

# PROPERTIES AND COMPOSITION OF THE $f_0(500)$ RESONANCE\*

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In this paper, we review our recent developments on the understanding of the nature of the  $f_0(500)$  resonance — or  $\sigma$  meson — coming from the  $N_c$  expansion, dispersion relations with Chiral Perturbation Theory, as well as finite energy sum rules and semi-local duality.

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## 1. Introduction

The  $f_0(500)$  isoscalar–scalar resonance — also known as  $\sigma$  meson, or the correlated two-pion exchange with those quantum numbers, plays a prominent role in nucleon–nucleon attraction as well as in the spontaneous chiral symmetry breaking of QCD. Its properties and nature are, therefore, very relevant for Nuclear and Particle Physics, and it even has implications for Cosmological and Anthropic considerations. However, even though a scalar–isoscalar field was first proposed [1] more than 65 years ago to explain nucleon–nucleon attraction, its description in terms of quarks and gluons is still the subject of an intense debate.

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Concerning the properties, namely its mass and width, it seems that the issue has been finally solved (see [2] for a brief review) using elaborated dispersive analyses of existing data, and this is why in the last edition of the Particle Data Tables (PDT) [3], the former  $f_0(600)$  meson, previously quoted with a huge mass uncertainty from 400 to 1200 MeV, has changed the name to  $f_0(500)$  and is now quoted with a mass between 400 and 550 MeV. A similar reduction has taken place in the PDT for the width. Nevertheless, the very PDT suggest in their “Note on light scalars below 2 GeV” that one could “take the more radical point of view and just average the most advanced dispersive analyses” [4–7], and obtain an estimated pole position at  $M - i\Gamma/2 \simeq \sqrt{s_\sigma} = (446 \pm 6) - (276 \pm 5) \text{ MeV}$ .

With the mass and width issue solved, we need to understand its composition in terms of quarks and gluons. Actually, the inverted mass hierarchy of the lightest scalar nonet, of which the  $\sigma$  is the lightest member, hints at a possible tetraquark nature, as suggested long ago in [8]. However, the sigma nature is still a matter of intense debate today, due to the poor quality of the data available, the use of some strong model dependent analyses, and the additional complication of mixing between different configurations, which is expected to be very relevant in the meson sector. Nevertheless, in view of the supporting evidence, there is a growing consensus that the tetraquark — or possibly molecular — component might be dominant although it could be mixed with components of a different nature. The results of our group, that we review next, support this interpretation.

## 2. The $f_0(500)$ predominant non- $\bar{q}q$ $N_c$ behavior

The  $1/N_c$  expansion of QCD [9],  $N_c$  being the number of colors, can be applied at all energies (not only at high energies as the usual expansion in the coupling) and, for our purposes, provides a prediction for the leading order behavior of  $\bar{q}q$  mesons, whose mass and width scale as  $O(1)$  and  $O(1/N_c)$ , respectively. The behavior of other configurations is also known [9, 10].

### 2.1. Model independent approach

If the pole  $s_R = m_R^2 - im_R\Gamma_R$  of an elastic resonance (like the  $\sigma$ ) behaves as a  $\bar{q}q$ , then the scattering phase shift where it appears satisfies [11]

$$\delta(m_R^2) = \frac{\pi}{2} - \frac{\text{Re } t^{-1}}{\sigma} \bigg|_{m_R^2} + O(N_c^{-3}) \quad , \quad \delta'(m_R^2) = -\frac{(\text{Re } t^{-1})'}{\sigma} \bigg|_{m_R^2} + O(N_c^{-1}) \quad ,$$

from which we have shown [12] that the adimensional observables

$$\frac{\frac{\pi}{2} - \text{Re } t^{-1}/\sigma}{\delta} \bigg|_{m_R^2} \equiv \Delta_1 = 1 + \frac{a}{N_c^3} \dots \quad , \quad -\frac{[\text{Re } t^{-1}]'}{\delta' \sigma} \bigg|_{m_R^2} \equiv \Delta_2 = 1 + \frac{b}{N_c^2} \dots$$

are equal to one up to corrections suppressed by more than just one power of  $1/N_c$ . Since  $a, b$  are naturally expected to be of the order of one *or less* (cancellations with higher order terms can substantially decrease their effective value, but not increase it), and are coefficients of very suppressed corrections, they are very sensitive to deviations from a  $\bar{q}q$  behavior.

Now, we can use the scattering phase shifts obtained in [13] within a model independent dispersive analysis of data and the  $\sigma$  and  $\rho(770)$  pole positions corresponding to that analysis, which are found in [6]. Altogether, these yield, for the  $\rho(770)$  (widely accepted as an ordinary  $\bar{q}q$  resonance):  $a_\rho = -0.06 \pm 0.01$  and  $b = 0.37^{+0.04}_{-0.05}$ , in good agreement with the expectations. In contrast, for the  $\sigma$ , we find:  $a_\sigma = -252^{+119}_{-156}$  and  $b_\sigma = 77^{+28}_{-24}$ . Two or more orders of magnitude larger than expected for a  $\bar{q}q$ . *This is a strong and model independent support for a predominant non- $\bar{q}q$  component for the  $f_0(500)$ , since it only makes use of the QCD leading behavior of  $\bar{q}q$  states and a dispersive data analysis.*

A glueball component [12], whose width scales as  $1/N_c^2$ , is more disfavored since the resulting  $a, b$  are one order of magnitude larger.

## 2.2. Unitarized Chiral Perturbation Theory

A different approach consists on using a partial wave dispersion relation for the inverse amplitude in order to describe the  $\pi\pi$  scattering data. The elastic Inverse Amplitude Method (IAM) [14] uses Chiral Perturbation Theory (ChPT) [15] to evaluate the subtraction constants and the left cut of the dispersion relation. The elastic right cut is exact in the elastic approximation, since the elastic unitarity condition  $\text{Im } t = \sigma |t|^2$ , fixes  $\text{Im } t^{-1} = -\sigma$ . Note that the IAM is derived only from exact elastic unitarity, analyticity in the form of a dispersion relation and ChPT, which is only used at low energies, and reproduces meson-meson scattering data up to energies  $\sim 1$  GeV. It can be analytically continued into the second Riemann sheet to find the poles associated to the  $\rho(770)$  and  $f_0(500)$  resonances, which are generated from the unitarization process. The dependence on the QCD number of colors is implemented [16, 17] through the model independent leading  $1/N_c$  scaling of the ChPT low energy constants (LECs) [15, 17].

Hence, by varying  $N_c$  in the LECs, we obtain the  $\rho(770)$  and  $f_0(500)$  behavior. This can be done within ChPT to one-loop,  $O(p^4)$  [16], or two-loops,  $O(p^6)$  [17]. Thus, we show in the left panel of Fig. 1 the  $\rho(770)$  pole movement in the complex plane. As expected for a  $\bar{q}q$  its mass barely moves, whereas the width decreases with  $1/N_c$ . In the center and right panels, we compare the behavior of the  $\Delta_1$  and  $\Delta_2$  observables defined above, *versus* their expected  $1/N_c^3$  or  $1/N_c$  behavior for ordinary mesons. Note the  $\rho$  is very consistent with the expected behavior, but the  $f_0(500)$  is completely at odds with it. Actually, from Fig. 2, which includes the uncertainty in

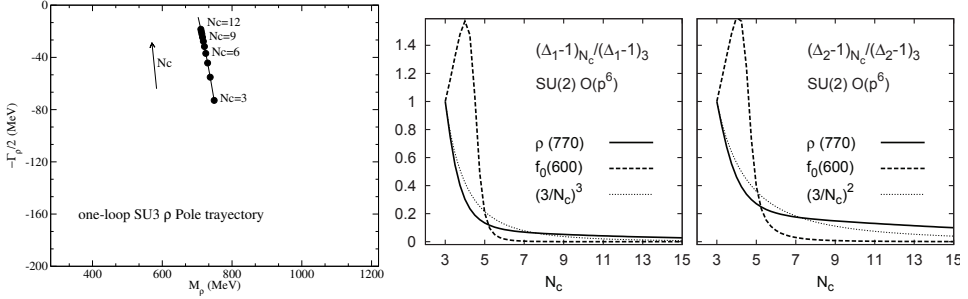


Fig. 1. Left:  $N_c$  behavior of the  $\rho(770)$  pole. Center and right: The  $\Delta_1$  and  $\Delta_2$  observables both for the  $\rho(770)$  and  $f_0(500)$ , *versus* the expected behavior for a  $\bar{q}q$ .

the one-loop calculation, parameterized by the renormalization scale  $\mu$  of the LECs [16, 18], we note that not far from  $N_c = 3$ , the width always grows as the pole moves deep in the lower complex plane as  $N_c$  increases, in contrast to the shrinking width observed for the  $\rho(770)$  in Fig. 1. This result, obtained not far from  $N_c = 3$  is very robust within uncertainties and has been confirmed by several authors [19]. Nevertheless, within the uncertainties, it is possible for the width to keep growing or decrease again. The latter could be interpreted as the effect of mixing with another  $\bar{q}q$  component, which is subdominant in the  $N_c = 3$  physical world, but becomes dominant at larger  $N_c$ . Note however, that this subdominant  $\bar{q}q$  component only arises *above* 1 GeV [17], unless one spoils the data description or the  $\rho(770)$   $\bar{q}q$  behavior. This scenario could be expected within the most common interpretation, which suggests that the lightest scalar nonet is of a non- $\bar{q}q$  nature, whereas

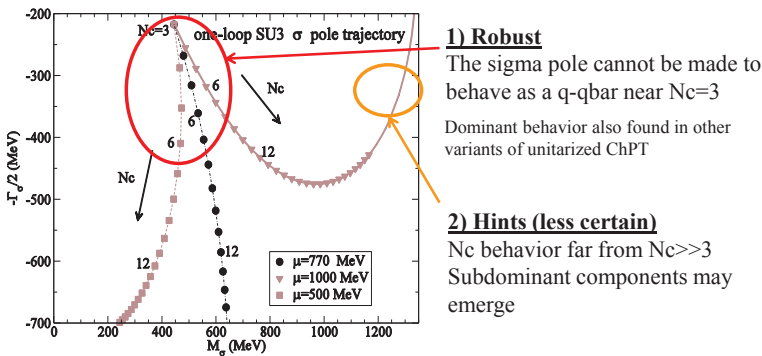


Fig. 2.  $N_c$  behavior of the  $f_0(500)$  pole, including uncertainties. Not far from  $N_c = 3$  the width increases dramatically with  $N_c$ , at odds with a  $\bar{q}q$  dominant component (compare the  $\rho(770)$  in Fig. 1). At two loops, the preferred solution comes close to the  $\mu = 1000$  MeV case. The figure is from [18].

the first ordinary  $\bar{q}q$  nonet appears around 1 to 1.5 GeV [20]. Furthermore, this “subdominant  $\bar{q}q$  component around 1–1.5 GeV” scenario is favored by the two-loop analysis [17]. Moreover, our picture is qualitatively consistent with recent lattice calculations [21] finding that the lightest scalar is about a factor 1.5 heavier than the  $\rho$  in the  $N_c \rightarrow \infty$  limit.

### 2.3. Semi local duality and the $N_c$ dependence of the $\rho(770)$ and $f_0(500)$

In the real world ( $N_c = 3$ ) and low energies, the scattering amplitude is well represented by a sum of resonances (with a background), but as the energy increases, the resonances (having more phase space for decay) become wider and increasingly overlap. This overlap generates a smooth Regge behavior described by a small number of crossed channel Regge exchanges. Thus,  $s$ -channel resonances are related “on the average” to Regge exchanges in the  $t$ -channel, a feature known as “semi-local duality”.

Now, data teach us that in the repulsive isospin  $I = 2$   $\pi\pi$  scattering channel, there are no resonances. Hence, semi-local duality means that the  $I = 2$   $t$ -channel amplitude should be small compared with other  $t$ -channels. But the  $I = 2$   $t$ -channel can be recast in terms of  $s$ -channel amplitudes as

$$\text{Im } A^{t2}(s, t) = \frac{1}{3} \text{Im } A^{s0}(s, t) - \frac{1}{2} \text{Im } A^{s1}(s, t) + \frac{1}{6} \text{Im } A^{s2}(s, t), \quad (1)$$

$A^{s2}$  being small. Hence, to have a small  $A^{t2}$  requires a strong cancellation between  $A^{s0}$  and  $A^{s1}$ . However, these channels are saturated at low energies by the  $f_0(500)$  and  $\rho(770)$  resonances, respectively. This “on the average cancellation” is properly defined via Finite Energy Sum Rules

$$F(t)_n^{21} = \frac{\int_{\nu_{\text{th}}}^{\nu_{\text{max}}} d\nu \text{Im } A^{t2}(s, t)/\nu^n}{\int_{\nu_{\text{th}}}^{\nu_{\text{max}}} d\nu \text{Im } A^{t1}(s, t)/\nu^n}, \quad \nu = (s - u)/2. \quad (2)$$

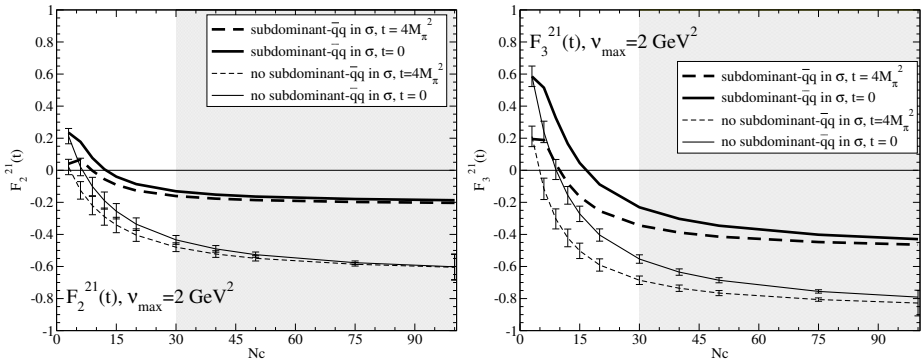


Fig. 3. Semi-local duality is well satisfied,  $|F(t)_n^{21}| \ll 1$ , for all  $N_c$  when a subdominant  $\bar{q}q$  component around 1 to 1.5 GeV is present in the  $f_0(500)$  [18].

Semi local duality implies  $|F(t)_n^{21}| \ll 1$ , which we have checked to be well satisfied for  $n > 1$  [18]. We expect the  $I = 2$  channel to remain repulsive and no resonances to appear even for  $N_c \neq 3$ . As a consequence, *all models where the  $\rho(770)$  and the  $f_0(500)$  resonances behave differently* are in potential conflict with semi-local duality.

We have recently shown [18] that this conflict actually occurs in those scenarios where the  $f_0(500)$  disappears deeply in the complex plane, *but this conflict is not present when there is a subdominant  $\bar{q}q$  component in the sigma in the 1 to 1.5 GeV region*. The results are summarized in Fig. 3.

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