

SCALING BEHAVIOR OF TRANSVERSE MOMENTA
DISTRIBUTIONS IN HADRONIC AND NUCLEAR
COLLISIONS*MACIEJ RYBCZYŃSKI[†], ZBIGNIEW WŁODARCZYK[‡]Institute of Physics, Jan Kochanowski University
Świętokrzyska 15, 25-406 Kielce, PolandGRZEGORZ WILK[§]

National Centre for Nuclear Research, Hoża 69, 00-681 Warszawa, Poland

(Received January 25, 2013)

It has been recently noticed that transverse momenta (p_T) distributions observed in high energy production processes exhibit remarkably universal scaling behavior. This is seen when they are in some suitable variable, replacing the usual p_T . On the other hand, it is also known that transverse momenta distributions, in general, follow a power-like Tsallis distribution, rather than an exponential Boltzmann–Gibbs, with a (generally energy dependent) nonextensivity parameter q . We now show that it is possible to choose a suitable variable such that p_T distributions of particles produced in proton–proton interactions in a wide energy range can be fitted by the same Tsallis distribution (with the same, energy independent, value of the q -parameter). Similar scaling behavior in nucleus–nucleus collisions is also observed. The possible dynamical origin of the q parameter used in these fits will be discussed.

DOI:10.5506/APhysPolBSupp.6.507

PACS numbers: 05.90.+m, 13.85.-t, 11.80.Fv, 13.75.Cs

It is well known [1–3] that p_T distributions, in general, follow a two-parameter power-like Tsallis distribution [4] characterized by some energy dependent nonextensivity parameter q , which for $q \rightarrow 1$ becomes the usual

* Presented at the International Symposium on Multiparticle Dynamics, Kielce, Poland, September 17–21, 2012.

[†] maciej.rybczynski@ujk.edu.pl

[‡] zbigniew.wlodarczyk@ujk.edu.pl

[§] wilk@fuw.edu.pl

one-parameter exponential Boltzmann–Gibbs (BG) distribution¹

$$h_q(p_T) = C_q \left[1 - (1 - q) \frac{p_T}{T} \right]^{\frac{1}{1-q}} \xrightarrow{q \rightarrow 1} h(p_T) = C_1 \exp\left(-\frac{p_T}{T}\right), \quad (1)$$

(where C_q is a normalization constant)². The transverse momenta distributions are then supposed to bring information on the thermodynamical properties of the production process, on the temperature T parameter in the case of exponential BG distributions, or, additionally, also on some intrinsic fluctuations presented in such systems and described by the nonextensivity parameter q in Tsallis distributions.

It was recently shown that transverse momenta (p_T) distributions observed in high energy production processes exhibit a universal scaling behavior when presented in suitable variables, for example:

- The variable p'_T [7]. It is defined by demanding that p_T at energy W should be connected with p'_T at energy W' via $p'_T = p_T (W'/W)^{\frac{\lambda}{\lambda+2}}$; it can be reproduced by a two-parameter Tsallis fit, $h_q(p'_T)$, cf. Fig. 1 (left panel);
- The variable $u = u(p_T) = \frac{p_T}{\langle p_T \rangle - b p_T}$. It is discussed in [8], cf. Fig. 1 (right panel) showing $h_q(u)$; here, b is an energy dependent parameter, $b = b(s)$.

In both cases, q is energy independent, *i.e.*, one obtains a kind of q -scaling phenomenon.

It must be stressed at this point that essentially *all data* observed in high energy production processes can be represented by a universal scaling distribution, $\psi(z)$, [12–14]³. In Fig. 2, we demonstrate that these curves can also be nicely reproduced by two-parameter Tsallis fits $h_q(z)$.

¹ In phenomenological fits, the Tsallis distribution is sometimes used interchangeably with the old two-parameter, purely phenomenological, power-like parametrization, the so-called “Hagedorn formula” [5]. Here, we shall not discuss this possibility.

² Actually, as shown in [6], $h_q(p_T)$ can fit *all* available data on p_T distributions from RHIC to LHC, *i.e.*, up to p_T 200 GeV/ c . The fact that data behave in such a way that they can be fitted by a simple two-parameter Tsallis formula for such a broad p_T range is really a phenomenon awaiting a proper understanding.

³ Where, in short: $\psi(z) = -\pi s / [(dN/d\eta)\sigma_{\text{in}}] J^{-1} E d^3\sigma / dp^3$, with J being the corresponding Jacobian of transformations from variables $\{p_z, p_T\}$ to $\{z, \eta\}$; $z = z_0 \Omega^{-1}$, where $z_0 = \sqrt{s_T} / [m (dN_{\text{ch}}/d\eta)_0^c]$ and Ω^{-1} is the minimal resolution at which a constituent subprocess can be singled out of the inclusive reaction, $\sqrt{s_T}$ is the transverse kinetic energy of the subprocess consumed on production of m_1 and m_2 , $dN_{\text{ch}}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$, c is a parameter interpreted as a “specific heat” of created medium and m is an arbitrary constant (usually fixed at the value of nucleon mass).

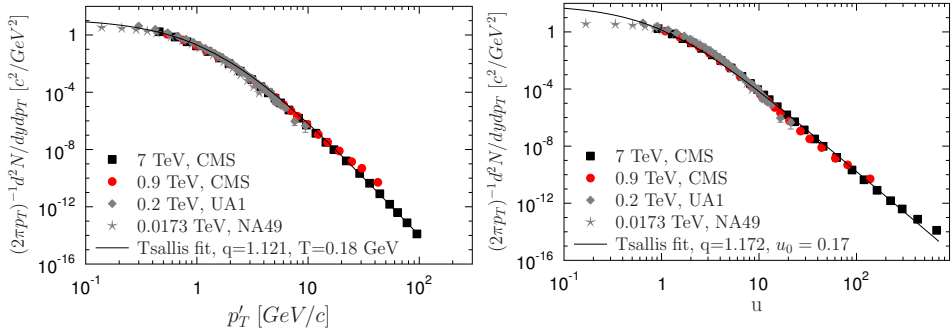


Fig. 1. (Color online) $p+p$, $(p+\bar{p})$ data for transverse momentum distributions for different energies [9–11] plotted by using as scaling variables p'_T (left panel) and u (right panel).

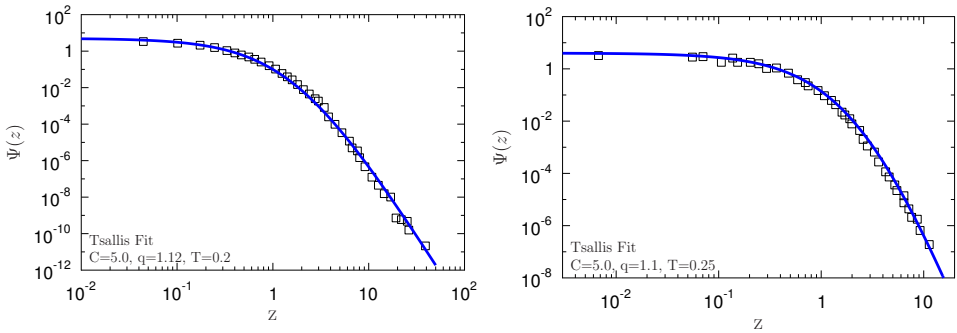


Fig. 2. (Color online) Tsallis fits to z -scaling plots presented by Tokarev. Left panel: spectra of charged hadrons produced in pp collisions at energies $\sqrt{s} = 19\text{--}2360$ GeV [14]; right panel: spectra of different particles produced at $\sqrt{s} = 200$ GeV (from Tokarev [13]).

In this way, we have reached our main point. Both $h_q(p'_T)$ and $h_q(u)$ (not to mention $h_q(z)$) depart from the usual connection of the nonextensive Tsallis distribution with a thermodynamical approach (justified and advocated in [15]). Instead, as was already mentioned in [8], we opt for some more dynamical source of h_q . As shown there, our scaling seems to indicate close connections with the so-called *preferential attachment* (where $T = T(p_T)$), seen in the scale-free networks and dynamics leading to it [8]. In the case of variable u , this means that the parameter b can be positive or negative, depending on circumstances. This, in turn, can result in growth or decrease of q . In our case, $q = 1.172$, higher than obtained so far by using the usual $h_q(p_T)$. The question then arises of whether it is possible to get $h_{q \rightarrow 1}(u) \sim \exp(-u/u_0)$ (by changing the sign of the parameter b in the def-

initiation of the variable u). In Fig. 3, we show a comparison of data for $p + p$ and $A + A$ for such scaling. They are different because of different u_0 and different normalizations used. However, both curves coincide when multiplied by u_0/A and when plotted for u/u_0 (they become independent of A), cf. Fig. 4. They correspond to, respectively, $b(\sqrt{s}) = -0.085 + 0.115(\sqrt{s})^{-0.2}$ for $p + p$ collisions and $b(\sqrt{s}) = -0.052 - 0.0002(\sqrt{s})^{0.7}$ for $A + A$ collisions.

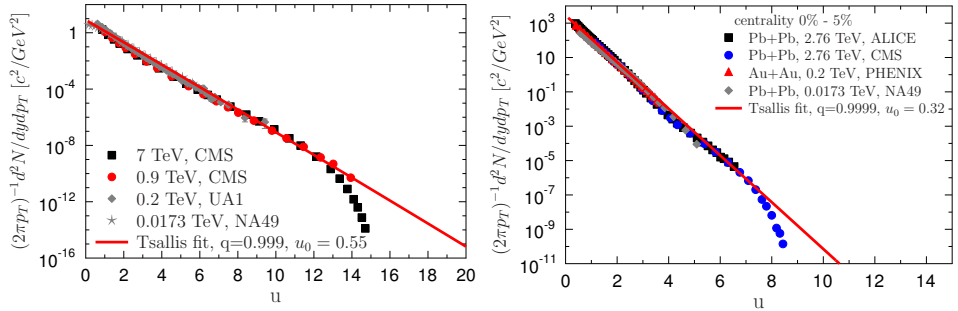


Fig. 3. (Color online) $p + p$, $(p + \bar{p})$ (left panel) and central $A + A$ (right panel) data (from, respectively, [9–11] and [16–19]) for transverse momentum distributions for different energies plotted by using the scaling variable u .

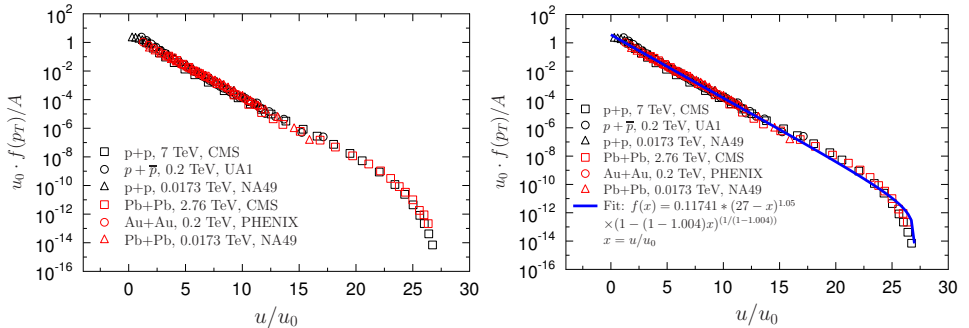


Fig. 4. (Color online) The same as in Fig. 3 but multiplied by u_0/A and plotted for u/u_0 . Notice that curves for pp and AA coincide (left panel) and can be fitted by the same formula (right panel). The observed departure from an exponent for large u is connected with the effect of limitation of phase space and “conditional probability” discussed in [2].

We close with some remarks concerning the variable u . As already said, by switching to this variable, we depart from the usual thermal-like description of such processes because u contains some dynamical input, for example:

- One can write $u/u_0 = p_T/T_{\text{eff}}$, where T_{eff} is an effective temperature: $T_{\text{eff}} = T_0 + T_v(p_T)$ with $T_0 = u_0 \langle p_T \rangle$ and $T_v = -b u_0 p_T$. Such T_{eff} could be related to the possible p_T transfer, additional to that resulting from a hard collision, perhaps proceeding by a kind of multiple scattering process⁴.
- T_{eff} also occurs in a description of the growth of the so-called complex free networks which can then be applied to hadronic production ([8], cf. also [21] and references therein).

Let us elaborate some more on the second proposition. Here, one associates p_T with the number of links in the quark-gluonic network assumed to be formed in the hadronization process. In this case, their actual original energy-momentum distributions would be of secondary importance, since, because of their mutual interactions, they connect to each other and this process of connection has its distinctive dynamical consequences. One can think of such a process in the following way: We start with some initial state consisting of a number n_0 of already existing ($q\bar{q}$) pairs (identified with vertices in the network) and we add to them, in each consecutive time step, another vertex (a new ($q\bar{q}$) pair), which can have k_0 possible connections (links in network language) to the old state. Quarks are dressed by interaction with surrounding gluons and, therefore, “excited” and each quark interacts with k other quarks (has k links). Assuming that the “excitation” of a quark is proportional to the number of links k (which is proportional to the number of gluons participating in “excitation”, *i.e.*, existing in the vicinity of a given quark), the chances to interact with a given quark grow with the number of links attached to it, k . The new links will be preferentially attached to quarks with k . This corresponds to building up a so-called preferential network, which evolves due to the occurrence of new ($q\bar{q}$) pairs from decaying gluons. Such networks always result in a power-like behavior of suitable variables, in our case in p_T .

This research was supported in part by the National Science Center, Poland (NCN) under contract DEC-2011/03/B/ST2/02617 (M.R. and Z.W.) and by the Polish Ministry of Science and Higher Education under contract DPN/N97/CERN/2009 (G.W.). We would like to warmly thank Dr. Eryk Infeld for reading this manuscript.

⁴ Similar, in a sense, to that proposed on a different occasion in [20].

REFERENCES

- [1] G. Wilk, Z. Włodarczyk, *Eur. Phys. J.* **A40**, 299 (2009); *Cent. Eur. J. Phys.* **10**, 568 (2012).
- [2] G. Wilk, Z. Włodarczyk, *Eur. Phys. J.* **A48**, 161 (2012).
- [3] K. Ürmösy, G.G. Barnaföldi, T.S. Biró, *Phys. Lett.* **B718**, 125 (2012); J. Cleymans, D. Worku, *Eur. Phys. J.* **A48**, 160 (2012).
- [4] C. Tsallis, *Stat. Phys.* **52**, 479 (1988); *Eur. Phys. J.* **A40**, 257 (2009); *Introduction to Nonextensive Statistical Mechanics*, Springer, 2009. For an updated bibliography on this subject, see <http://tsallis.cat.cbpf.br/biblio.htm>
- [5] C. Michael, L. Vanryckeghem, *J. Phys. G* **3**, L151 (1977); C. Michael, *Prog. Part. Nucl. Phys.* **2**, 1 (1979); G. Arnison *et al.* [UA1 Collab.], *Phys. Lett.* **B118**, 167 (1982); R. Hagedorn, *Riv. Nuovo Cim.* **6**, 1 (1984).
- [6] C.-Y. Wong, G. Wilk, *Acta Phys. Pol. B* **42**, 2047 (2012) [arXiv:1210.3661 [hep-ph]].
- [7] L. McLerran, M. Praszalowicz, *Acta Phys. Pol. B* **41**, 1917 (2010); *Acta Phys. Pol. B* **42**, 99 (2011); M. Praszalowicz, *Acta Phys. Pol. B* **42**, 1557 (2011); *Phys. Rev. Lett.* **106**, 142002 (2011).
- [8] M. Rybczyński, Z. Włodarczyk, G. Wilk, *J. Phys. G* **39**, 095004 (2012).
- [9] V. Khachatryan *et al.* [CMS Collab.], *J. High Energy Phys.* **1002**, 041 (2010); *Phys. Rev. Lett.* **105**, 022002 (2010); *J. High Energy Phys.* **1108**, 086 (2011).
- [10] C. Albajar *et al.* [UA1 Collab.], *Nucl. Phys.* **B335**, 261 (1990).
- [11] C. Alt *et al.* [NA49 Collab.], *Eur. Phys. J.* **C45**, 343 (2006); T. Anticic *et al.* [NA49 Collab.], *Phys. Rev.* **C70**, 034902 (2004).
- [12] M. Tokarev, I. Zborovský, *J. Phys. G* **37**, 085008 (2010); *Int. J. Mod. Phys.* **A24**, 417 (2009); *Phys. Rev.* **D75**, 094008 (2007).
- [13] M. Tokarev, I. Zborovský, *Acta. Phys. Pol. B Proc. Suppl.* **6**, 531 (2013) these proceedings.
- [14] M. Tokarev, I. Zborovský, *Prog. Theor. Phys. Suppl.* **193**, 264 (2012).
- [15] T.S. Biró, *Is There a Temperature? Conceptual Challenges at High Energy, Acceleration and Complexity*, Springer, 2011; O.J.E. Maroney, *Phys. Rev.* **E80**, 061141 (2009); T.S. Biró, P. Ván, *Phys. Rev.* **E83**, 061147 (2011); J. Cleymans, D. Worku, *J. Phys. G* **39**, 025006 (2012).
- [16] K. Aamodt *et al.* [ALICE Collab.], *Phys. Lett.* **B693**, 53 (2010); *Eur. Phys. J.* **C71**, 1594 (2011).
- [17] P. Kurt *et al.* [CMS Collab.], *J. Phys. Conf. Ser.* **347**, 012004 (2012).
- [18] S.S. Adler *et al.* [PHENIX Collab.], *Phys. Rev.* **C69**, 034909 (2004); A. Adare *et al.* [PHENIX Collab.], *Phys. Rev.* **C81**, 034911 (2010).
- [19] C. Alt *et al.* [NA49 Collab.], *Phys. Rev.* **C77**, 024903 (2008).
- [20] C.-Y. Wong, *Phys. Rev.* **C84**, 024901 (2011).
- [21] G. Wilk, Z. Włodarczyk, *Acta Phys. Pol. B* **35**, 2141 (2004).