QUANTIFYING INITIAL STATE FLUCTUATIONS IN HEAVY ION COLLISIONS*

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Quantum fluctuations induce interesting structures in the initial state profiles for hydrodynamic calculations of the hot and dense quark-gluon plasma phase of heavy ion collisions. Especially, after the discovery of non-zero odd higher flow coefficients, there has been a lot of theoretical effort to quantify the correlation lengths of energy density fluctuations. A new method to characterize initial state profiles based on a 2D Fourier analysis is presented and an outlook on how this analysis can lead to a better quantitative understanding of the properties of hot and dense nuclear matter is given.

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1. Introduction

Heavy ion collisions offer the opportunity to study strongly interacting matter under extreme conditions. At high temperatures and/or net baryon densities, a phase transition to a new state of matter, where quarks and gluons are the relevant degrees of freedom is expected. One of the major

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goals in the investigation of heavy ion collisions is to extract transport properties of this new phase, the quark-gluon plasma (QGP). Bulk observables in heavy ion collisions such as *e.g.* particle yields, spectra and collective flow are well described by hybrid approaches based on hydrodynamics for the hot and dense stage and hadron transport theory for the late stages where the matter is diluted and runs out of equilibrium [1]. The major unknowns to achieve a complete effective description for the dynamics of a heavy ion reaction are the initial conditions [2].

Currently, there are many different models on the market to provide initial state profiles, *e.g.* parametrizations based on Monte Carlo Glauber approaches, gluon saturation or transport approaches. So far, 1D Fourier decompositions of the azimuthal energy density distribution have been used to characterize the initial state profiles in terms of ϵ_n coefficients. The motivation for our study was to gain a more comprehensive understanding of the structure of fluctuations and provide a basis for detailed comparisons to extract differences and similarities [4].

2. 2D Fourier decomposition

To characterize initial state profiles in a more comprehensive way, a two-dimensional Fourier decomposition of the energy density distribution in coordinate space is introduced. The basic idea is to make use of the radial direction in addition to the azimuthal information. Since the main interest is to quantify the amount and structure of fluctuations in the initial state, the averaged smooth distribution has been subtracted from the profile and the decomposition is applied only to the fluctuations around that mean distribution.

We propose a set of orthogonal basis functions that factorize the radial and angular information. The radial decomposition in terms of Bessel functions and the usual angular Fourier decomposition for the azimuthal direction lead to

$$\phi_{m,n}(r,\theta) := \frac{1}{J_{|m|+1}\left(\lambda_{m,n}\right)} J_m\left(\frac{r}{r_0}\lambda_{m,n}\right) e^{im\theta}\,. \tag{1}$$

As such, any well behaved function f can be expanded

$$f(r,\theta) = \sum_{m,n} A_{m,n}\phi_{m,n}(r,\theta)\,, \tag{2}$$

in terms of these basis functions, with generalized Fourier coefficients $A_{m,n} \in \mathbb{C}$ given by

$$A_{m,n} = \frac{1}{\pi r_0^2} \int f(r,\theta)\phi^{\star}_{m,n}(r,\theta) r dr d\theta\,. \tag{3}$$

Figure 1 represents the real part of the first 15 basis functions $\phi_{m,n}$ ($m = 1, \ldots, 5$ and $n = 1, 2, 3$). The lumpy structures in the basis functions provide confidence that the fluctuations in the initial conditions can be captured well with this decomposition.

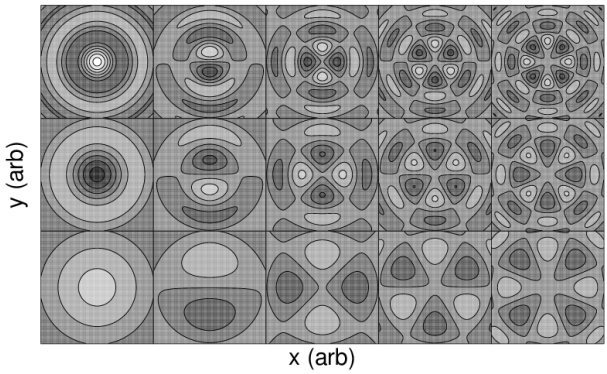

Fig. 1. Real parts of the first 15 basis functions $\phi_{m,n}$.

The first reality check for this new method to quantify initial state profiles is shown in Fig. 2. On the left, there is one initial energy density distribution generated with the hadron transport approach Ultra-relativistic Quantum Molecular Dynamics (UrQMD) [5] for a central Au+Au collision at $\sqrt{s_{NN}} = 200A$ GeV. The right-hand side shows the contour plot of the reconstructed profile from the two-dimensional Fourier decomposition that has been applied. All the structures are reproduced with of the order of ~ 35 generalized Fourier coefficients.

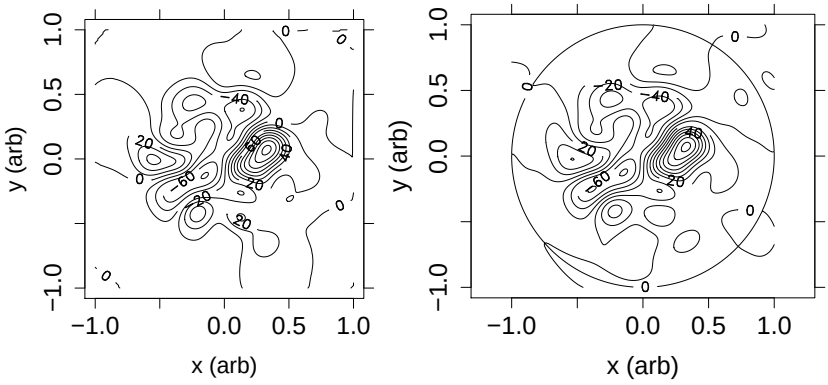

Fig. 2. Left: The original UrQMD initial energy density distribution from one event. Right: The reconstructed profile of the fluctuations using the 2D Fourier decomposition.

2.1. Results for test cases

To condense the information further to only a few numbers, three different norms are defined. The $L_2(f)$ norm is a measure of the total mass of f

$$L_2(f) := \langle f, f \rangle^{1/2} = \left[\sum |A_{m,n}|^2 \right]^{1/2} , \tag{4}$$

where $\langle a, b \rangle = \frac{1}{\pi r_0^2} \int_0^{r_0} a(r,\theta) b^\star(r,\theta) r dr d\theta$ is the inner product. The Sobolev $H_1(f)$ norm contains radial gradients

$$H_1(f) = \left\langle \left(-\ell^2 \nabla^2 + I\right) f, f \right\rangle^{1/2} = \left[\sum \left(\frac{\ell^2 \lambda_{m,n}^2}{r_0^2} + 1 \right) |A_{m,n}|^2 \right]^{1/2} , \tag{5}$$

where ℓ is a characteristic length scale introduced to maintain unit consistency (we use $\ell = 1$ fm). A variation on the Sobolev norm gives the angular variation $M_1(f)$ which quantifies angular gradients

$$M_1(f) := \left\langle \partial_\theta^2 f, f \right\rangle^{1/2} = \left[\sum m^2 |A_{m,n}|^2 \right]^{1/2} . \tag{6}$$

In Fig. 3 the results for these three norms for three different representative initial state models are shown. The UrQMD includes Boltzmann hadronic transport before the hydro begins, while the MC-Glauber code [3] includes simple streaming transport of the nucleons after interaction and KNO scaling of the multiplicity fluctuations per binary collision. The MC-KLN model [6] is based on gluon saturation and has different degrees of freedom than the other two approaches. 100 initial states for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for two different centralities $b = 2, 7$ fm are considered.

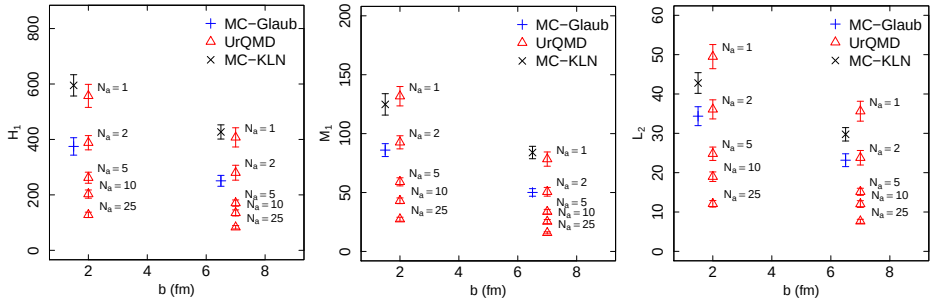

Fig. 3. The ensemble averages of the Sobolev norm H_1 (left) the angular variation M_1 (center) and the L_2 norm (right). The labels on the UrQMD glyphs give the number of averages used to generate the ensemble members. The MC-Glauber and MC-KLN results are plotted with an artificial offset in b to permit easier comparison.

To have a systematic handle on the granularity, a series of ensembles of UrQMD events by averaging successively larger samples of independent raw events together before subtracting out the ensemble average [7] has been generated ($N_a = 1, 2, 5, 10, 25$). The first observation is that the norms all decrease for smoother (higher N_a) initial conditions and, therefore, are useful to quantify fluctuations. For all norms, we see good agreement between the $N_a = 2$ UrQMD events and the MC-Glauber code because they are both based on hadronic degrees of freedom and rather similar in the scale and nature of their fluctuations. The MC-KLN model exhibits a larger H_1 and comparable M_1 indicating that these events exhibit larger radial gradients than the other models. This may be attributed to the very rapid spatial falloff of the gluon density near the edges of the nucleons.

To remove the sensitivity to the overall scale of the fluctuations that is visible in the dependence of the norms defined above on the value of N_a, it is useful to define the "roughness ratio" as an overall scale invariant measure of the roughness in an event

$$\mathcal{R}^2 = \frac{H_1^2}{L_2^2} - 1 = \frac{\langle -\ell^2 \nabla^2 f, f \rangle}{\langle f, f \rangle} = \frac{\ell^2 \sum \lambda_{m,n}^2 |A_{m,n}|^2}{r_0^2 \sum |A_{m,n}|^2}, \tag{7}$$

a weighted average of the scale-free squared characteristic inverse lengths $(\lambda_{m,n}\ell/r_0)^2$, weighted by the squared coefficients $|A_{m,n}|^2$.

Figure 4 shows the probability distribution of the roughness measure $\mathcal{R}^2$ for the same test cases as considered above. Using this scale-independent measure provides a clear separation of the UrQMD and Glauber calculation and the MC-KLN parametrization and, therefore, allows to distinguish between different initial degrees of freedom.

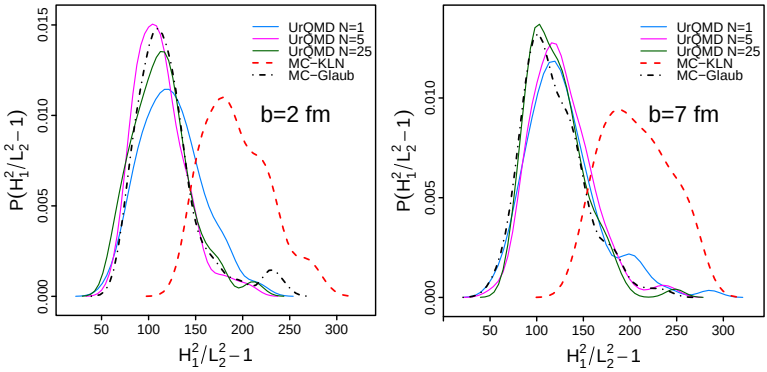

Fig. 4. The distribution of $\mathcal{R}^2$ (left, right) for the UrQMD $N_a = \{1, 5, 25\}$, MC-KLN and MC-Glauber. The left figure shows events at $b = 2$ fm, the right figure shows $b = 7$ fm.

3. Summary and outlook

The two-dimensional Fourier decomposition of spatial initial energy density profiles has been applied to three test cases that represent different classes of initial state models currently used in heavy ion physics phenomenology. The norms and roughness measure are useful to characterize the nature of the fluctuations and distinguish different physics assumptions.

This new method to quantitatively describe initial state profiles is applicable to analytical and Monte Carlo calculations in the same manner. It provides a good tool for necessary apples-to-apples comparisons by extracting essential features, differences and similarities. It is easy to generalize to 3D and other quantities, *e.g.* initial velocity profiles. The next step to be pursued in future work is to connect the initial state properties quantified by the 2D Fourier decomposition with final state observables. This work will be carried out as a multi-parameter analysis applied to a full dynamical hybrid approach at the highest RHIC beam energy with an extensive set of experimental data for bulk observables.

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