COLOUR DISSIPATION BY PROPAGATION THROUGH THE QCD VACUUM*

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In the framework of QCD stochastic vacuum model, we show that colour particle losses its colour propagating through the QCD vacuum medium. At large distances colour disappears completely.

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1. QCD stochastic vacuum as environment

The model of QCD stochastic vacuum is one of the popular phenomenological models which explains quark confinement [1, 3–5]. It is based on the assumption that one can calculate vacuum expectation values of gauge-invariant quantities as expectation values with respect to some well-behaved stochastic gauge field. Instead of considering nonperturbative dynamics of Yang–Mills fields, one introduces external stochastic field and average over its implementations. It is known that such vacuum provides confining properties, giving rise to QCD strings with constant tension at large distances.

On the other hand, suppose that we have some quantum system which interacts with the environment in quantum-optical language. Interactions with the environment can be effectively represented by additional stochastic terms in the Hamiltonian of the system. The density matrix of the system in this case is obtained by averaging with respect to these stochastic terms [6–9]. Interactions with the environment result in decoherence and relaxation of quantum superpositions [8, 9]. Information on the initial state of the quantum system is lost after sufficiently large time.

QCD stochastic vacuum can be considered as the environment for colour quantum particles [10–13] with the averaging over external QCD stochastic

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vacuum implementations and, as a consequence, decoherence, relaxation of quantum superpositions and the information loss appear. Here, the analogy between QCD stochastic vacuum and the environment is expanded to confinement of colour states phenomenon.

Most frequently, the model is used to calculate Wilson loops, string tensions and field configurations around static charges [3, 4]. In this paper, we consider the colour states of quarks themselves. Usually, white wave functions of hadrons are constructed as gauge-invariant superpositions. We show that white objects can be obtained as white mixtures of states described by the density matrix as a result of evolution in the QCD stochastic vacuum as environment.

2. Colour dissipation and Wilson loop disappearing (confinement)

Consider propagation of heavy spinless particle along some fixed path γ. The amplitude is obtained by parallel transport [11–13]

$$|\phi(\gamma)\rangle = \hat{P}\exp\left(i\int\limits_{\gamma}\hat{A}_\mu dx^\mu\right)|\phi_{\rm in}\rangle\,, \tag{1}$$

where kets are colour state vectors, $\hat{P}$ is the path-ordering operator and $\hat{A}_\mu$ is the gauge field vector. In order to consider mixed states, we introduce the colour density matrix taking into account both colour and QCD stochastic vacuum (environment)

$$\rho(\text{loop},\gamma\bar{\gamma}) = \langle\phi(\gamma)\rangle\langle\phi(\gamma)|\rangle\,, \tag{2}$$

here we average over all implementations of stochastic gauge field (environment degrees of freedom). In the model of QCD stochastic vacuum, only expectation values of path ordered exponents over closed paths are defined. Closed path corresponds to a process in which the particle–antiparticle pair is created, propagates and finally annihilates. With the help of (1) and (2), we can obtain the next expression [11, 13]

$$\rho(\text{loop},\gamma\bar{\gamma}) = N_c^{-1} + \left(|\phi_{\rm in}\rangle\langle{}_{\rm in}\phi| - N_c^{-1}\right)W_{\rm adj}(\text{loop},\gamma\bar{\gamma})\,. \tag{3}$$

Here N_c — is a number of colours, $W_{\rm adj}(\text{loop},\gamma\bar{\gamma})$ is the Wilson loop in the adjoint representation and we have used the property that colour density matrix in colour neutral stochastic vacuum can be decomposed into the pieces transform under trivial and adjoint representations [11, 13], and the Wilson loop in which fundamental representation is [14]

$$W_{\rm fund}(\text{loop},\gamma\bar{\gamma}) = \left\langle T_r\hat{P}\exp\left(\int\limits_{\text{loop},\gamma\bar{\gamma}} i\hat{A}_\mu dx^\mu\right)\right\rangle\,. \tag{4}$$

As it is known due to Casimir scaling, decay rates of the Wilson loop in different representations are proportional to each other [4], in particular, $W_{\rm fund}(\text{loop}, \gamma\bar{\gamma})$ and $W_{\rm adj}(\text{loop}, \gamma\bar{\gamma})$. Decay of $W_{\rm fund}(\text{loop}, \gamma\bar{\gamma})$ points out at confinement of colour charges. Simultaneously, we have decay of $W_{\rm adj}(\text{loop}, \gamma\bar{\gamma})$ that means from (3) that colour density matrix obtained as a result of parallel transport along the loop, $\gamma\bar{\gamma}$ tends under the confinement regime to colour density matrix of white colourless mixture $\rho = N_c^{-1}$. Here, all colour states are mixed with equal probabilities and all information on initial colour state is lost. The stronger are the colour states confined the stronger their states transform in white mixture. So as the Wilson area law (confinement criterion) holds for the Wilson loop [14], we can obtain an explicit expression for the density matrix if we choose the rectangular loop $\gamma\bar{\gamma}$ spanned on γ in terms of time T and distance R [11, 12]. When R or T are of the order of 1 fm (for SU(3) theory), Wilson loops decay exponentially with the area spanned on loop $\gamma\bar{\gamma}$

$$\rho(\text{loop}, \gamma\bar{\gamma}) = N_c^{-1} + \left(\rho_{\rm in} - N_c^{-1}\right) \exp(-\sigma_{\rm adj} RT)\,, \tag{5}$$

where $\sigma_{\rm adj} = \sigma_{\rm fund} G_{\rm adj} G_{\rm fund}^{-1}$ — is string tension in the adjoint representation and $G_{\rm adj}$, $G_{\rm fund}$ — are the eigenvalues of quadratic Casimir operators. Under the condition of Gaussian dominance, string tensions is

$$\sigma_{\rm fund} = \frac{g^2}{2} l_{\rm corr}^2 F^2\,, \tag{6}$$

where g is coupling constant, $l_{\rm corr}$ — correlation length in the QCD stochastic vacuum, F^2 — average of the second cumulant of curvature treason, when $g^2 F^2 l_{\rm corr}^2 \ll 1$ [7, 11].

The decoherence rate of transition from pure colour states to white mixture can be estimated on the base of purity [8] $P = T_r \rho^2$ [11]

$$P = N_c^{-1} + \left(1 - N_c^{-1}\right) \exp(-2\sigma_{\rm fund} G_{\rm adj} G_{\rm fund}^{-1} RT)\,, \tag{7}$$

when T or R tends to 0, $P \to 1$, that corresponds to pure state with the density matrix $\rho_{\rm in} = |\phi_{\rm in}\rangle\langle\phi_{\rm in}|$. When composition RT tends to infinity, the purity tends to N_c^{-1}, that corresponds to the white mixture state with the density matrix N_c^{-1}. The rate of purity decreasing is $T_{\rm dec}^{-1} = -2\sigma_{\rm fund} G_{\rm adj} G_{\rm fund}^{-1}$, where $T_{\rm dec}$ — is characteristic time of decoherence proportional to QCD string tension and distance R. It can be inferred from (3) and (7) that the stronger is particle–antiparticle pair coupled by QCD string or the larger is the distance between particle and antiparticle the quicker information about colour states lost as a result of interaction with the QCD stochastic vacuum. Thus, white states can be obtained as a result of decoherence process which allows to conjecture analogy with colour particle confinement.

3. Confinement and fidelity of colour particle motion

Wilson loop definition in QCD is similar with the definition of fidelity [14], the quantity which describes the stability of quantum motion of the particles [9]. Using the analogy between the theory of gauge fields and the theory of holonomic quantum computation [12, 15, 16], we define the fidelity as a scalar product of vector states in colour space of QCD stochastic vacuum

$$f = \langle\langle\langle \phi(\gamma_1)\phi(\gamma_2)\rangle\rangle\rangle\,. \tag{8}$$

We consider the motion of colour particles in different paths starting from the point x and join in the point y. In the initial point x, state vectors are $|\phi_{\rm in}\rangle$. For large particle mass and because of Hermitian character of A_μ, operator (1) is unitary. We can rewrite (8) as integral over the closed loop, travelling from point x to the point y

$$f = \left\langle\left(\left(\langle \phi_{\rm in}|\hat{P}\exp\int igA_\mu dx^\mu|\phi_{\rm in}\rangle\right)\right)\right\rangle \tag{9}$$

in the path γ_1 and back to the point x in the path γ_2 and obtain integral proportional to the identity due to the colour neutrality of stochastic vacuum.

The final expression for the fidelity of the particle moving in the Gaussian-dominated stochastic vacuum is

$$f = \exp\left(-\tfrac{1}{2}\,g^2 l_{\rm corr}^2 F^2 S_\gamma\right)\,, \tag{10}$$

where S_γ is the area of the surface spanned on the contour loop, $\gamma_1\gamma_2$. Thus the fidelity for colour particle mowing along contour decays exponentially with the surface spanned on the contour S_γ the decay rate being equal to the string tension (6).

Another situation, more close to the standard treatment of the fidelity, is realised when γ_1, and γ_2 are two random paths in the Minkowski space, closed to each other. The corresponding expression for the fidelity is similar to (8), but now the averaging is performed with respect to all random paths which are close enough. The final expression is

$$f = \exp\left(-\tfrac{1}{2}\,g^2 l_{\rm corr}\int\limits_{\gamma_1} dx^\upsilon F_{\chi\alpha}\widetilde{F}_{\upsilon\beta}\upsilon^\chi\left\langle\delta\chi^\alpha\delta\chi^\beta\right\rangle\right)\,, \tag{11}$$

where $\delta\chi^\alpha$ — is the deviation of the path γ_2 from the path γ_1, υ^χ — is the four-dimensional velocity and $l_{\rm corr}$ is the correlation length of perturbation of the particle path expressed in units of world line length. If unperturbed path is parallel to the time axis in the Minkowski space, the particle moves randomly around some point in three dimensional space. The fidelity in this case decays exponentially with time.

This hints give close connection between confinement and instability of colour particle motion and can be related to possible mechanisms of colour particle confinement.

The increasing of instability of motion in the confinement region is also connected with existence of chaotic solutions of the Yang–Mills field [1, 16], possible chaos onset [17], entanglement as a probe of confinement [18–21] and quantum squeezing of states [22].

4. Conclusions

We show that in QCD stochastic vacuum white states of colour charges can be obtained as a result of decoherence of pure colour states into a mixed white state. Decoherence rate is found to be proportional to the tension of QCD string and to the distance between colour charges. The purity of colour states is calculated and is shown to tend to N_c^{-1} that means appearance of white mixture states (information about pure colour states disappears). It is shown that in confinement regime (Wilson loop goes to zero) the fidelity decreases which means the motion of colour charges becomes more and more unstable.

Thus in QCD stochastic vacuum, there exists a direct connection between confinement of colour charges, colour dissipation (decoherence, purity decreasing) and unstable motion of colour particle (fidelity decreasing).

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