ELECTROWEAK HADRON STRUCTURE WITHIN A POINT-FORM APPROACH* **

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We present a relativistic point-form approach for the calculation of electroweak form factors of few-body bound states. As an example, the transition form factors for the semileptonic weak decay $B \to D^* e \bar{\nu}_e$ are discussed and it is sketched how they can be extracted unambiguously from the invariant transition amplitude that describes the process. It is shown how these form factors go over into one universal function, the Isgur–Wise function, in the heavy-quark limit, $m_Q \to \infty$, and comparison with the available experimental data is made.

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1. The point form of relativistic quantum mechanics

The point form is one of the three prominent forms proposed by Dirac in his seminal paper of 1949 [1] to formulate relativistic Hamiltonian dynamics. It has the nice feature that the whole Lorentz group (rotations and boosts) is kinematical, *i.e.* is not affected by interactions. This allows to boost and rotate bound-state wave functions in a simple way. As a price, all components of the 4-momentum operator become interaction dependent. The formalism presented here is based on the point form of relativistic quantum mechanics and makes use of the Bakamjian–Thomas construction [2, 3] for introducing interactions in a fully Poincaré invariant manner. As a consequence, the 4-momentum operator factorizes into an interacting mass operator and a free velocity operator so that it suffices to consider only an eigenvalue problem for the mass operator. The formalism presented here has been applied successfully to the study of electromagnetic properties of spin-0 and spin-1 two-body bound states consisting of equal-mass particles [4–6], as well as to the electroweak structure of mesons consisting of constituents with different masses [7–9].

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The starting point of all these calculations is the physical processes in which the form factors can be measured, *i.e.* electromagnetic scattering or weak decays. To describe these processes in a fully Poincaré invariant manner, a multichannel version of the Bakamjian–Thomas construction [2, 3] is employed. As mentioned already, the 4-momentum operator then factorizes into an interacting mass operator and a free 4-velocity operator

$$\hat{P}^\mu = \hat{P}^\mu_{\rm free} + \hat{P}^\mu_{\rm int} = \hat{M}\hat{V}^\mu_{\rm free} = \left(\hat{M}_{\rm free} + \hat{M}_{\rm int}\right)\hat{V}^\mu_{\rm free}\,. \tag{1}$$

The (free) 4-velocity operator $\hat{V}^\mu_{\rm free}$ is defined by $\hat{V}^\mu_{\rm free} := \hat{P}^\mu_{\rm free}/\hat{M}_{\rm free} = \hat{P}^\mu/\hat{M}$ and describes the overall motion of the system. The mass operator $\hat{M}$, depending on internal variables only, is the quantity of interest, since it contains the information on the internal structure of the system.

2. Extracting electroweak currents and form factors

Electron–meson scattering, *e.g.*, is then formulated on a Hilbert space consisting of a $eq\bar{q}$ and a $eq\bar{q}\gamma$ sector. A convenient basis consists of, so-called, *velocity states*. These are multiparticle states characterized by the overall 4-velocity and the center-of-mass momenta and spins of its components [10]. The mass eigenvalue equation to be solved has the form

$$\begin{pmatrix} \hat{M}^{\rm conf}_{eq\bar{q}} & \hat{K} \\ \hat{K}^\dagger & \hat{M}^{\rm conf}_{eq\bar{q}\gamma} \end{pmatrix} \begin{pmatrix} |\psi_{eq\bar{q}}\rangle \\ |\psi_{eq\bar{q}\gamma}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{eq\bar{q}}\rangle \\ |\psi_{eq\bar{q}\gamma}\rangle \end{pmatrix}. \tag{2}$$

The diagonal elements of the mass matrix contain the relativistic kinetic energies and an instantaneous confining interaction between the quarks. The transition between both channels is caused by $\hat{K}^\dagger$ and $\hat{K}$, which are vertex operators that account for the creation and annihilation of one photon, respectively. They are uniquely related to the interaction Lagrangian density of QED [11]. Eliminating the $eq\bar{q}\gamma$ channel, one ends up with an equation for the $eq\bar{q}$ component

$$\left(\hat{M}^{\rm conf}_{eq\bar{q}} - m\right)|\psi_{eq\bar{q}}\rangle = \underbrace{\hat{K}\left(\hat{M}^{\rm conf}_{eq\bar{q}\gamma} - m\right)^{-1}\hat{K}^\dagger}_{\hat{V}_{\rm opt}(m)}|\psi_{eq\bar{q}}\rangle\,. \tag{3}$$

$\hat{V}_{\rm opt}(m)$ is an optical potential that describes the (dynamical) 1-photon exchange between electron and (anti)quark. On-shell matrix elements of $\hat{V}_{\rm opt}(m)$ between (velocity) states of a confined $q\bar{q}$ pair with quantum numbers of the meson M provide the invariant 1-photon-exchange amplitude from which the electromagnetic current of the meson M can be extracted

$$\left\langle V'; \vec{k}'_e, \mu'_e; \vec{k}'_M, \mu'_M \right| \hat{V}_{\mathrm{opt}}(m) \left| V; \vec{k}_e, \mu_e; \vec{k}_M, \mu_M \right\rangle_{\text{on-shell}}$$
$$\propto V^0 \delta^3 \left(\vec{V} - \vec{V}'\right) \frac{j_\mu \left(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e\right) J^\mu \left(\vec{k}'_M, \mu'_M; \vec{k}_M, \mu_M\right)}{\left(k'_e - k_e\right)^2} . \qquad (4)$$

This relation determines the hadron current and thus the electromagnetic form factors in a unique way and it fixes also the normalization of the form factors. It can be shown [4, 6] that the resulting electromagnetic current transforms covariantly under Lorentz transformations and it is conserved for pseudoscalar mesons. However, because of cluster separability problems inherent in the Bakamjian–Thomas construction [3] one observes that the current obtained in this way cannot be decomposed in terms if hadronic covariants only, but one needs additional covariants built with the sum of the incoming and outgoing electron 4-momenta [5–7]. The need of such additional, spurious covariants resembles the situation in the covariant front-form formalism [12]. There one also encounters spurious dependencies of the currents on a 4-vector that specifies the orientation of the light front. Remarkably, if we let the invariant mass of the electron–meson system go to infinity our electromagnetic form factors turn out to agree with the front-form results computed in the $q^+ = 0$ frame [4–6][1].

It is quite obvious, how this formalism can be generalized to semileptonic weak decays in order to calculate transition amplitudes, currents and decay form factors [7]. Decay processes involve time-like momentum transfers. Unlike scattering the covariant decomposition of decay currents does not require the introduction of spurious covariants, neither in pseudoscalar-to-pseudoscalar nor in pseudoscalar-to-vector meson decays. Form factors can be extracted unambiguously in a frame-independent way [7]. Analytical and numerical studies of electromagnetic and weak form factors of heavy-light systems show that the predictions of heavy-quark symmetry are respected when one of the constituent masses goes to infinity [7]. As it should be, one ends up with the, so-called, Isgur–Wise function [13, 14], *i.e.* one single, universal, spin-independent form factor that does not depend on the mass of the heavy quark. Within our approach the Isgur–Wise function acquires a simple analytical form

[1] For a comprehensive discussion of how to handle cluster problems in electromagnetic form factors of pseudoscalar and vector bound states of equal-mass constituents, see Ref. [6]. A more detailed analysis of cluster problems in heavy-light systems, their elimination in the heavy quark limit and the connection of point- and front-form results can be found in Ref. [7].

$$\xi\left(v\cdot v'\right)=\int\frac{d^3\tilde{k}'_{\bar{q}}}{4\pi}\sqrt{\frac{\omega_{\tilde{k}_{\bar{q}}}}{\omega_{\tilde{k}'_{\bar{q}}}}}\,\mathcal{S}\,\psi^*\left(\left|\vec{\tilde{k}}'_{\bar{q}}\right|\right)\,\psi\left(\left|\vec{\tilde{k}}_{\bar{q}}\right|\right)\,, \tag{5}$$

with v and v' denoting the initial and final meson 4-velocities, respectively. $\mathcal{S}=\left(m_{\bar{q}}+\omega_{\tilde{k}'_{\bar{q}}}+\tilde{k}'^{1}_{\bar{q}}\sqrt{\frac{(v\cdot v')-1}{(v\cdot v')+1}}\right)\Big/\left((m_{\bar{q}}+\omega_{\tilde{k}_{\bar{q}}})(m_{\bar{q}}+\omega_{\tilde{k}'_{\bar{q}}})\right)^{1/2}$ is a spin-rotation factor and $\omega_{\tilde{k}_{\bar{q}}}=\tilde{k}'^{1}_{\bar{q}}\sqrt{(v\cdot v')^2-1}+\omega_{\tilde{k}'_{\bar{q}}}\left(v\cdot v'\right)$ with $\omega_{\tilde{k}'_{\bar{q}}}=\sqrt{m_{\bar{q}}^2+\vec{\tilde{k}}'^2}$ [2].

3. The $B\to D^*e\bar{\nu}_e$ transition form factors

As an example, we present numerical results for the transition form factors of the semileptonic $B\to D^*e\bar{\nu}_e$ decay. Calculations are done with a simple harmonic-oscillator wave function (see Ref. [7]) with oscillator parameter $a=0.55$ GeV. The most general covariant decomposition of the weak $B\to D^*$ transition current is usually written in the form

$$\begin{aligned}
J^{\nu}_{B\to D^*}\left(\underline{\vec{p}}'_{D^*},\underline{\sigma}'_{D^*};\underline{\vec{p}}_B\right)&=\frac{2i\epsilon^{\nu\mu\rho\sigma}}{m_B+m_{D^*}}\,\epsilon^*_{\mu}\left(\underline{\vec{p}}'_{D^*},\underline{\sigma}'_{D^*}\right)\,\underline{p}'_{D^*\rho}\,\underline{p}_{B\sigma}\,V\left(\underline{q}^2\right)\\
-2m_{D^*}\frac{\epsilon^*\left(\underline{\vec{p}}'_{D^*},\underline{\sigma}'_{D^*}\right)\cdot\underline{q}}{\underline{q}^2}\,\underline{q}^{\nu}A_0\left(\underline{q}^2\right)&-(m_B+m_{D^*})\,\epsilon^{*\nu}\left(\underline{\vec{p}}'_{D^*},\underline{\sigma}'_{D^*}\right)A_1\left(\underline{q}^2\right)\\
+\frac{\epsilon^*\left(\underline{\vec{p}}'_{D^*},\underline{\sigma}'_{D^*}\right)\cdot\underline{q}}{m_B+m_{D^*}}\left(\underline{p}_B+\underline{p}'_{D^*}\right)^{\nu}A_2\left(\underline{q}^2\right)&+2m_{D^*}\frac{\epsilon^*\left(\underline{\vec{p}}'_{D^*},\underline{\sigma}'_{D^*}\right)\cdot\underline{q}}{\underline{q}^2}\,\underline{q}^{\nu}\,A_3\left(\underline{q}^2\right)\,,
\end{aligned} \tag{6}$$

where $2m_{D^*}A_3(\underline{q}^2)=(m_B+m_{D^*})A_1(\underline{q}^2)-(m_B-m_{D^*})A_2(\underline{q}^2)$.

As mentioned above, our microscopic expression for the $B\to D^*$ transition current is not plagued by spurious contributions and the covariant decomposition (6) can be applied directly to extract the decay form factors [7]. Introducing the shorthand notation $J^{\nu}(\underline{\mu}'_{D^*}):=J^{\nu}_{B\to D^*}(\underline{\vec{k}}'_{D^*},\underline{\mu}'_{D^*};\underline{\vec{k}}_B)$ and adopting the same kinematics as in Ref. [7], one encounters 10 non-vanishing spin matrix elements of the current, namely $J^2(0)$, $J^3(0)$, and $J^{\mu}(\pm1)$, $\mu=0,1,2,3$. Taking into account that $J^{\mu}(1)$ and $J^{\mu}(-1)$ are related by parity, one is left with only 6 different matrix elements, 4 of them being independent. One can see that A_0 and A_2 enter only $J^0(1)$ and $J^1(1)$. Thus the set $J^2(0)$, $J^3(0)$, $J^0(1)$ and $J^1(1)$ can be used to extract all the $B\to D^*$ decay form factors.

[2] A numerical analysis carried out in Ref. [7] reveals that the spin-rotation factor $\mathcal{S}$ is by no means negligible, which emphasizes the need of an appropriate relativistic treatment of the spin rotation when boosting bound states.

The numerical results for the form factors as a function of $v \cdot v'$ (multiplied by appropriate kinematical factors [13, 14]) are shown and compared with the Isgur–Wise function and with the available experimental data in Fig. 1. Our numerical values for the Isgur–Wise function agree with those obtained with a front-form quark model [15]. Discrepancies between the point- and front-form approach show up as soon as the decay form factors are calculated for finite, physical masses of the heavy quarks. Most likely, these differences can be attributed to the different roles played by Z-graphs, *i.e.* non-valence contributions, in either approach. In the heavy-quark limit, Z-graphs do not contribute to the current, neither in the front form nor in the point form, which explains why the results agree for the Isgur–Wise function. For finite quark masses, however, the inclusion of Z-graphs seems to be crucial for the frame independence of the decay form factors in front form [15, 16], whereas this is not the case in point form (as discussed above).

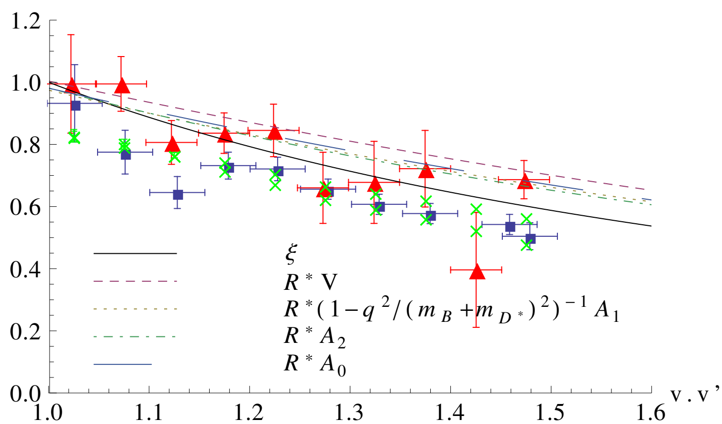

Fig. 1. Weak $B^- \to D^{0*}$ decay form factors (multiplied by appropriate kinematical factors, $R^* = 2\sqrt{m_B m_D}/(m_B + m_D)$) in comparison with the Isgur–Wise function $\xi(v \cdot v')$ and with the available experimental data. The values taken for the physical quark masses are $m_u = 0.25$ GeV, $m_b = 4.8$ GeV and $m_c = 1.6$ GeV. Experimental data are taken from Belle [17] (dots), CLEO [18] (triangles) and BaBar [19] (crosses) assuming that $|V_{cb}| = 0.0409$, *i.e.* the central value given by the Particle Data Group [20].

4. Summary and conclusions

A Poincaré invariant description of electromagnetic and weak currents and form factors of two-body systems has been presented. The predictions of heavy-quark symmetry are respected by this approach and a simple analytical expression for the Isgur–Wise function in terms of the initial and final wave function has been derived. As an example, numerical results for the semileptonic $B \to D^* e\bar{\nu}_e$ decay have been given. Direct comparison of the Isgur–Wise function with the transition form factors that are obtained for physical masses of the heavy quarks revealed that heavy-quark symmetry is broken by about 20%. It remains to be seen, whether Z-graph contributions to the decay process may restore the equivalence of our point-form approach with front-form calculations that has already been found for electromagnetic form factors in the space-like momentum-transfer region [4–6].

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