

QUARKONIUM AND GLUEBALL ADMIXTURES OF THE SCALAR–ISOSCALAR RESONANCES $f_0(1370)$, $f_0(1500)$ AND $f_0(1710)^*$

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Using the $U(3)_R \times U(3)_L$ extended Linear Sigma Model with the ordinary (pseudo)scalar and (axial)vector mesons as well as a scalar glueball, we study the vacuum phenomenology of the scalar–isoscalar resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. We present here a solution, based only on the masses and not yet on decays, in which the resonances $f_0(1370)$ and $f_0(1500)$ are predominantly nonstrange and strange $\bar{q}q$ states respectively, and the resonance $f_0(1710)$ is predominantly a scalar glueball.

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1. Introduction

The experimental verification of many scalar–isoscalar resonances, $I^G(J^{PC}) = 0^+(0^{++})$, up to the energy of 2 GeV [1] reenforced the idea that the scalar glueball lies among them. However, the remaining open question is which of these scalars–isoscalars is predominantly the glueball.

In this paper, we address this issue by studying the vacuum phenomenology of scalar–isoscalar states within the framework of the so-called extended Linear Sigma Model (eLSM) [2–4]. Our numerical calculations of the effective Lagrangian (1) are explicitly done in the case of three quark flavours. For $N_f = 3$ two scalar–isoscalar $\bar{q}q$ states exist, the nonstrange $\sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ and the strange $\sigma_S \equiv \bar{s}s$ mesons. The scalar glueball $G \equiv gg$ is implemented in the eLSM from the very beginning as the fluctuation of the dilaton field. We describe G as well as the generation of the scale anomaly of the pure Yang–Mills Lagrangian at the quantum level

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by the usual logarithmic dilaton Lagrangian, first introduced in Refs. [5, 6]. Due to the same quantum numbers, a mixing between the scalar–isoscalar quarkonia and glueball in our model takes place.

In this study, we focus on the determination of the mixing matrix by using the mass eigenvalues of the scalar–isoscalar states and identify them with the physical masses of the resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ [1].

The outline of this proceeding is as follows: In Sec. 2 we present the Lagrangian of the extended Linear Sigma Model with a scalar glueball. In Sec. 3 we discuss our results and finally in Sec. 4 we give a summary as well as an outlook for further research.

2. The effective Lagrangian

In order to study the vacuum phenomenology of the scalar–isoscalar resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, we use the eLSM including a scalar glueball [2–4]. Its compact representation for an optional number of flavours is as follows:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{dil}} + \text{Tr} \left[(D^\mu \Phi)^\dagger (D_\mu \Phi) - m_0^2 \left(\frac{G}{G_0} \right)^2 \Phi^\dagger \Phi - \lambda_2 \left(\Phi^\dagger \Phi \right)^2 \right] \\ & - \lambda_1 \left(\text{Tr} \left[\Phi^\dagger \Phi \right] \right)^2 + c_1 \left(\det \Phi^\dagger - \det \Phi \right)^2 + \text{Tr} \left[H \left(\Phi^\dagger + \Phi \right) \right] \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} \left(\frac{G}{G_0} \right)^2 + \Delta \right) (L^{\mu 2} + R^{\mu 2}) \right] \\ & - \frac{1}{4} \text{Tr} [L^{\mu \nu 2} + R^{\mu \nu 2}] + \frac{h_1}{2} \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [L_\mu L^\mu + R_\mu R^\mu] \\ & + h_2 \text{Tr} [\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr} [\Phi R_\mu \Phi^\dagger L^\mu] + \dots, \quad (1) \end{aligned}$$

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) \quad (2)$$

and

$$\mathcal{L}_{\text{dil}} = \frac{1}{2} (\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right). \quad (3)$$

The dilaton Lagrangian (3) describes the scale anomaly of the pure Yang–Mills sector where the logarithmic term with the energy scale Λ breaks the dilatation symmetry explicitly. The field G is the scalar dilaton and after performing the shift $G \rightarrow G_0 + G$, a particle with the mass m_G arises which we interpret as the scalar glueball.

Our model underlies the following symmetries and their several breakings:

- (i) The global chiral symmetry, $U(N_f)_R \times U(N_f)_L$, which is broken explicitly due to the quantum effects and the nonvanishing quark masses as well as spontaneously. As a consequence of breaking of the latter one a nonvanishing quark condensate, $\langle \bar{q}q \rangle \neq 0$, arises.
- (ii) A crucial symmetry in our model is the already mentioned dilatation symmetry or scale invariance, $x^\mu \rightarrow \lambda^{-1}x^\mu$, which is realized at the classical level of the Yang–Mills sector of QCD, but explicitly broken by the loop corrections. This is known as scale or trace anomaly, respectively, and leads to the nonvanishing gluon condensate, $\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \rangle \neq 0$. Taking into account the dilatation symmetry, we can constrain the number of possible terms in our model. This implies that in the chiral limit with the exception of the logarithmic term in Eq. (3) and the term generating the $U(1)_A$ anomaly, all parameters entering in the model are dimensionless. (Note, using the chiral anomaly it was also possible to couple the pseudoscalar glueball to the model, for details and results see Refs. [8]).
- (iii) Our model is also in agreement with discrete symmetries of QCD, *e.g.* parity P and charge conjugation C .

The multiplet of the ordinary scalar and pseudoscalar mesons in the case of $N_f = 3$, containing the two out of three bare scalar–isoscalar states σ_N and σ_S , reads [2]:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star0} + iK^0 \\ K_0^{\star-} + iK^- & \bar{K}_0^{\star0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}. \quad (4)$$

The explicit form of the left-handed and the right-handed (axial)vector mesons multiplets, L^μ and R^μ can be found in Ref. [2].

3. Results and discussion

A reasonable approach to study the scalar–isoscalar sector of the eLSM with a scalar glueball is to use the values of the global fit performed in Ref. [2] and only to determine the additional three free parameters which enter in this sector. Two of them arise directly from the dilaton Lagrangian (3), namely the bare mass of the scalar glueball m_G and the energy scale Λ . The third one is λ_1 which couples the ordinary (pseudo)scalar mesons and was in Ref. [2] only determined as a part of combination.

In order to obtain the numerical values of the corresponding parameters, we use the bare masses of the scalar–isoscalar states and the experimental masses of the resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. Accordingly, we consider the potential of the corresponding states in the matrix representation

$$V(\sigma_N, G, \sigma_S) = \frac{1}{2}(\sigma_N, G, \sigma_S) \begin{pmatrix} m_{\sigma_N}^2 & z_{G\sigma_N} & z_{\sigma_S\sigma_N} \\ z_{G\sigma_N} & M_G^2 & z_{G\sigma_S} \\ z_{\sigma_S\sigma_N} & z_{G\sigma_S} & m_{\sigma_S}^2 \end{pmatrix} \begin{pmatrix} \sigma_N \\ G \\ \sigma_S \end{pmatrix}. \quad (5)$$

The corresponding bare mass equations are

$$m_{\sigma_N}^2 = C_1 + 2\lambda_1\phi_N^2 + \frac{3}{2}\lambda_2\phi_N^2, \quad (6)$$

$$M_G^2 = \frac{m_0^2}{G_0^2}(\phi_N^2 + \phi_S^2) + \frac{m_G^2}{\Lambda^2} \left(1 + 3 \ln \left| \frac{G_0}{\Lambda} \right| \right) G_0^2, \quad (7)$$

$$m_{\sigma_S}^2 = C_1 + 2\lambda_1\phi_S^2 + 3\lambda_2\phi_S^2. \quad (8)$$

After diagonalization of the bare mass matrix (see Eq. (5)), we obtain the matrix containing the physical masses of the scalar–isoscalar states, $M' = BMB^T$, where B is the mixing matrix which transforms the bare states into the physical ones and *vice versa*

$$\begin{pmatrix} f_0(1370) \\ f_0(1710) \\ f_0(1500) \end{pmatrix} = B \begin{pmatrix} \sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \\ G \equiv gg \\ \sigma_S \equiv \bar{s}s \end{pmatrix}. \quad (9)$$

The numerical values of the free parameters are presented in Table I. They have been obtained by requiring that the three following masses of the scalar–isoscalar states [1] hold: $M_{f_0(1370)} = (1350 \pm 150)$ MeV, $M_{f_0(1500)} = (1505 \pm 6)$ MeV and $M_{f_0(1710)} = (1720 \pm 6)$ MeV.

TABLE I

Values of the free parameters.

Parameter	Value
λ_1	2.03
m_G	1580 [MeV]
Λ	930 [MeV]

The numerical solution of the mixing matrix reads:

$$B = \begin{pmatrix} 0.92 & -0.39 & 0.05 \\ -0.33 & -0.83 & -0.45 \\ -0.22 & -0.40 & 0.89 \end{pmatrix}. \quad (10)$$

It turns out that the resonances $f_0(1370)$ and $f_0(1500)$ are predominantly nonstrange and strange $\bar{q}q$ states, and that the resonance $f_0(1710)$ is predominantly a scalar glueball. For other results of the mixing matrix B using different theoretical models, we refer to Refs. [7] and references therein. The solution in Eq. (10) relies only on calculations of masses of the scalar–isoscalar states. In order to give final statements regarding the composition of the f_0 resonances, evaluation of the corresponding decays is essential. Nevertheless, it is astonishing that by increasing the number of flavours in our model, $N_f = 2 \rightarrow N_f = 3$, the phenomenology of the f_0 resonances seems to change (see Ref. [3]).

4. Conclusions and outlook

We have evaluated the numerical values of the mixing matrix B by using the masses of the scalar–isoscalar states between 1 and 2 GeV within a chiral model, the so-called extended Linear Sigma Model with a scalar glueball [2, 3]. We have found that the resonances $f_0(1370)$ and $f_0(1500)$ are predominantly nonstrange and strange $\bar{q}q$ states, and the resonance $f_0(1710)$ is predominantly a scalar glueball.

Study in progress are calculations of the decay processes appropriate to the solution presented in this work and further search for a solution with the assignment where $f_0(1500)$ is predominantly a glueball and $f_0(1710)$ predominantly a strange $\bar{q}q$ state [9].

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