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# SCALAR MESONS, $\pi\pi$ $D$-WAVE PHOTO-PRODUCTION AND THRESHOLD PARAMETERS — ALL FROM THE ROY-LIKE DISPERSION RELATIONS*

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Once subtracted dispersion relations with imposed crossing symmetry condition for the $\pi\pi$ $S$-, $P$-, $D$- and $F$-wave scattering amplitudes were recently derived. Together with the well known Roy equations with two subtractions, they allowed *e.g.* for unambiguous and very precise determination of parameters of the $f_0(500)$ and $f_0(980)$ resonances in the $S$-wave. Analytic continuation of the amplitude to the complex energy plane led to finding the poles related with these resonances at $(457^{+14}_{-13} - i\,279^{+11}_{-7})$ MeV and at $(996 \pm 7 - i\,25^{+10}_{-6})$ MeV respectively. These results led to significant changes in section of Particle Data Tables 2012 for light scalar mesons in comparison with previous editions. In this short paper, general mathematical structure of these dispersion relations is presented. It is shown that they produce output amplitudes with very small errors what significantly increases the accuracy of determined amplitudes. It can be very beneficial in practical applications *e.g.* in description of final state interactions in two-pion photo-production processes.

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## 1. Introduction

Recently the once subtracted dispersion relations with imposed crossing symmetry condition for the $S$, $P$, $D$ and $F$ $\pi\pi$ partial waves in energy range from the threshold to 1100 MeV were derived and analyzed [1, 2]. The output amplitudes (OUT) given by dispersion relations for the $S$- and $P$-waves (*i.e.* by the so-called GKPY equations) have been confronted with the real parts of the input amplitudes (IN) fitted to the $\pi\pi$ scattering data (including very recent, $K_{l4}$ experimental results).

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The once subtracted dispersion relations read

$$\mathrm{Re}\ t_\ell^{I(\mathrm{OUT})}(s) = ST_\ell^I + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int\limits_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}\left(s,s'\right)\ \mathrm{Im}\ t_{\ell'}^{I'(\mathrm{IN})}\left(s'\right), \qquad (1)$$

where $s = m_{\pi\pi}^2$, $ST_\ell^I$ are subtraction terms being combinations of the $S$-wave scattering lengths and $K_{\ell\ell'}^{II'}(s,s')$ are kernels derived by imposing $s \leftrightarrow t$ crossing symmetry conditions on the $\pi\pi \leftrightarrow \pi\pi$ amplitudes. Relation of the unitary amplitudes $t_\ell^I(s)$ with experimentally determined phase shifts $\delta_\ell^I(s)$ and inelasticities $\eta_\ell^I(s)$ is given by

$$t_\ell^I(s) = \frac{\eta_\ell^I(s) e^{2i\delta_\ell^I(s)} - 1}{2i\sqrt{1-4m_\pi^2/s}}\,. \qquad (2)$$

Integration part in Eq. (1) consists of the kernel terms $KT_\ell^I(s)$ and driving ones $DT_\ell^I(s)$. The former account for contributions of all partial waves ($\ell = 0\ldots 3$) at effective two pion mass $\sqrt{s'_{\mathrm{max}}} < 1.42$ GeV and are given by model independent phenomenological parameterizations. The latter terms enclose contributions from the higher $\sqrt{s'}$ region and are given by Regge parameterizations (for details, see [1, 2]).

## 2. Method and results

The smaller difference $\Delta(s) = \mid \mathrm{Re}\ t_\ell^{I(\mathrm{OUT})}(s) - \mathrm{Re}\ t_\ell^{I(\mathrm{IN})}(s) \mid$ the better crossing symmetry for given amplitude $\ell I$ is satisfied. Consistency check of the fit with all theoretical constraints (GKPY and Roy equations, forward dispersion relations FDR and Olsson sum rules SR) has been done by minimization of the sum

$$\chi_{\mathrm{tot}}^2 = \chi_{\mathrm{data}}^2 + \bar{d}_{\mathrm{Roy}}^2 + \bar{d}_{\mathrm{GKPY}}^2 + \bar{d}_{\mathrm{FDR}}^2 + \bar{d}_{\mathrm{SR}}^2\,, \qquad (3)$$

where $\bar{d}_i^2$ are averaged distances of $\Delta_i(s)$ ($i = \mathrm{Roy}, \mathrm{GKPY}\ldots$) taken with uncertainties calculated using Monte Carlo method (for details, see [1, 2]).

In the fit only to experimental data, values of $\bar{d}_i^2$ were: $\bar{d}_{\mathrm{Roy}}^2 = 0.87$, $\bar{d}_{\mathrm{GKPY}}^2 = 1.9$ and $\bar{d}_{\mathrm{FDR}}^2 = 2.0$, while corresponding values in the final fit (*i.e.* with dispersion relations) were 0.14, 0.32, 0.4. Difference between $\bar{d}_{\mathrm{Roy}}^2$ and $\bar{d}_{\mathrm{GKPY}}^2$ is caused by much smaller uncertainties above about 400 MeV in the GKPY equations than in the Roy ones.

The curves in Figs. 1 and 2 present the input and output amplitudes (in fact, their real parts) for the GKPY equations and components $ST_\ell^I, KT_\ell^I(s)$ and $DT_\ell^I(s)$ of the output amplitudes for the $S$-, $P$-, $D$- and $F$-waves. As one can easily see, the differences $\Delta(s)$ for the $S$- and $P$-waves are smaller than their uncertainties in the full energy range.

In the case of the $D$- and $F$-waves, which have not been fitted directly to dispersion relations, quite good agreement between input and output is visible below $\sim$ 800 MeV. For the $D0$ wave, this agreement is sufficiently good even at $m_{\pi\pi} > 1000$ MeV. As was shown in [3], it allows to use this amplitude in *e.g.* photo-production processes to parameterize final state interactions between two pions.

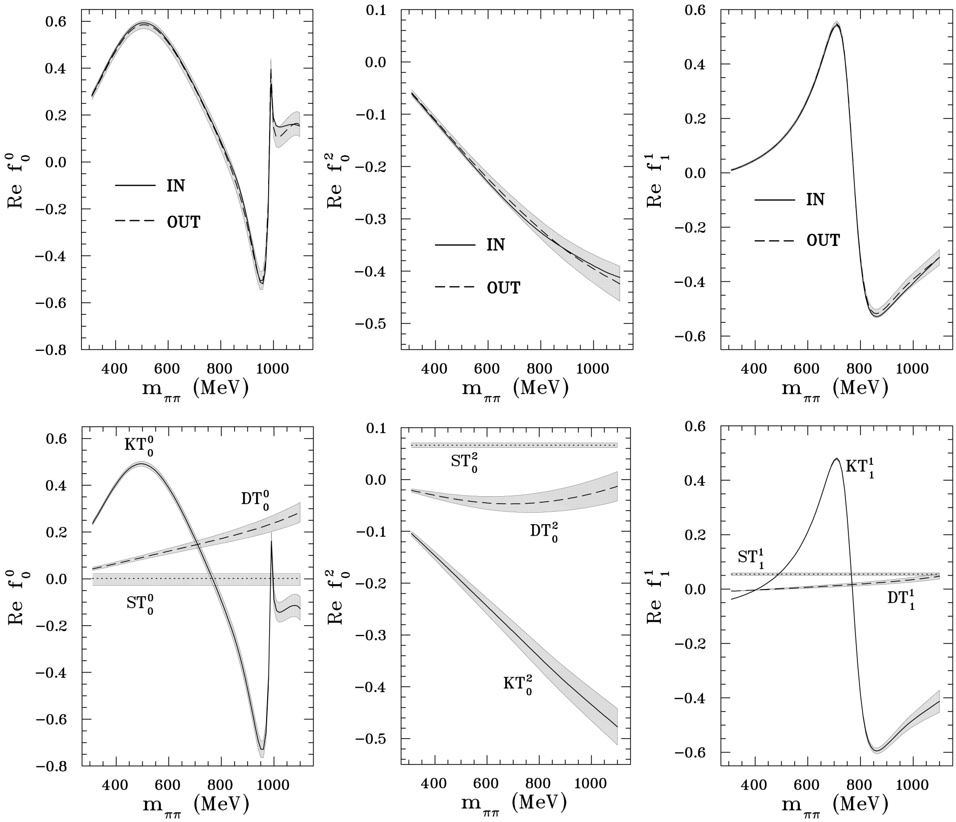


Fig. 1. Three upper figures: $\pi\pi$ input (solid line) and output (dashed line) amplitudes for the $S0$, $S2$ and $P1$ waves together with output error band. Lower figures: components of the $S0$, $S2$ and $P1$ output amplitudes — subtracting ($ST_l^I$), kernel ($KT_l^I$) and driving terms ($DT_l^I$) together with corresponding error bands.

Very slow increase of the output uncertainties is due to the fact that, contrary to the Roy equations, the subtracting terms in the GKPY ones are constant and their errors do not propagate with increasing energy.

Making analytical continuation of the output amplitudes from the Roy and GKPY equations to the second Riemann sheet in the complex energy plane, the poles related with resonances $f_0(500)$, $f_0(980)$ and $\rho(770)$ have

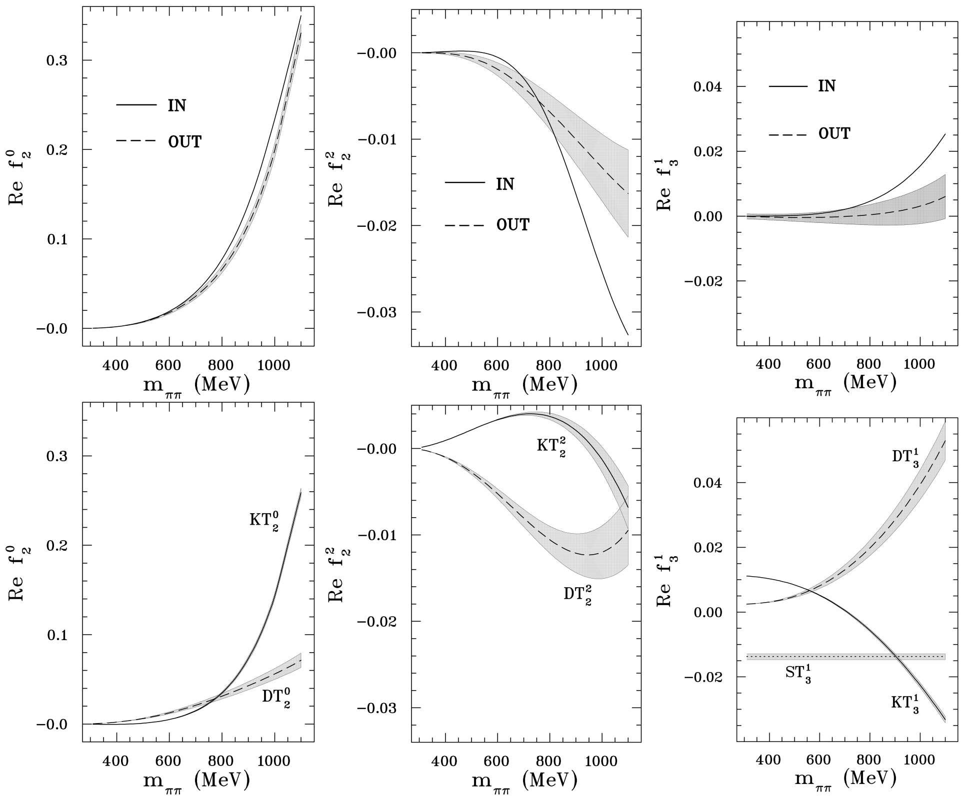


Fig. 2. As in Fig. 1 but for waves $D0$, $D2$ and $F1$.

been found [4] and used to calculate couplings of these resonances to the $\pi\pi$ channel given by residues of the poles (for details, see [4]). The parameters of the resonances and their couplings are presented in Table I. The

TABLE I

Poles and residues from Roy and GKPY equations.

| | $\sqrt{s_{\rm pole}}$ [MeV] | $\lvert g \rvert$ [GeV] |
|---|---|---|
| $f_0(500)^{\rm GKPY}$ | $\left(457^{+14}_{-13}\right) - i\left(279^{+11}_{-7}\right)$ | $3.59^{+0.11}_{-0.13}$ |
| $f_0(500)^{\rm Roy}$ | $(445 \pm 25) - i\left(278^{+22}_{-18}\right)$ | $3.4 \pm 0.5$ |
| $f_0(980)^{\rm GKPY}$ | $(996 \pm 7) - i\left(25^{+10}_{-6}\right)$ | $2.3 \pm 0.2$ |
| $f_0(980)^{\rm Roy}$ | $\left(1003^{+5}_{-27}\right) - i\left(21^{+10}_{-8}\right)$ | $2.5^{+0.2}_{-0.6}$ |
| $\rho(770)^{\rm GKPY}$ | $\left(763.7^{+1.7}_{-1.5}\right) - i\left(73.2^{+1.0}_{-1.1}\right)$ | $6.01^{+0.04}_{-0.07}$ |
| $\rho(770)^{\rm Roy}$ | $\left(761^{+4}_{-3}\right) - i\left(71.7^{+1.9}_{-2.3}\right)$ | $5.95^{+0.12}_{-0.08}$ |

central values for the Roy and GKPY equations well agree within the errors. The only sizable differences are in the errors of positions of poles and couplings what is caused by much smaller uncertainties of the GKPY equations than those of the Roy equations. This agreement confirms compatibility of the twice and once subtracted dispersion relations used in the fits.

Figure 3 presents comparison of the $f_0(500)$ poles taken from Particle Data Group Tables 2010 and 2012 [5, 6]. Also position of the $f_0(500)$ pole found in this analysis is indicated there. Very well seen is significant difference between parameters estimated in the new tables and those in the previous ones. The mass of the $f_0(500)$ has changed from $M = 400$–$1200$ MeV to 400–550 MeV and the width from $\Gamma = 500$–$1000$ MeV to 400–700 MeV. Apart of the changes in the central values of the parameters, very significant changes are introduced in the accuracy of presented estimations caused by the recent dispersive analyzes of the ππ amplitudes which incorporate either Roy or GKPY equations together with other theoretical constraints [1, 7]. In the presented here analysis [1, 4] the greatest impact on reduction of the errors had GKPY equations.

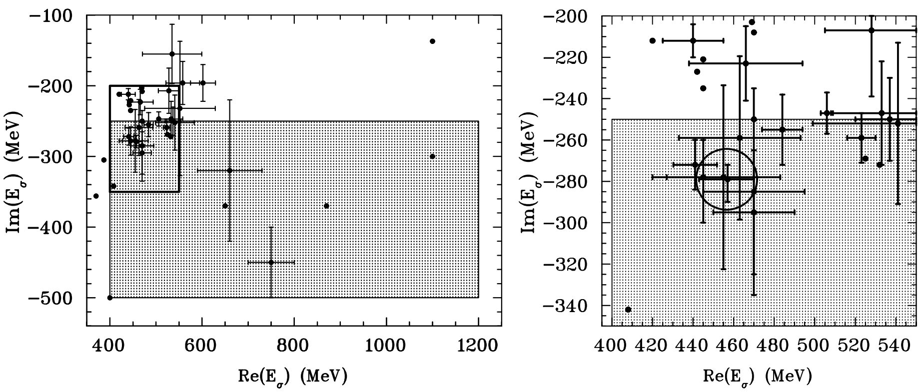


Fig. 3. Positions of the poles (black dots) related with the $f_0(500)$ cited in [5], energy $E_\sigma = \sqrt{s_\sigma}$. Big gray rectangle represents errors of mass and half of the width of the $f_0(500)$ in [5]. The smaller rectangle in the left figure indicates the magnified area shown on the right drawing and corresponds to the errors of mass and half of the width of the $f_0(500)$ in [6]. The pole calculated in presented work lies in the middle of the circle.

For the next scalar–isoscalar resonance $f_0(980)$, the estimated values of parameters in PDGT 2012 are: $M_{f_0(980)} = 990 \pm 20$ MeV and $\Gamma_{f_0(980)} = 40$–$100$ MeV. As one can see in Table I, the position of the pole in presented here analysis completely agrees with this estimation.

## 3. Conclusions

In this short note the main points of new dispersive analysis of the $\pi\pi$ data [1, 2, 4] have been presented. Shortly discussed is the method of simultaneous analysis of theoretical constraints expressed by set of dispersion relations with imposed crossing symmetry condition and experimental data. This method seems to be very demanding, efficient, precise and easy to use. Importance of presented here results of dispersive analysis can be seen, for example, in the new edition of the particle data tables [6], where parameters of the two lightest scalar–isoscalar mesons $f_0(500)$ and $f_0(980)$ have been changed (in the case of the $f_0(500)$ — very significantly). In the case of the $f_0(500)$, even the name has been changed (previously was $f_0(600)$).

Apart of presented here results of dispersive analyzes [1, 2, 4], one has to point out other works in which the Roy equations have been used (*e.g.* [7]). Very important is to mention here that all predictions of those analyzes on the $S$- and $P$-wave amplitudes agree with those obtained in presented in this note and with other analyzed in [1, 4].

One can hope that briefly presented here theoretical method will be widely accepted and used in various analyzes to determine or to correct the $\pi\pi$ theoretical and experimental amplitudes in many partial waves and in wide $m_{\pi\pi}$ range. The unitary and model independent amplitudes parameterized in [1] can be very useful in, for example, analyzes of final state interactions in heavy meson decays and photo-production of pion–pion pairs.

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## REFERENCES

[1] R. Garcia-Martin *et al.*, *Phys. Rev.* **D83**, 074004 (2011).
[2] R. Kamiński, *Phys. Rev.* **D83**, 076008 (2011).
[3] Ł. Bibrzycki, R. Kamiński, *Acta Phys. Pol. B Proc. Suppl.* **6**, 833 (2013), this issue.
[4] R. Garcia-Martin, R. Kamiński, J. Pelaez, J. Ruiz de Elvira, *Phys. Rev. Lett.* **107**, 072001 (2011).
[5] K. Nakamura *et al.* [Particle Data Group], *J. Phys. G* **37**, 075021 (2010).
[6] J. Beringer *et al.* [Particle Data Group], *Phys. Rev.* **D86**, 010001 (2012).
[7] I. Caprini, G. Colangelo, H. Leutwyler, *Phys. Rev. Lett.* **96**, 132001 (2006).