NEUTRINO MASS SPECTRUM FROM THE SEESAW EXTENSION*

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The Standard Model includes neutrinos as massless particles, but neutrino oscillations showed that neutrinos are not massless. A simple extension of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos. The smallness of the neutrino masses can be well understood within the seesaw mechanism. We analyse two cases of the minimal extension of the Standard Model when one or two right-handed fields are added to the three left-handed fields. A second Higgs doublet is included in our model. We calculate the one-loop radiative corrections to the mass parameters which produce mass terms for the neutral leptons. In both cases, we numerically analyse light neutrino masses as functions of the heavy neutrinos masses. Parameters of the model are varied to find light neutrino masses that are compatible with experimental data of solar $\Delta m^2_{\odot}$ and atmospheric $\Delta m^2_{\rm atm}$ neutrino oscillations for normal and inverted hierarchy.

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1. The model

We extend the Standard Model (SM) by adding a second Higgs doublet and right-handed neutrino fields. The Yukawa Lagrangian of the leptons is expressed by

$$\mathcal{L}_{\rm Y} = -\sum_{k=1}^{2} \left(\varPhi_k^\dagger \bar{\ell}_{\rm R} \varGamma_k + \tilde{\varPhi}_k^\dagger \bar{\nu}_{\rm R} \varDelta_k \right) D_{\rm L} + \text{h.c.} \tag{1.1}$$

in a vector and matrix notation, where $\tilde{\varPhi}_k = i\tau_2 \varPhi_k^*$. In expression (1.1), $\ell_{\rm R}$, $\nu_{\rm R}$, and $D_{\rm L} = (\nu_{\rm L}\ \ell_{\rm L})^T$ are the vectors of the right-handed charged

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leptons, of the right-handed neutrino singlets, and of the left-handed lepton doublets, respectively, and Φ_k, $k = 1, 2$ are the two Higgs doublets. The Yukawa coupling matrices Γ_k are $n_{\rm L} \times n_{\rm L}$, while the Δ_k are $n_{\rm R} \times n_{\rm L}$.

In this model, spontaneous symmetry breaking of the SM gauge group is achieved by the vacuum expectation values $\langle \Phi_k \rangle_{\rm vac} = \left(0, v_k/\sqrt{2}\right)^T$, $k = 1, 2$. By a unitary rotation of the Higgs doublets, we can achieve $\langle \Phi_1^0 \rangle_{\rm vac} = v/\sqrt{2} > 0$ and $\langle \Phi_2^0 \rangle_{\rm vac} = 0$ with $v \simeq 246$ GeV. The charged-lepton mass matrix M_ℓ and the Dirac neutrino mass matrix $M_{\rm D}$ are

$$\hat{M}_\ell = \frac{v}{\sqrt{2}}\,\Gamma_1 \qquad \text{and} \qquad M_{\rm D} = \frac{v}{\sqrt{2}}\,\Delta_1 \tag{1.2}$$

with the assumption that $\hat{M}_\ell = \operatorname{diag}(m_e, m_\mu, m_\tau)$. The hat indicates that $\hat{M}_l$ is a diagonal matrix. The mass terms for the neutrinos can be written in a compact form with a $(n_{\rm L} + n_{\rm R}) \times (n_{\rm L} + n_{\rm R})$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} 0 & M_{\rm D}^T \\ M_{\rm D} & \hat{M}_{\rm R} \end{pmatrix} . \tag{1.3}$$

M_ν can be diagonalized as

$$U^T M_\nu U = \hat{m} = \operatorname{diag}(m_1, m_2, \ldots, m_{n_{\rm L}+n_{\rm R}}) \,, \tag{1.4}$$

where the m_i are real and non-negative. In order to implement the seesaw mechanism [1, 2], we assume that the elements of $M_{\rm D}$ are of the order of $m_{\rm D}$ and those of $M_{\rm R}$ are of the order of $m_{\rm R}$, with $m_{\rm D} \ll m_{\rm R}$. Then the neutrino masses m_i with $i = 1, 2, \ldots, n_{\rm L}$ are of the order of $m_{\rm D}^2/m_{\rm R}$, while those with $i = n_{\rm L} + 1, \ldots, n_{\rm L} + n_{\rm R}$ are of the order of $m_{\rm R}$. It is useful to decompose the $(n_{\rm L} + n_{\rm R}) \times (n_{\rm L} + n_{\rm R})$ unitary matrix U as $U = (U_{\rm L}, U_{\rm R}^*)^T$, where the submatrix $U_{\rm L}$ is $n_{\rm L} \times (n_{\rm L} + n_{\rm R})$ and the submatrix $U_{\rm R}$ is $n_{\rm R} \times (n_{\rm L} + n_{\rm R})$ [3, 4]. With these submatrices, the left- and right-handed neutrinos are written as linear superpositions of the $n_{\rm L} + n_{\rm R}$ physical Majorana neutrino fields χ_i: $\nu_{\rm L} = U_{\rm L} P_{\rm L} \chi$ and $\nu_{\rm R} = U_{\rm R} P_{\rm R} \chi$, where $P_{\rm L}$ and $P_{\rm R}$ are the projectors of chirality.

It is possible to express the couplings of the model in terms of the mass eigenfields, where three neutral particles namely, the Z boson, the neutral Goldstone boson G^0 and the Higgs bosons H_b^0 couple to neutrinos. The full formalism for the scalar sector of the multi-Higgs-doublet SM is given in Refs. [3, 4].

Once the one-loop corrections are taken into account the neutral fermion mass matrix is given by

$$M_\nu^{(1)} = \begin{pmatrix} \delta M_{\rm L} & M_{\rm D}^T + \delta M_{\rm D}^T \\ M_{\rm D} + \delta M_{\rm D} & \hat{M}_{\rm R} + \delta M_{\rm R} \end{pmatrix} \approx \begin{pmatrix} \delta M_{\rm L} & M_{\rm D}^T \\ M_{\rm D} & \hat{M}_{\rm R} \end{pmatrix} , \tag{1.5}$$

where the $0_{3\times3}$ matrix appearing at tree level (1.3) is replaced by the contribution $\delta M_{\rm L}$. This correction is a symmetric matrix, it dominates among all the sub-matrices of corrections. Neglecting the sub-dominant pieces in (1.5), one-loop corrections to the neutrino masses originate via the self-energy function $\Sigma^{\rm S}_{\rm L}(0) = \Sigma^{{\rm S}(Z)}_{\rm L}(0) + \Sigma^{{\rm S}(G^0)}_{\rm L}(0) + \Sigma^{{\rm S}(H^0)}_{\rm L}(0)$, where the $\Sigma^{{\rm S}(Z,G^0,H^0)}_{\rm L}(0)$ functions arise from the self-energy Feynman diagrams and are evaluated at zero external momentum squared. Each diagram contains a divergent piece but when summing up the three contributions the result turns out to be finite.

The final expression for one-loop corrections is given by [5]

$$\begin{aligned}\delta M_{\rm L} &= \sum_b \frac{1}{32\pi^2}\,\Delta_b^T U_{\rm R}^* \hat{m}\left(\frac{\hat{m}^2}{m^2_{H^0_b}} - \mathbb{1}\right)^{-1} \ln\left(\frac{\hat{m}^2}{m^2_{H^0_b}}\right) U_{\rm R}^\dagger \Delta_b \\ &\quad + \frac{3g^2}{64\pi^2 m_W^2}\, M_{\rm D}^T U_{\rm R}^* \hat{m}\left(\frac{\hat{m}^2}{m_Z^2} - \mathbb{1}\right)^{-1} \ln\left(\frac{\hat{m}^2}{m_Z^2}\right) U_{\rm R}^\dagger M_{\rm D} \qquad (1.6)\end{aligned}$$

with $\Delta_b = \sum_k b_k \Delta_k$, where b are two-dimensional complex unit vectors and the sum $\sum_b$ runs over all neutral physical Higgses H^0_b.

2. Case $n_{\rm R} = 1$

First, we consider the minimal extension of the Standard Model by adding only one right-handed field $\nu_{\rm R}$ to the three left-handed fields contained in $\nu_{\rm L}$.

For this case, we use the parametrization $\Delta_i = \left(\sqrt{2}\, m_{\rm D}/v\right) \vec{a}_i^{\,T}$, where $\vec{a}_1^{\,T} = (0,0,1)$ and $\vec{a}_2^{\,T} = (0,1,e^{i\phi})$. Diagonalization of the symmetric mass matrix M_ν (1.3) in block form is

$$U^T M_\nu U = U^T \begin{pmatrix} 0_{3\times3} & m_{\rm D}\vec{a}_1 \\ m_{\rm D}\vec{a}_1^{\,T} & \hat{M}_{\rm R}\end{pmatrix} U = \begin{pmatrix} \hat{M}_{\rm l} & 0 \\ 0 & \hat{M}_{\rm h}\end{pmatrix}. \qquad (2.1)$$

The non-zero masses in $\hat{M}_{\rm l}$ and $\hat{M}_{\rm h}$ are determined analytically by finding eigenvalues of the Hermitian matrix $M_\nu M_\nu^\dagger$. These eigenvalues are the squares of the masses of the neutrinos $\hat{M}_{\rm l} = {\rm diag}(0,0,m_{\rm l})$ and $\hat{M}_{\rm h} = m_{\rm h}$. Solutions $m^2_{\rm D} = m_{\rm h} m_{\rm l}$ and $m^2_{\rm R} = (m_{\rm h} - m_{\rm l})^2 \approx m^2_{\rm h}$ correspond to the seesaw mechanism.

The diagonalization matrix U for the tree level is constructed from a rotation matrix and a diagonal matrix of phases $U_{\rm tree} = U_{34}(\beta) U_i$, where the angle β is determined by the masses $m_{\rm l}$ and $m_{\rm h}$.

For the calculation of radiative corrections, we use the following set of orthogonal complex vectors: $b_Z = (i, 0)$, $b_1 = (1, 0)$, $b_2 = (0, i)$ and $b_3 = (0, 1)$. Diagonalization of the mass matrix after calculation of one-loop corrections is performed with a unitary matrix $U_{\rm loop} = U_{\rm egv} U_{\varphi}(\varphi_1, \varphi_2, \varphi_3)$, where $U_{\rm egv}$ is an eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$ and U_φ is a phase matrix. The second light neutrino obtains its mass from radiative corrections. The third light neutrino remains massless.

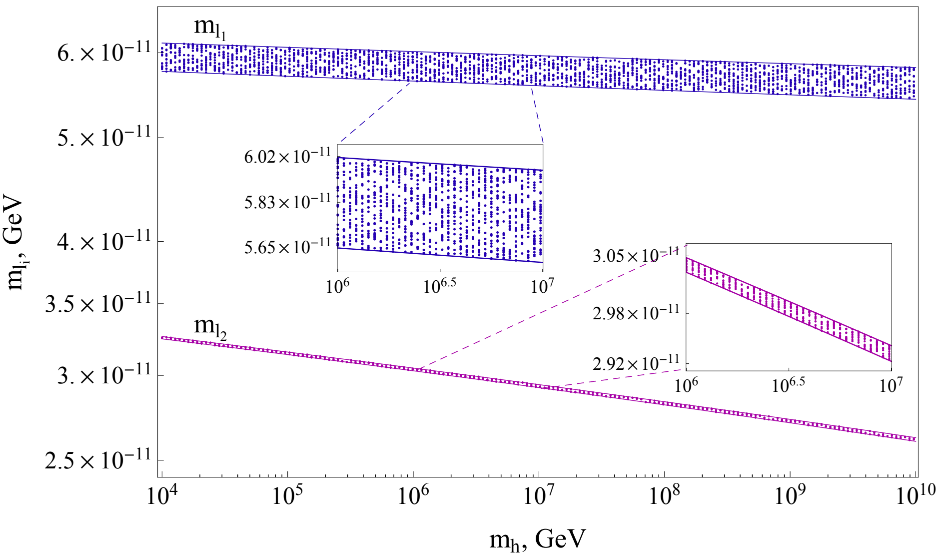

Fig. 1. Calculated masses of two light neutrinos as a function of the heavy neutrino mass $m_{\rm h}$. Solid lines show the boundaries of allowed neutrino mass ranges when the model parameters are constrained by the experimental data on neutrino oscillations with $\theta_{\rm atm} = 45°$. The allowed values of m_{l_1} and m_{l_2} form bands, their scattered values are shown separately in the middle plots.

The masses of the neutrinos are restricted by experimental data of solar and atmospheric neutrino oscillations [6] and by cosmological observations. The numerical analysis shows that we can reach the allowed neutrino mass ranges for a heavy singlet with the mass close to 10^4 GeV and with the angle of oscillations fixed to $\theta_{\rm atm} = 45°$, see Fig. 1. The free parameters $m_{H_2^0}, m_{H_3^0}$, and ϕ are restricted by the parametrization used and by oscillation data. Figure 2 illustrates the allowed values of Higgs masses for different values of the heavy singlet.

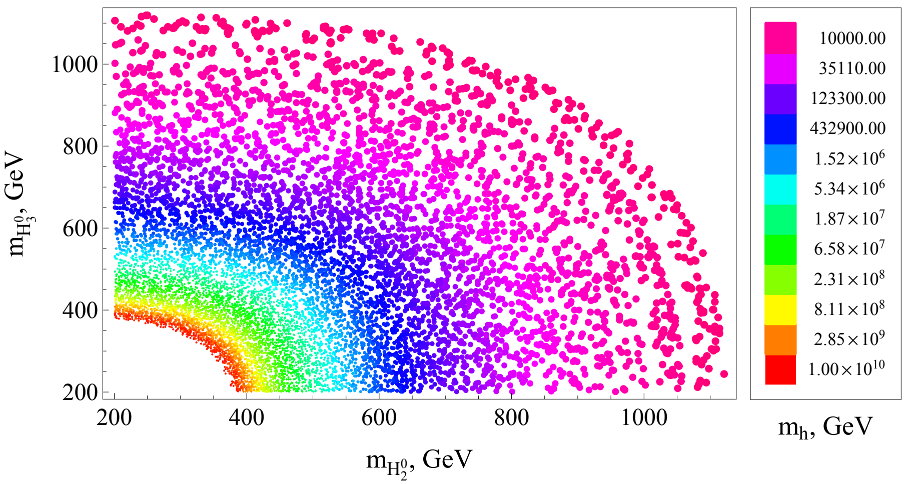

Fig. 2. The values of the free parameters $m_{H_2^0}$ and $m_{H_3^0}$ as functions of the heaviest right-handed neutrino mass m_{h}, for the case $n_{\mathrm{R}} = 1$. The mass of the SM Higgs boson is fixed to $m_{H_1^0} = 125$ GeV and the angle of oscillations is $\theta_{\mathrm{atm}} = 45°$.

3. Case $n_{\mathrm{R}} = 2$

When we add two singlet fields ν_{R} to the three left-handed fields ν_{L}, the radiative corrections give masses to all three light neutrinos.

Now we parametrize $\Delta_i = \frac{\sqrt{2}}{v}\left(m_{\mathrm{D}_2}\vec{a}_i^T, m_{\mathrm{D}_1}\vec{b}_i^T\right)^T$ with $|\vec{a}_1| = 1$, $|\vec{b}_1| = 1$, $|\vec{a}_2| = 1$ and $|\vec{b}_2| = 1$. Diagonalizing the symmetric mass matrix M_ν (1.3) in block form, we write

$$U^T M_\nu U = U^T \begin{pmatrix} 0_{3\times3} & m_{\mathrm{D}_2}\vec{a} \;\; m_{\mathrm{D}_1}\vec{b} \\ m_{\mathrm{D}_2}\vec{a}^T & \\ m_{\mathrm{D}_1}\vec{b}^T & \hat{M}_{\mathrm{R}} \end{pmatrix} U = \begin{pmatrix} \hat{M}_{\mathrm{l}} & 0 \\ 0 & \hat{M}_{\mathrm{h}} \end{pmatrix}. \tag{3.1}$$

The non-zero masses in $\hat{M}_{\mathrm{l}}$ and $\hat{M}_{\mathrm{h}}$ are determined by the seesaw mechanism: $m^2_{\mathrm{D}_i} \approx m_{\mathrm{h}_i} m_{\mathrm{l}_i}$ and $m^2_{\mathrm{R}_i} \approx m^2_{\mathrm{h}_i}$, $i = 1, 2$. Here we use $m_1 > m_2 > m_3$ ordering of masses. The third light neutrino is massless at tree level.

The diagonalization matrix for tree level $U_{\mathrm{tree}} = U^{\mathrm{tree}}_{\mathrm{egv}} U_\phi(\phi_i)$ is composed of an eigenmatrix of $M_\nu M_\nu^\dagger$ and a diagonal phase matrix, respectively.

For calculation of radiative corrections, we use the same set of orthogonal complex vectors b_i as in the first case. Diagonalization of the mass matrix including the one-loop corrections is performed with a unitary matrix $U_{\mathrm{loop}} = U^{\mathrm{loop}}_{\mathrm{egv}} U_\varphi(\varphi_i)$, where $U^{\mathrm{loop}}_{\mathrm{egv}}$ is the eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$ and U_φ is a phase matrix.

A broader description of the case $n_{\mathrm{R}} = 2$ and graphical illustrations of the obtained light neutrino mass spectra is given in Ref. [7]. Both normal and inverted neutrino mass orderings are considered.

4. Conclusions

For the case of $n_{\mathrm{R}} = 1$, we can match the differences of the calculated light neutrino masses to $\Delta m^2_{\odot}$ and $\Delta m^2_{\mathrm{atm}}$ with the mass of a heavy singlet close to 10^4 GeV. The parametrization used for this case and restrictions from the neutrino oscillation data limit the values of free parameters. Only normal ordering of neutrino masses is possible.

In the case of $n_{\mathrm{R}} = 2$, we obtain three non-vanishing masses of light neutrinos for normal and inverted hierarchies. The numerical analysis [7] shows that the values of light neutrino masses (especially of the lightest mass) depend on the choice of the heavy neutrino masses. The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.

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