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TOWARDS A SELF-CONSISTENT SOLUTION OF THE LANDAU GAUGE QUARK-GLUON VERTEX DYSON–SCHWINGER EQUATION*

Andreas Windisch, Markus Hopfer, Reinhard Alkofer

Institut für Physik, Karl-Franzens Universität Graz
Universitätsplatz 5, 8010 Graz, Austria

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The quark-gluon vertex in Landau gauge QCD is investigated in the Dyson–Schwinger approach. The aim is to obtain a fully self-consistent solution of the quark propagator and the quark-gluon vertex Dyson–Schwinger equation (DSE) in the vacuum using the gluon propagator as input from other calculations. The truncation scheme used in an earlier study is systematically improved and the first decisive step towards a full solution is presented. First results for the quark propagator with a fully dressed quark-gluon vertex in a truncation that incorporates all twelve vertex tensor-structures are shown.

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1. Motivation

Prominent features of QCD like confinement and dynamical chiral symmetry breaking (DχSB) are still not satisfactorily understood. One possibility to explore the non-perturbative regime of QCD is to employ DSEs, an infinite tower of coupled integral equations, see *e.g.*, Ref. [1] and references therein. Their non-perturbative nature makes them an appropriate tool for exploring the low-energy domain of the theory. Careful truncations have to be imposed on the system in order to evaluate the equations. In this paper, we extend a prior study of the quark-gluon vertex [2] by systematically improving on the used truncation. The aim is to shed more light on a possible relation between confinement and DχSB by studying the quark-gluon vertex.

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The starting point is the system of equations for the quark propagator and the quark-gluon vertex in Landau gauge using fits for the gluon propagator as input [2, 3]. The three-gluon vertex is modeled as in Ref. [2][1]. The infrared suppression of the gluon propagator relates on the one hand to an infrared enhancement of the ghost propagator (see *e.g.*, Ref. [6]) and, on the other hand, to the positivity violation for transverse gluons. The latter property indicates the role of the transverse gluons in a non-perturbative BRST quartet mechanism [7] and, therefore, a certain aspect of gluon confinement: In Landau gauge QCD transverse gluons are confined and, therefore, not confining. Thus, given the infrared suppression of the gluon propagator, the quark-gluon vertex has to acquire a certain strength in the infrared to provide DχSB and, eventually, quark confinement. In Ref. [2], it has been found that within the imposed truncation (and at vanishing temperatures and densities) DχSB and quark confinement occur either together in a self-consistent way or are both absent. By improving on the truncation of the quark-gluon vertex DSE, we hope to get deeper insights into the possible relation between quark confinement and DχSB. In addition, isolating important tensor structures of the quark-gluon vertex will likely lead to improved phenomenological applications. Especially, corresponding studies at non-vanishing temperatures and/or quark chemical potentials can benefit from a more detailed understanding of the quark-gluon vertex, see *e.g.*, Ref. [8].

2. The coupled system

The coupled system of the quark propagator and the quark-gluon vertex is shown in Fig. 1. This system serves as a starting point for this study.

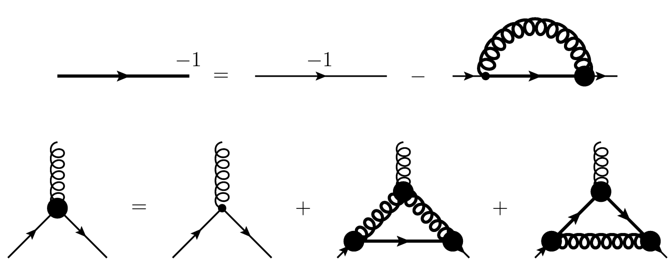

Fig. 1. The quark propagator and the quark-gluon vertex DSE. All internal propagators are dressed. The vertex equation has been derived from a 3PI effective action [9].

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[1] These fits relate to the so-called scaling solution, one of two types of solutions for Landau gauge Green functions when classified by their infrared behavior, see *e.g.*, Refs. [4, 5] and references therein. However, we want to emphasize here the evidence that the difference between these types of solutions is irrelevant for phenomenology.

The vertex equation in Fig. 1 has been derived from a 3PI effective action and corresponds to the DSE truncated at three-point level [9]. Note that, as usual in an nPI-based approach, all vertices are dressed. For obvious reasons, the last two diagrams on the right-hand side of the vertex equation are referred to as the *non-Abelian* and the *Abelian* diagram. Upon performing the color traces one finds that the non-Abelian diagram has a prefactor of $N_c/2$, whereas the Abelian diagram is suppressed by a factor of $-1/2N_c$. This suppression has been found to be even more severe due to the dynamics of the system [2]. Thus, as a first step to solve the system of the quark propagator and the quark-gluon vertex DSE, only the non-Abelian diagram has been taken into account.

2.1. A first step

The system as depicted in Figs. 2 and 3 has been considered as a first step towards the full solution of the system of Fig. 1. Within this approximation, one vertex of the non-Abelian diagram remains bare while the second is dressed such that the full vertex experiences feed-back from all twelve tensor structures. This vertex also enters the propagator equation, Fig. 2. In the vacuum, twelve tensor structures are needed as a basis to span the vertex. Instead of employing the widely-used Ball–Chiu basis [10] it turned out to

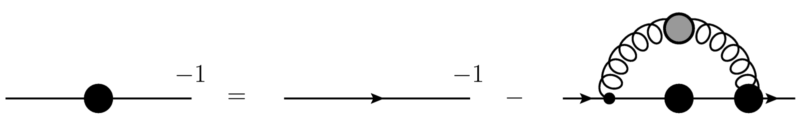

Fig. 2. Quark propagator DSE. Full blobs denote fully dressed quantities, shaded blobs are fits from other studies.

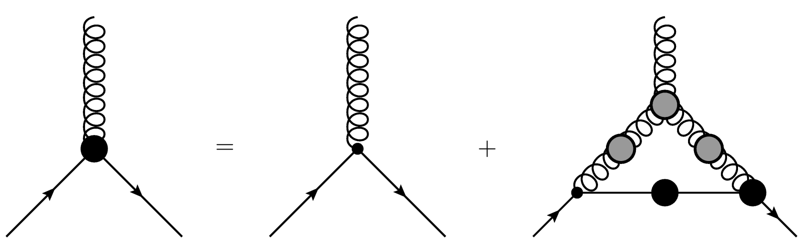

Fig. 3. The truncated quark-gluon vertex DSE. Full blobs denote fully dressed quantities, shaded blobs are fits from other studies. For the three-gluon vertex a model has been used which is adopted from [2].

be numerically advantageous to use a simpler basis given by

$$\Gamma^\mu(p_1,p_2) \propto \left\{ \begin{array}{c} \mathbb{1} \\ \not{p}_1 \\ \not{p}_2 \\ \frac{1}{2}[\not{p}_1,\not{p}_2] \end{array} \right\} \otimes \left\{ \begin{array}{c} \gamma^\mu \\ p_1^\mu \\ p_2^\mu \end{array} \right\},$$

where p_1 and p_2 are the in- and outgoing quark momenta. As input from the Yang–Mills sector the gluon propagator has been taken from earlier DSE studies[2]. The three-gluon vertex is represented by its tree-level tensor structure combined with the following ansatz [2]

$$\Gamma^{3g}(x) = \left(\frac{x}{d_1+x}\right)^{-3\kappa} \left(d_3 \frac{d_1}{d_1+x} + d_2 \log\left[\frac{x}{d_1}+1\right]\right)^{17/44}, \qquad (1)$$

where $x = p_1^2 + p_2^2 + p_3^2$ is the sum over the squared gluon momenta entering the vertex and $\kappa \approx 0.595$. This ansatz reflects the strong IR enhancement known from Yang–Mills theory (see, however, the remark in Sec. 3) as well as ensures the correct running in the UV region. The residual parameters d_1, d_2 and d_3 have to be fixed to physical observables, *e.g.* the chiral condensate.

2.2. Obtaining the vertex dressing functions

As a first step, the dressing functions have to be disentangled to get them in an explicit form. Since the employed basis is neither orthonormal nor orthogonal one obtains a linear system of equations for the vertex dressing functions in terms of the twelve right-hand side projections. This system can be solved in advance, giving rise to explicit expressions for the dressing functions as linear combinations of the right-hand side projections[3]. In each iteration step, these projections are evaluated numerically and are used subsequently to obtain the dressing functions by inserting them into the solution of the pre-calculated linear system of equations.

2.3. Renormalization and numerical treatment

The quark wave function renormalization constant Z_2, the mass renormalization constant Z_{m} as well as the quark-gluon vertex renormalization constant Z_{1F} have been fixed within a MOM scheme. In particular, the renormalization conditions for the quark dressing function $A(\mu^2) = 1$ and the tree-level vertex dressing function $\lambda_1(\mu^2,\mu^2,2\mu^2) = 1$ have been employed, where $\mu^2 = 170$ GeV^2 denotes the renormalization scale. Only the

[2] Here, a scaling-type fit obtained from a self-consistent solution of the corresponding ghost–gluon system has been used [3]. The fit parameter values are taken from [2].

[3] The algebraic manipulations have been carried out with the program `FORM` [11].

tree-level tensor γ^μ has to be concerned in this procedure since all other tensor structures lead to UV finite expressions. Z_m has been fixed by the condition $B(\mu^2) = m_R$, where m_R is the mass at the renormalization scale.

The evaluation of the coupled system of the quark propagator and the quark-gluon vertex as depicted in Fig. 2 and Fig. 3 is numerically demanding. The actual calculations are thus performed on a cluster featuring Graphics Processing Units (GPUs). GPUs have already been applied successfully to solve DSEs [12]. They will be vital once the full non-Abelian diagram and/or the Abelian diagram is taken into account.

3. First results

The system described in Sec. 2.1 has been solved self-consistently, taking all twelve tensor structures for the quark-gluon vertex into account. Figure 3 shows the behavior of the mass function $M(p^2)$ as obtained from the quark propagator dressing functions in the chiral limit. Due to stability issues it was not yet possible to take the full IR strength of the three-gluon vertex $(p^2)^{-3\kappa}$ into account[4].

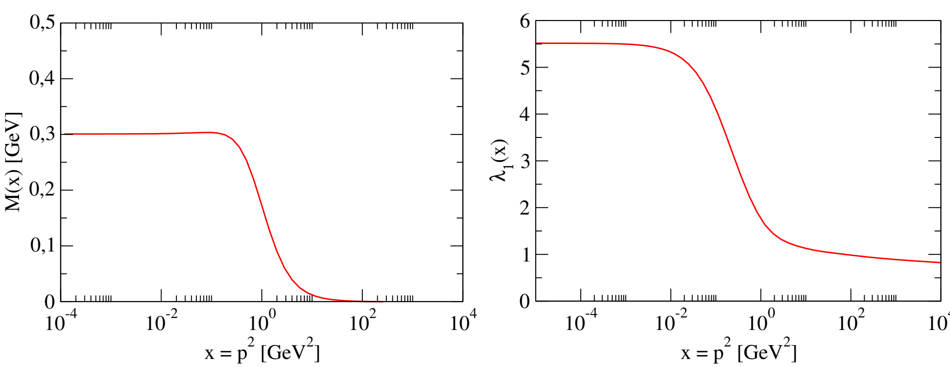

Fig. 4. Left: The mass function $M(p^2) = B(p^2)/A(p^2)$ as obtained from the coupled system described in Sec. 2.1. Right: The vertex dressing function $\lambda_1(p^2)$ of the tree-level part γ^μ of the quark-gluon vertex evaluated at $p_1^2 = p_2^2 = p^2$ and $p_1 \cdot p_2 = 0$.

Figure 3 shows the vertex dressing function λ_1 which corresponds to the tree-level structure γ^μ evaluated at the symmetric point $p_1^2 = p_2^2 = p^2$ and $p_1 \cdot p_2 = 0$. The significant IR enhancement seen in this function is also present in most other tensor structures, independent whether they have been generated by DχSB or are chirally symmetric.

[4] In our calculations, the corresponding IR exponent has been varied from 0 to -2κ resulting in moderate changes of the dynamical generated mass.

4. Conclusions and outlook

The coupled DSEs for the quark propagator and the quark-gluon vertex in Landau gauge has been investigated. Results for the quark propagator coupled to a partially dressed quark-gluon vertex including all twelve tensor structures have been presented. Isolating the important tensor structures will become vital for subsequent calculations taking the full non-Abelian as well as the Abelian diagram into account, aiming towards a complete solution of the system as depicted in Fig. 1. Such a reduction of complexity is also mandatory in studies involving the quark-gluon vertex, especially those at non-vanishing temperatures and/or quark chemical potentials.

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REFERENCES

[1] R. Alkofer, L. von Smekal, *Phys. Rep.* **353**, 281 (2001) [arXiv:hep-ph/0007355]; P. Maris, C.D. Roberts, *Int. J.Mod. Phys.* **E12**, 297 (2003) [arXiv:nucl-th/0301049].

[2] R. Alkofer, C.S. Fischer, F.J. Llanes-Estrada, K. Schwenzer, *Ann. Phys.* **324**, 106 (2009) [arXiv:0804.3042 [hep-ph]].

[3] C.S. Fischer, R. Alkofer, *Phys. Lett.* **B536**, 177 (2002) [arXiv:hep-ph/0202202]; C.S. Fischer, R. Alkofer, *Phys. Rev.* **D67**, 094020 (2003) [arXiv:hep-ph/0301094].

[4] C.S. Fischer, A. Maas, J.M. Pawlowski, *Ann. Phys.* **324**, 2408 (2009) [arXiv:0810.1987 [hep-ph]].

[5] F.J. Llanes-Estrada, R. Williams, *Phys. Rev.* **D86**, 065034 (2012) [arXiv:1207.5950 [hep-th]].

[6] P. Watson, R. Alkofer, *Phys. Rev. Lett.* **86**, 5239 (2001) [arXiv:hep-ph/0102332].

[7] N. Alkofer, R. Alkofer, *Phys. Lett.* **B702**, 158 (2011) [arXiv:1102.2753 [hep-th]]; *PoS* **QCD-TNT-II**, 002 (2011) [arXiv:1112.4483 [hep-th]].

[8] M. Hopfer, M. Mitter, B.-J. Schaefer, R. Alkofer, *Acta Phys. Pol. B Proc. Suppl.* **6**, 353 (2013), this issue.

[9] J. Berges, *Phys. Rev.* **D70**, 105010 (2004) [arXiv:hep-ph/0401172].

[10] J.S. Ball, T.-W. Chiu, *Phys. Rev.* **D22**, 2542 (1980).

[11] J.A.M. Vermaseren, arXiv:math-ph/0010025.

[12] M. Hopfer, R. Alkofer, G. Haase, arXiv:1206.1779 [hep-ph].