GLUEBALLS IN THE BETHE–SALPETER FORMALISM*

E.S. Swanson

Department of Physics and Astronomy, University of Pittsburgh
Pittsburgh, 15260 PA, USA

(Received July 10, 2013)

The Schwinger–Dyson Bethe–Salpeter approach to the bound state problem is applied to the spin zero glueball spectrum. Quark and ghost propagators are obtained from the lattice gluonic two-point function and used as an input to the glueball bound state problem. Although reasonably good results are obtained for all quantities, inconsistencies in the gauge couplings point to a moderate truncation sensitivity.

DOI:10.5506/APhysPolBSupp.6.859
PACS numbers: 11.10.St, 12.38.Lg, 12.39.Mk

1. Introduction

The Bethe–Salpeter (BS) formalism is a field-theoretic method for obtaining information on bound state systems in a covariant framework. It is, however, technically challenging to implement and most applications to hadronic physics have been made in simple models or in two-body systems [1]. Because the Bethe–Salpeter formalism is covariant, it is especially useful for describing dynamical quantities such as form factors and distribution functions. It is also a many-body approach and thus can incorporate chiral symmetry breaking, which is crucial to obtaining reliable predictions in the light hadron spectrum.

Unfortunately, these benefits come at a cost. The Bethe–Salpeter formalism hinges on the assumed form of a two-body irreducible scattering kernel, and this form must be obtained with some truncation that must be made with no apparent small parameter in sight. This issue can become acute because truncated kernels often lead to bizarre predictions such as 'ghost states' [1], nonsensical spectra, or unphysical parameter-dependence of observables. Even this undesirable situation can be a distant goal. Often

* Presented at the Workshop "Excited QCD 2013", Bjelašnica Mountain, Sarajevo, Bosnia–Herzegovina, February 3–9, 2013.

one simply postulates a kernel and tests the accuracy of its predictions. Of course, this removes all putative connections to Quantum Chromodynamics (QCD) and the Bethe–Salpeter formalism devolves to a (quite sophisticated) quark model.

The extension of the Bethe–Salpeter approach to the gluonic sector of the strong interactions is described here [2]. To our knowledge, this is the first attempt at computing glueball masses with the Bethe–Salpeter formalism. It is also a rare example of a hadronic computation that uses QCD to build the interaction kernel. The ingredients necessary for this investigation are gluon, ghost, and quark propagators and the three-gluon, four-gluon, quark-gluon, and ghost-gluon vertices. The ghost and quark propagators are obtained by solving the relevant Schwinger–Dyson equations with a model gluon propagator that is taken from lattice Landau gauge computations. Vertices are modeled with Ansatze that incorporate constraints from gauge invariance and multiplicative renormalisability. All ingredients are then combined with an assumed interaction kernel to obtain the scalar glueball spectrum.

2. Propagators and vertices

Quark, gluon, and ghost propagators are defined in terms of dressing functions as:

$$S(k) \equiv \frac{i}{A\not{k} - B}\,, \tag{1}$$

$$D_{\mu\nu}(k) \equiv -i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) G(k) - i\xi \frac{k^\mu k^\nu}{k^4 + i\epsilon}\,, \tag{2}$$

$$H^{ab}(k) \equiv i\delta^{ab}\, H(k) \equiv i\delta^{ab}\, \frac{h(k)}{k^2}\,. \tag{3}$$

The exact quark and ghost gap equations are shown in Fig. 1. As discussed above, these are truncated by employing vertex models.

Fig. 1. Quark and ghost gap equations.

The primary ingredient in this investigation is the gluon dressing function, G. The unrenormalised propagator has been computed in the Landau gauge in pure SU(3) gauge theory on large lattices [3], with results shown in Fig. 2.

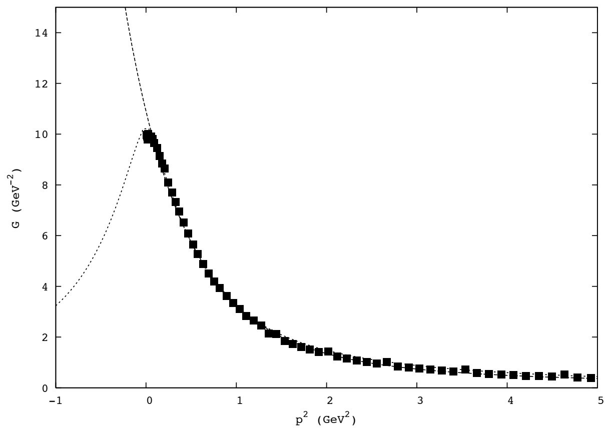

Fig. 2. Lattice results [3] for the Landau gauge gluon propagator and model fits. Error bars are smaller than the points.

Solving the ghost gap equation yields the results shown in Fig. 3, where it is seen that the agreement with a similar lattice computation is quite good. We note that the gap equation has been renormalised and that the gauge coupling has been adjusted to provide a good fit to the lattice results.

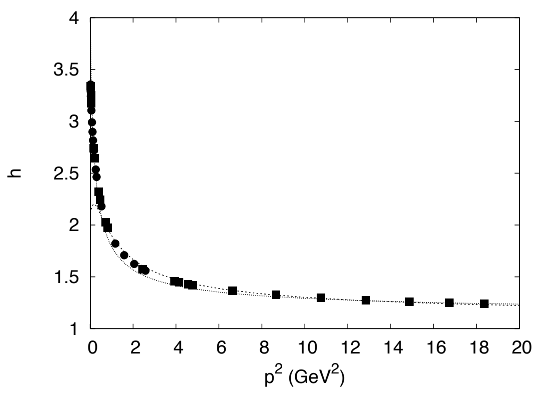

Fig. 3. The renormalised ghost dressing function and lattice results [3]. Error bars are smaller than the data points. Dashed line: bare vertex model; dotted line: full vertex model.

The quark dressing functions are compared to lattice data in Fig. 4; again, the agreement is quite good except for a 20% discrepancy in the wavefunction normalisation in the infrared.

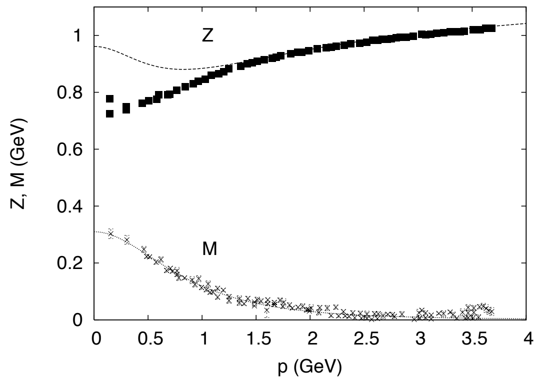

Fig. 4. The quark wavefunction renormalisation, $Z = 1/A$, and quark running mass $M = B/A$, along with lattice results [4].

3. Glueball Bethe–Salpeter equation and results

The form of the gluonic Bethe–Salpeter amplitude is constrained by parity and transversality. The negative parity amplitude contains one scalar function and is given by

$$\chi^{\mu\nu}(k_+, k_-) = \epsilon^{\mu\nu\alpha\beta} k_\alpha P_\beta F(k, P) \,. \tag{4}$$

Alternatively, the scalar glueball is described by two scalar amplitudes as follows

$$\chi^{\mu\nu}(k_+, k_-) = A(k, P) A^{\mu\nu} + B(k, P) B^{\mu\nu} \,. \tag{5}$$

Here,

$$A_{\mu\nu} \equiv \frac{k_\mu^{\perp +} k_\nu^{\perp -}}{k^{\perp +} \cdot k^{\perp -}} \,, \qquad B_{\mu\nu} \equiv g_{\mu\nu} - \frac{k_\mu^- k_\nu^+}{k^+ \cdot k^-} \,, \tag{6}$$

and

$$k_\mu^{\perp +} = k_\mu^+ - k_\mu^- \frac{(k^+)^2}{k^+ \cdot k^-} \,, \qquad k_\mu^{\perp -} = k_\mu^- - k_\mu^+ \frac{(k^-)^2}{k^+ \cdot k^-} \,. \tag{7}$$

The Bethe–Salpeter equation is represented in terms of coupled gluon and ghost BS amplitudes as illustrated in Figs. 5 and 6. Coupling to quarks has been neglected to facilitate comparison to quenched lattice gauge theory computations of the glueball spectrum.

The equation for the pseudoscalar amplitude simplifies because the coupling to the ghost channel is zero and because the contact diagram vanishes. Alternatively, the coupled three-channel problem must be solved for the scalar glueball. The numerical task is made cumbersome because of the $O(10^2)$ terms that contribute to the kernels. The ground state and excited state masses are shown in Fig. 7, along with quenched lattice results

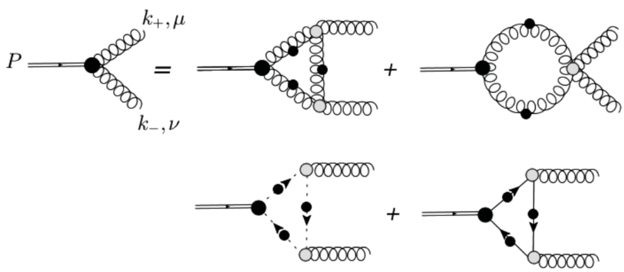

Fig. 5. Gluonic Bethe–Salpeter equation. Dots represent full propagators and model vertices. Crossed diagrams are not shown.

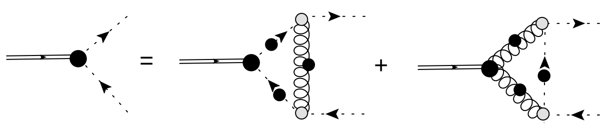

Fig. 6. Ghost Bethe–Salpeter equation.

from Ref. [5]. The figure shows the Bethe–Salpeter eigenvalue, which obtains unity at the Minkowski momentum that corresponds to the relevant state's mass. These results are obtained by fitting the gauge coupling to reproduce the ground state scalar mass. The other three masses are predictions. One sees that the excited scalar and ground state pseudoscalar are reasonably accurately reproduced, while the excited state pseudoscalar (right most dotted line) is approximately 500 MeV too light.

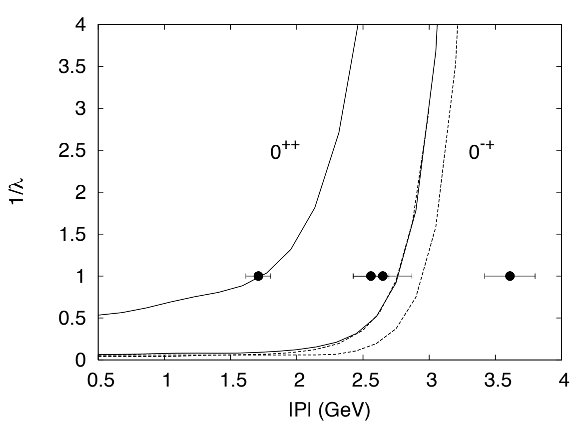

Fig. 7. Glueball Bethe–Salpeter eigenvalues. Lattice data [5] are represented by the horizontal bars. Solid lines: ground states; dotted lines: first excited states. $\mu = 1$ GeV, $g(\mu) = 0.41$.

4. Conclusions

While a reasonably good description of the glueball masses and ghost and quark dressing functions was obtained, the couplings required to obtain such descriptions were drastically different. In particular, $g(\mu = 1\ \text{GeV}) = 4.8$ for the quark propagator, $g(\mu = 1) = 2.4$ for the ghost propagator, and $g(\mu = 1) = 0.41$ for the glueball spectrum (μ is the renormalisation scale). It thus appears that a consistent description of simple properties of QCD remains to be achieved. In particular, the effective strength of the gluonic interaction in the model glueball kernel is much too large. Presumably the problems arise due to inadequate truncations of the Schwinger–Dyson and Bethe–Salpeter equations. Finding alternate truncation schemes that can provide robust approximations to low energy QCD is clearly an important goal.

This research was supported by the U.S. Department of Energy under contract DE-FG02-00ER41135.

REFERENCES

[1] E.E. Salpeter, H.A. Bethe, *Phys. Rev.* **84**, 1232 (1951); M. Gell-Mann, F.E. Low, *Phys. Rev.* **84**, 350 (1951); N. Nakanishi, *Suppl. Prog. Theor. Phys.* **43**, 1 (1969); P. Maris, C.D. Roberts, *Int. J. Mod. Phys.* **E12**, 297 (2003).

[2] J. Meyers, E.S. Swanson, *Phys. Rev.* **D87**, 036009 (2013).

[3] I.L. Bogolubsky, E.M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, *Phys. Lett.* **B676**, 69 (2009).

[4] P.O. Bowman *et al.*, *Phys. Rev.* **D71**, 054507 (2005); P.O. Bowman, U.M. Heller, A.G. Williams, *Phys. Rev.* **D66**, 014505 (2002).

[5] Y. Chen *et al.*, *Phys. Rev.* **D73**, 014516 (2006).