost dominance approximation is only valid in the IR regime of YM theory).

Finally, we remark that for $D = 2$ dimensions ($\epsilon = 0$) the two fixed points in Eq. (6) coincide and the resulting trivial fixed point is IR unstable. Consistently, lattice calculations in two dimensions [27] do not find decoupling behavior for the propagators (but rather scaling behavior; however, we have not yet been able to construct the corresponding IR stable fixed point).

For reasons of space, we can only briefly comment that the other scaling solution of the DS equations (*cf.* the discussion after Eq. (6)) also arises in our RG approach if one implements the so-called horizon condition [13] in its simplest form as IR divergence of the ghost dressing function. The RG analysis can then be carried through as before. However, this second scaling solution turns out to be IR unstable, too, unless the horizon condition is enforced. For further details, the reader is refered to Ref. [28].

In summary, a renormalization group analysis gives considerable insight into the deep infrared regime of Yang–Mills theory when a gluonic mass term from the breaking of BRST symmetry due to the restriction of the functional integral to the first Gribov region is taken into account. All the different solutions of the Dyson–Schwinger equations are reproduced, but only the decoupling solution is found to be infrared stable, in agreement with the results of lattice simulations. The application of perturbation theory is justified through the triviality of the stable fixed point.

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