# NON-EQUILIBRIUM DYNAMICS OF QGP IN AdS/CFT FRAMEWORK*

Przemyslaw Witaszczyk

The Marian Smoluchowski Institute of Physics, Jagiellonian University
Reymonta 4, 30-059 Kraków, Poland

*(Received January 24, 2013)*

Recently developed numerical method in AdS/CFT correspondence allows for simulation of strongly coupled non-equilibrium expanding $\mathcal{N}=4$ SYM fluid. This system serves as a testbed for RHIC and LHC quark-gluon plasma research, as well as a well-defined toy model of strongly coupled, non-perturbative non-Abelian gauge theory at finite temperature. Application of numerical tools allowed to gain some insight into the process of transition to hydrodynamic regime and also gave a few surprisingly simple characteristics of hydronization and thermalization process.

DOI:10.5506/APhysPolBSupp.6.53
PACS numbers: 11.25.Tq, 25.75.–q, 04.25.D–

## 1. Introduction

Non-Abelian gauge theories lay at the very heart of modern understanding of interactions in the Nature. From elementary particles to gravity, all the forces are mediated by some sort of a gauge field. In the case of the Standard Model, one can efficiently test the theory by very precise experiments, in which one finds new particles and states of matter. One of the most recent and most exciting discoveries of this kind, is the observation of deconfined phase of QCD, namely the celebrated Quark-Gluon Plasma [1]. This new state of matter is focusing a lot of attention due to surprising properties. It is very heavily coupled, despite high energy, it has very short thermalization time after which it is well described by hydrodynamics, and most it is by far the best (near-)perfect fluid known [2].

Unfortunately, although well described by hydrodynamic models, QGP is still in the domain of non-Abelian gauge theories, and those are notoriously difficult to understand at strong coupling. Any insight into the non-perturbative properties of such theories is desired. Here enters the AdS/CFT correspondence. The duality [3] originates from superstring theory and

* Presented at the Light Cone 2012 Conference, Kraków, Poland, July 8–13, 2012.

states, that certain classes of gauge theories are fully equivalent to some gravitational theories. One of the most widly known is the duality between $\mathcal{N} = 4\, d = 4$ Super-Yang–Mills in the 't Hooft limit and Einstein gravity with negative cosmological constant $\Lambda = -6$. To obtain information about processess in the field theory one considers evolution of 5-dimensional metric and other relevand fields, and from its Taylor expansions reads off VEVs of the operators. The application of the duality to the problem of QGP relies on the possibility of obtaining the full time evolution of quantum expectation values of SYM operators at strong coupling from gravity. Basic steps are the following. We consider 5-dimensional geometry in Poincare patch $(A, B = 0, \dots, 4)$

$$ds^2 = G_{AB}(t, x_1, x_2, x_3, z) dx^A dx^B = \frac{g_{\mu\nu} dx^\mu dx^\nu + dz}{z}\,, \tag{1}$$

subject to Einstein equations

$$R_{AB} - \tfrac{1}{2} R G_{AB} - 6 G_{AB} = 0\,. \tag{2}$$

The solution is pursued perturbatively in $z$ and exactly in time $t$, by expanding the metric in the holographic (or bulk) variable, $z$, ranging from conformal boundary at $z = 0$ to the interior of AdS, $z \to \infty$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^6 g^{(6)}_{\mu\nu} + \dots \tag{3}$$

To solve so expanded Einstein equations, boundary conditions are imposed on the metric, such that at the boundary $z = 0$ the metric is Minkowski, $g_{\mu\nu} = \eta_{\mu\nu}$, and fourth expansion coefficient is equal to the boundary theory stress-energy tensor VEV. This also gives the desired relation between the VEV and the metric solution

$$\langle T_{\mu\nu} \rangle = \frac{N_c^2}{2\pi^2} g^{(4)}_{\mu\nu}\,. \tag{4}$$

Equations can be solved with generic ansatz for the VEV, and then from properties of general relativity (like non-singularity of the geometry) specific solution should be picked. Then, by expanding the metric, one can retrieve the desired VEV of gauge theory at strong coupling.

For the specific case of QGP, one very well motivated model of fluid can be given, going back to Bjorken [4]. Since the collisions involve an extent medium (the blob is large compared to the micro scale of parton collisions) and occur at very high energies, one can simplify things by considering boost invariant, translation and rotation symmetric stress tensor, in proper time-rapidity coordinates

$$\langle T_{\mu\nu} \rangle = \mathrm{Diag}\left( \epsilon(\tau), \tau^2 p_{\mathrm{L}}(\tau), p_{\mathrm{T}}(\tau), p_{\mathrm{T}}(\tau) \right)\,. \tag{5}$$

Here $p_{\rm L}$ and $p_{\rm T}$ are longitudinal and transverse pressures. It is conserved, $\nabla_\mu \langle T^{\mu\nu}\rangle = 0$ and since $\mathcal{N} = 4\, d = 4$ SYM is conformal, also traceless: $\langle T^\mu_\mu\rangle = 0$. Eventually, one is left with only one unknown function, namely the energy density $\epsilon(\tau)$ in the rest frame

$$\langle T_{\mu\nu}\rangle = \mathrm{Diag}\left(\epsilon(\tau), -\tau^2(\epsilon(\tau)+\tau\epsilon'(\tau)), \epsilon(\tau)+\tfrac{1}{2}\tau\epsilon'(\tau), \epsilon(\tau)+\tfrac{1}{2}\tau\epsilon'(\tau)\right) . \tag{6}$$

Time dependence of this quantity should be specified by the dual gravity theory.

Previous developments in this direction gave several analytic results. In particular, late and early time evolution of the metric (in proper time $\tau$) was considered by perturbative means [5, 6]. From those, we know the initial non-equilibrium stage of the evolution, and we know that at the end, the field theory should be described by expanding perfect fluid. Very interesting, intermediate stage of transition to some sort of equilibrium remains a challange, due to non-linear partial differential equations that needs to be solved (despite the physically justified simplifications). This motivated us to develop numerical approach that could solve the full non-linear equations in a non-perturbative manner (without any further expansions in time or metric coefficients). It turned out that there is a formulation of general relativity which is very suitable for numerical simulations, and at the same time makes it possible to use initial data constructed for the analytic early time investigation. It is the ADM formalism. Another very advantageous feature of this approach is related to the fact, that the initial data are always singular [6], which signals the presence of a horizon from the onset. The ADM nevertheless has an 'ability' to cease the evolution at some given point, effectively excising singular portion of spacetime from the numerical domain [9]. We thus consider the metric

$$ds^2 = \frac{-a^2(u)\alpha^2(t,u)dt^2 + t^2a^2(u)b^2(t,u)dy^2 + c^2(t,u)dx_\perp^2}{u} + \frac{d^2(t,u)du^2}{4u^2}\,, \tag{7}$$

where $a = \cos(u)$, $\alpha$ is so-called lapse, the shift $\beta$ is zero, and the extrinsic curvature reads

$$K_{ij} = \mathrm{Diag}\left(ta(u)L(t,u), M(t,u), M(t,u), P(t,u)/(4u)\right)/\sqrt{u}\,. \tag{8}$$

The space of initial data solving the constraints is parametrized by just one arbitrary function of $u$ [6, 9], and one can perform numerical integration in time. Since the metric is time-dependent, we expect the presence of apparent horizon. Its position in the bulk was tracked with an aid of so-called expansions. Those functions describe null congruence of geodesics and reflect the focusing of spacetime. Apparent horizon is defined by the values of outgoing and ingoing expansions, obeying

$$\theta_l = 0\,, \qquad \theta_n < 0\,. \tag{9}$$

This information was crucial for the numerics, since as mentioned the domain of integration should not contain singularites. Thus the horizon location was used as the cut-off.

## 2. The results

The outcome of integration can be seen in Fig. 1. The bulk variable $u$ runs horizontally to the right and time flows upwards. The arrows indcate null geodesics escaping towards the boundary $u = 0$. Dashed line represents the apparent horizon, and thickened/red line represents marginally trapped geodesics, which asymptotically touches the horizon. The color represents the value of curvature through invariant, $\mathfrak{R}^2$. At the top, one can see the formation of singularity, covered by the horizon as it should be.

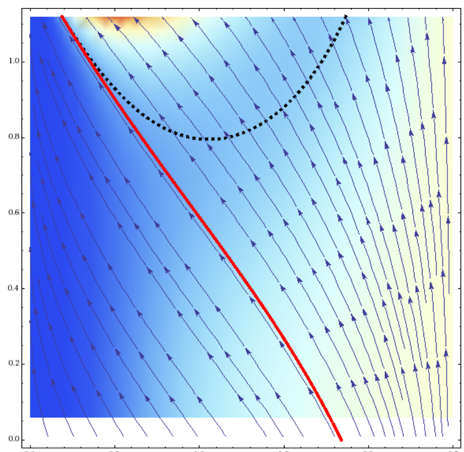


Fig. 1. (Color online) AdS dynamic black hole, obtained from numerics in ADM coordinates.

One of the most interesting results is related to the observation of transition to hydrodynamics (now so-called hydronization), which as it appears, does not go along with perfect isotropisation [7]. By introducing an effective temperature and dimensionless parameter

$$\langle T_{\tau\tau}\rangle = \epsilon(\tau) = N_c^2 \tfrac{3}{8}\pi^2 T_{\rm eff}^4(\tau)\,, \qquad w(\tau) = T_{\rm eff}(\tau)\tau\,, \tag{10}$$

the general equation of hydrodynamics, of arbitrary order in gradients, can be recast in the dimensionless form

$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{\rm hydro}(w)}{w}\,. \tag{11}$$

The right-hand side defines this differential equation, so if one would put in a known solutions, it should always evaluate to the same value on them. Thus for all the solutions $\epsilon(\tau)$ in the hydrodynamic phase, it should give the same numbers. One can clearly see this in Fig. 2. Initially, the system is in a highly non-equlibrium stage, and then (for very different initial data) all the lines merge in approximately close range of the parameter, $w_* \sim 0.65$. At the same time, the system is not yet isotropic: pressure anisotropy

$$\Delta p = 1 - \frac{3p_{\mathrm{L}}}{\epsilon} = 12\frac{F(w)}{w} - 8 \tag{12}$$

evaluated at $w_*$ is of the order of 0.6, and agrees with analytic 3rd order hydrodynamics [7]. Quantity $F(w)/w$ obtained from numerics represents 'all-order hydrodynamics', namely, since it is computed from full non-linear Einstein equations, it incorporates all the infinite number of gradients, as well as transport coefficients. It may be thought of as a sort of resummation of hydrodynamic series, and as it is manifest from Fig. 2, it is finite at all times. The function $F(w)/w$ is known analytically only up to third order, in boost-invaraint case [8]

$$\frac{F_{\mathrm{hydro}}}{w} = \frac{2}{3} + \frac{1}{9\pi w} + \frac{1-\ln(2)}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45\ln(2) + 24\ln^2(2)}{972\pi^3 w^3} + \dots \tag{13}$$

The properties of the 'all-order' expression $F(w)/w$ clearly deserve further attension. In particular, it would be very interesting, to extract from it higher order transport coefficients, as well as other phenomenologically motivated data. It is a subject of ongoing research.

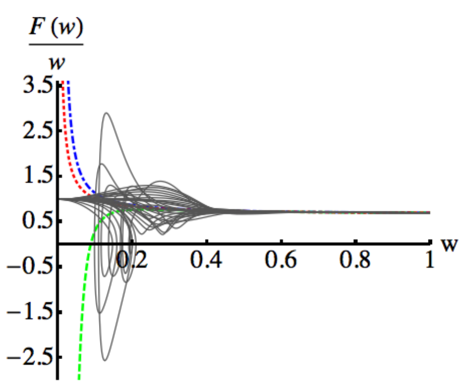


Fig. 2. Transition to ('all-order') hydrodynamics, observed for all the initial data.

Other characteristics of plasma evolution, that were observed during this study, are entropy density production $\Delta s$, thermalization time $\tau^{\mathrm{th}}T_{\mathrm{eff}}^{(i)}$

(in units of effective initial temperature) and thermalization temperature $T_{\rm eff}^{(\rm th)}/T_{\rm eff}^{(i)}$, all as a functions of initial non-equlibrium entropy density. Detailed discussion can be found in [7, 9].

## 3. Conclusions

The numerical scheme used to study full evolution of holographic fluid turned out to be very fruitful. Several results, inaccessible untill now, could be brough up to the attention, and some may even cause a shift in paradigm of phenomenological approach to the QGP physics. It is the idea that fluid may be well described by viscous hydrodynamics, even at manifestly anisotropic stage. Thus, one should distinguish between the thermalization, *i.e.* stage where thermodynamic equilibrium has been reached, and the state, where hydrodynamics can be successfully applied ('hydronization'). Further, the introduced resummed hydrodynamics may help to establish a better criterion for the transition to hydrodynamic regime.

The method also had its drawbacks. The coordinates used suffered from manifestly present horizons (apparent and event), which causes problems in numerical integration. Those problems were cured with the aid of ADM scheme, but not without effort. Alternative approach would be to use Eddington–Finkelstein coordinates, in which one uses natural characteristics of the equations, namely null observers, as coordinates and removes explict horizons from the equations [10]. Nevertheless, it seems that there is a lot of potential in numerical research in AdS/CFT, in the upcoming years.

The work presented and discussed here was performed in collaboration with Romuald Janik and Michał Heller, for which the author is deeply grateful. Earlier numerical studies of AdS/CFT in QGP context involve *e.g.* [11] and later works. The work was financially supported by the Jagiellonian University grant No. K/DSC/000705.

## REFERENCES

[1] U.W. Heinz, R. Snellings, `arXiv:1301.2826 [nucl-th]`.
[2] U.W. Heinz, *J. Phys. A* **42**, 214003 (2009).
[3] J.M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
[4] J.D. Bjorken, *Phys. Rev.* **D27**, 140 (1983).
[5] R.A. Janik, R.B. Peschanski, *Phys. Rev.* **D73**, 045013 (2006).
[6] G. Beuf, M.P. Heller, R.A. Janik, R. Peschanski, *J. High Energy Phys.* **0910**, 043 (2009).
[7] M.P. Heller, R.A. Janik, P. Witaszczyk, *Phys. Rev. Lett.* **108**, 201602 (2012).
[8] I. Booth, M.P. Heller, M. Spalinski, *Phys. Rev.* **D80**, 126013 (2009).
[9] M.P. Heller, R.A. Janik, P. Witaszczyk, *Phys. Rev.* **D85**, 126002 (2012).
[10] M.P. Heller *et al.*, *Phys. Rev. Lett.* **102**, 041601 (2009).
[11] P.M. Chesler, L.G. Yaffe, *Phys. Rev. Lett.* **102**, 211601 (2009).