Vol. 6 (2013) *Acta Physica Polonica B Proceedings Supplement* No 3

MESON VACUUM PHENOMENOLOGY IN A THREE-FLAVOR LINEAR SIGMA MODEL WITH (AXIAL-)VECTOR MESONS: INVESTIGATION OF THE $\mathrm{U}(1)_A$ ANOMALY TERM*

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(Received July 10, 2013)

Zero temperature properties of an (axial-) vector meson extended linear σ-model are discussed, concerning the possible different realizations of the axial anomaly term. The different anomaly terms are compared with each other on the basis of a χ^2 minimalization process. It is found that there is no essential difference among the different realizations. This means that any of them can be equally used from phenomenological point of view.

DOI:10.5506/APhysPolBSupp.6.853
PACS numbers: 12.39.Fe, 12.40.Yx, 14.40.Be, 14.40.Df

1. Introduction

According to our knowledge, the commonly believed fundamental theory of strong interaction is Quantum Chromodynamics (QCD), which is up to now proved to be unsolvable in the low energy regime, where the basic degrees of freedom are the observable mesons and hadrons. Since the original degrees of freedom in QCD are quarks and gluons and the construction of mesons and hadrons is unknown, in this regime, one possibility is to build up some effective theory [1], which reflects some of the original properties of QCD. One of the most important such property is the approximate global $\mathrm{U}(3)_\mathrm{L} \times \mathrm{U}(3)_\mathrm{R}$ symmetry (if we consider three flavors), the chiral symmetry. This symmetry is isomorphic to the $\mathrm{U}(1)_V \times \mathrm{SU}(3)_V \times \mathrm{U}(1)_A \times \mathrm{SU}(3)_A$, which is broken down — explicitly due to non-zero quark masses, and spontaneously due to non-zero quark condensates [2] — to $\mathrm{U}(1)_V \times \mathrm{SU}(3)_V$, if the isospin symmetric case is considered.

* Presented at the Workshop "Excited QCD 2013", Bjelašnica Mountain, Sarajevo, Bosnia–Herzegovina, February 3–9, 2013.

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Since if only the chiral symmetry is considered alone, the Lagrangian can still contain infinitely many terms, it is necessary to impose other restrictions as well. One natural choice is renormalizability, however, in an effective model this is not totally necessary. Moreover, since we would also like to include (axial-) vector mesons, the renormalizability is violated anyway. Instead of renormalizability, we have chosen dilaton symmetry to restrict the number of terms (for more details, see [3] and references therein).

In order to maintain the above mentioned symmetry breaking pattern ($\mathrm{U}(1)_V \times \mathrm{SU}(3)_V \times \mathrm{U}(1)_A \times \mathrm{SU}(3)_A \longrightarrow \mathrm{U}(1)_V \times \mathrm{SU}(3)_V$), besides the chiral and dilaton symmetric terms, symmetry breaking terms are also needed in the effective Lagrangian. The symmetry is broken explicitly and spontaneously, and concerning the $\mathrm{U}(1)_A$ violation, the spontaneous breaking is realized through the so-called axial/chiral anomaly [4]. Because of this, an anomaly term should be introduced into the effective Lagrangian, which can have different forms, as will be discussed shortly.

The specific form of the anomaly term will affect the form of the tree level masses of the pseudoscalar meson sector. Thus, through a χ^2 minimalization process, which maintains a comparison between the model predictions and the physical spectrum, the 'goodness' of the different anomaly terms can be investigated.

The paper is organized as follows. In Sec. 2 we briefly discuss the model, which is described in more detail in our previous works [3, 5, 6]. In Sec. 3 we investigate the different anomaly terms and describe the χ^2 minimalization process, while in Sec. 4 we conclude.

2. The model

For the Lagrangian, we use, apart from a modified anomaly term $\mathcal{L}_{\mathrm{U}(1)_A}$, the same as in [3], where we have neglected the dilaton field, since it is irrelevant in the current investigation. Thus, our Lagrangian takes the following form

$$
\begin{aligned}
\mathcal{L} =& \operatorname{Tr}\left[(D_\mu\Phi)^\dagger(D_\mu\Phi)\right] - m_0^2 \operatorname{Tr}\left(\Phi^\dagger\Phi\right) - \lambda_1\left[\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 - \lambda_2 \operatorname{Tr}\left(\Phi^\dagger\Phi\right)^2 \\
&-\frac{1}{4}\operatorname{Tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \operatorname{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)\left(L_\mu^2+R_\mu^2\right)\right] + \operatorname{Tr}\left[H\left(\Phi+\Phi^\dagger\right)\right] \\
&+\mathcal{L}_{\mathrm{U}(1)_A} + i\frac{g_2}{2}\left(\operatorname{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \operatorname{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}\right) \\
&+\frac{h_1}{2}\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\operatorname{Tr}\left(L_\mu^2+R_\mu^2\right) + h_2\operatorname{Tr}\left[|L_\mu\Phi|^2 + |\Phi R_\mu|^2\right] \\
&+2h_3\operatorname{Tr}\left(L_\mu\Phi R^\mu\Phi^\dagger\right) + g_3[\operatorname{Tr}(L_\mu L_\nu L^\mu L^\nu) + \operatorname{Tr}(R_\mu R_\nu R^\mu R^\nu)]
\end{aligned}
$$

$$+g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)] + g_5\,\mathrm{Tr}\,(L_\mu L^\mu)\;\mathrm{Tr}\,(R_\nu R^\nu)$$
$$+g_6[\mathrm{Tr}(L_\mu L^\mu)\;\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\;\mathrm{Tr}(R_\nu R^\nu)]\,, \tag{2.1}$$

where

$$\begin{aligned} D^\mu \Phi &\equiv \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA^\mu[T_3, \Phi]\,, \\ L^{\mu\nu} &\equiv \partial^\mu L^\nu - ieA^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu[T_3, L^\mu]\}\,, \\ R^{\mu\nu} &\equiv \partial^\mu R^\nu - ieA^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu[T_3, R^\mu]\}\,. \end{aligned}$$

The quantities Φ, R^μ, and L^μ represent the scalar and vector nonets:

$$\Phi = \sum_{i=0}^{8}(S_i + iP_i)T_i\,, \qquad L^\mu = \sum_{i=0}^{8}(V_i^\mu + A_i^\mu)T_i\,, \qquad R^\mu = \sum_{i=0}^{8}(V_i^\mu - A_i^\mu)T_i\,, \tag{2.2}$$

where $T_i\,(i = 0,\ldots,8)$ denote the generators of U(3), while S_i represents the scalar, P_i the pseudoscalar, V_i^μ the vector, A_i^μ the axial-vector meson fields, and A^μ is the electromagnetic field.

It is worth to note that in the (0–8) sector there is a particle mixing (see [3]) and we use the $\varphi_\mathrm{N} = \frac{1}{\sqrt{3}}\left(\sqrt{2}\,\varphi_0 + \varphi_8\right)$, $\varphi_\mathrm{S} = \frac{1}{\sqrt{3}}\left(\varphi_0 - \sqrt{2}\,\varphi_8\right)$, $\varphi \in (S_i, P_i, V_i^\mu, A_i^\mu)$ non-strange–strange basis, which is more suitable for our calculations. Moreover, H and Δ are constant external fields defined as $H = H_0T_0 + H_8T_8 = \mathrm{diag}\left(h_{0\mathrm{N}}/2, h_{0\mathrm{N}}/2, h_{0\mathrm{S}}/\sqrt{2}\right)$, $\Delta = \Delta_0 T_0 + \Delta_8 T_8 = \mathrm{diag}\,(\delta_\mathrm{N}, \delta_\mathrm{N}, \delta_\mathrm{S})$

Finally, for the $\mathcal{L}_{\mathrm{U}(1)_A}$ anomaly term we use three different terms

$$\begin{aligned} \mathcal{L}_{\mathrm{U}(1)_A} &= c_1\left(\det\Phi + \det\Phi^\dagger\right) + c_2\left(\det\Phi - \det\Phi^\dagger\right)^2 \\ &\quad + c_m\left(\det\Phi + \det\Phi^\dagger\right)\mathrm{Tr}\left(\Phi\Phi^\dagger\right) \end{aligned} \tag{2.3}$$

which should be understood in a sense that from the c_1, c_2, c_m parameters only one is different from zero at the same time. The first two terms are approximations of the "original" axial anomaly term, which is $\propto (\ln\det\Phi - \ln\det\Phi^\dagger)$ (see the original term *e.g.* in [7] and the approximation in [8]), while the third term is a mixed term. Our concept was to choose different anomaly terms in which the power of the Φ field is no more than six. This can be regarded as a first approximation used to compare the effects of different anomaly terms on the spectrum.

3. Comparison of the different anomaly terms

For the analysis, we used a χ^2 method (for more details, see [3]) in which we calculated some physical quantities — masses and decay widths — at tree-level, and compared to experimental data taken from the PDG [9]. χ^2 is defined as

$$\chi^2(x_1,\dots,x_N)=\sum_{i=1}^{M}\left[\frac{Q_i(x_1,\dots,x_N)-Q_i^{\rm exp}}{\delta Q_i}\right]^2 , \tag{3.1}$$

where $(x_1,\dots,x_N)=(m_0,\lambda_1,\lambda_2,\dots)$ are the unknown parameters of the model, $Q_i(x_1,\dots,x_N)$ are the calculated physical quantities, while $Q_i^{\rm exp}\pm\delta Q_i$ are the experimental values taken from the PDG. In the process, we minimalize the χ^2 and determine the 11 unknown parameters of the model, which are $C_1(\equiv m_0^2+\lambda_1\left(\phi_{\rm N}^2+\phi_{\rm S}^2\right)), C_2(\equiv m_1^2+\frac{h_1}{2}\left(\phi_{\rm N}^2+\phi_{\rm S}^2\right)), \delta_{\rm S}, g_1, g_2$, $\phi_{\rm N}$, $\phi_{\rm S}$, h_2, h_3, λ_2 and one from c_1, c_2, c_m. The determined parameters belonging to the minimal χ^2 give the best description of the experimental data. For the physical quantities, we have chosen the following 21 observables [3], f_π, f_K, m_π, m_K, m_η, $m_{\eta'}$, m_ρ , $m_{K^\star}$, $m_{\omega_S\equiv\varphi(1020)}$, $m_{f_{1S}\equiv f_1(1420)}$, m_{a_1}, $m_{a_0\equiv a_0(1450)}$, $m_{K_0^\star\equiv K_0^\star(1430)}$, $\Gamma_{\rho\to\pi\pi}$, $\Gamma_{K^\star\to K\pi}$, $\Gamma_{\phi\to KK}$, $\Gamma_{a_1\to\rho\pi}$, $\Gamma_{a_1\to\pi\gamma}$, $\Gamma_{f_1(1420)\to K^\star K}$, $\Gamma_{a_0(1450)}$, $\Gamma_{K_0^\star(1430)\to K\pi}$ [1].

In Table I, we summarized the χ^2 and the reduced $\chi^2_{\rm red}$ values for the different cases. In the last row, $c_{\rm all}$ means that we included all the three anomaly terms in the fit. It can be seen that the c_1 and c_2 terms give basically the same description of the experimental data, while the c_m term is not as good as the first two. Even if we include all the three terms and extend the number of free parameters, we cannot get any closer to describe better the experimental data. We can, therefore, conclude that in the linear

TABLE I

The total χ^2 and the reduced $\chi^2_{\rm red}=\chi^2/N_{\rm dof}$ for the different anomaly terms, where $N_{\rm dof}$ is the difference between the number of experimental quantities and the number of fit parameters (10 for the first three row and 8 for the last).

Term	χ^2	$\chi^2_{\rm red}$
c_1	59.38	5.94
c_2	62.40	6.24
c_m	110.93	11.09
$c_{\rm all}$	50.19	6.27

[1] It is worth to note that according to the different anomaly terms, the functional forms of some of the observables are different that in [3].

sigma model one can use either the c_1 or the c_2 type of term for the anomaly, while the use of some mixed term is not favored. A detailed analysis shows that either the c_1 or c_2 term is in good agreement with the experimental data.

4. Conclusion

We presented an (axial-) vector meson extended linear sigma model with different axial anomaly terms. Global χ^2 fits were performed in order to compare the different anomaly terms. All the model parameters were fixed in this χ^2 minimalization process. For the different anomaly terms, we found that two of them — which are emanating from the originally suggested anomaly term — describe basically the same physics and are in good agreement with the experimental data, while the third one, the mixed term, is not as good as the others.

We should point out, however, that the presented investigation is only a small part of the meson phenomenology, which aims to understand better the mechanisms of strong interaction at low energies.

The authors were supported by the Hungarian OTKA funds T71989 and T101438. Furthermore, the authors would like to thank the collegues F. Giacosa, D. Parganlija and D.H. Rischke for the useful discussions.

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