DIFFERENTIATING BETWEEN Δ- AND Y-STRING CONFINEMENT: CAN ONE SEE THE DIFFERENCE IN BARYON SPECTRA?*

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We use $O(4) \simeq O(3) \times O(3)$ algebraic methods to calculate the energy-splitting pattern of the $K = 2$, 3 excited states of the Y-string in two dimensions. To this purpose we use the dynamical O(2) symmetry of the Y-string in the shape space of triangles and compare our results with known results in three dimensions and find qualitative agreement.

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1. Introduction

QCD seems to demand a genuine three-quark confining potential: the so-called Y-junction string three-quark potential, defined by

$$V_{\rm Y} = \sigma \min_{\boldsymbol{x}_0} \sum_{i=1}^{3} |\boldsymbol{x}_i - \boldsymbol{x}_0| \,, \tag{1}$$

or, explicitly

$$V_{\rm string} \;=\; V_{\rm Y} = \sigma \sqrt{\tfrac{3}{2} \left(\boldsymbol{\rho}^2 + \boldsymbol{\lambda}^2 + 2|\boldsymbol{\rho} \times \boldsymbol{\lambda}| \right)} \,. \tag{2}$$

The complete Y-string potential contains "additional" two-body terms that are valid only in certain parts of the three-particle configuration space, and which we shall ignore here. The $|\boldsymbol{\rho} \times \boldsymbol{\lambda}|$ term is proportional to the area of the triangle subtended by the three quarks. The Y-string potential was proposed as early as 1975, see Refs. [1, 2] and the first schematic calculation (using perturbation theory) of the baryon spectrum for $K \leq 2$ followed soon

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thereafter, Ref. [3]. References [4–6] elaborated on this. The first non-perturbative calculations (variational approximation) of the $K = 3$ band with the Y-string potential were published in the early 1990s, Ref. [7] and extended to the $K = 4$ band later in that decade, Ref. [8]. Yet, some of the most basic properties, such as the ordering of the low-lying states in the spectrum of this potential, without the "QCD hyperfine interaction" and/or relativistic kinematics, remain unknown.

The first systematic attempt to solve the Y-string spectrum, albeit only for the $K \leq 2$ states, can be found in Ref. [9]. That paper used the hyper-spherical harmonics formalism, where the Y-string potential can be written as a function of hyper-angles

$$V_{\mathrm{Y}} \; = \; \sigma \sqrt{\tfrac{3}{2} R^2 \left(1 + \sin 2\chi |\sin\theta|\right)} \,. \tag{3}$$

This led to the discovery, see Ref. [10], of a new dynamical O(2) symmetry in the Y-string potential, with the permutation group $S_3 \subset O(2)$ as the subgroup of the dynamical O(2) symmetry. That symmetry was further elaborated in Ref. [11]. The present report is a continuation of that line of work.

The three-body sum of two-body potentials has only the three-body permutation group S_3 as its symmetry. When one changes variables from the hyper-angles (χ, θ) to $z^{'} = z = \cos 2\chi$ (vertical axis), and $x^{'} = x\sqrt{1 - z^2} = \cos\theta \sin 2\chi$, one can see the full S_3 symmetry, Fig. 1. The area of the triangle $\frac{\sqrt{3}}{2} |\boldsymbol{\rho} \times \boldsymbol{\lambda}|$ and the hyper-radius R are related to the Smith–Iwai variables α, ϕ as follows

$$(\cos\alpha)^2 \; = \; \left(\frac{2\boldsymbol{\rho} \times \boldsymbol{\lambda}}{R^2}\right)^2 \,, \tag{4}$$

$$\tan\phi \; = \; \left(\frac{2\boldsymbol{\rho} \cdot \boldsymbol{\lambda}}{\boldsymbol{\rho}^2 - \boldsymbol{\lambda}^2}\right) \,. \tag{5}$$

The Y-string potential becomes

$$V_{\mathrm{Y}} \; = \; \sigma \sqrt{\tfrac{3}{2} R^2 \left(1 + |\cos\alpha|\right)} \,. \tag{6}$$

Independence of the potential on the variable ϕ is equivalent to its invariance under (infinitesimal) "kinematic rotation" O(2) transformations $\delta x^{'} = 2\varepsilon z^{'}, \delta z^{'} = -2\varepsilon x^{'}$ or, in terms of the original Jacobi variables, $\delta\boldsymbol{\rho} = \varepsilon\boldsymbol{\lambda}, \delta\boldsymbol{\lambda} = -\varepsilon\boldsymbol{\rho}$, in the six-dimensional hyper-space. This invariance leads to the new integral of motion $G_3 = \frac{1}{2}\left(\boldsymbol{p}_\rho \cdot \boldsymbol{\lambda} - \boldsymbol{p}_\lambda \cdot \boldsymbol{\rho}\right)$, References [10, 11], associated with the dynamical symmetry (Lie) group $O(2)$ that is a subgroup of the (full hyper-spherical) $O(6)$ Lie group.

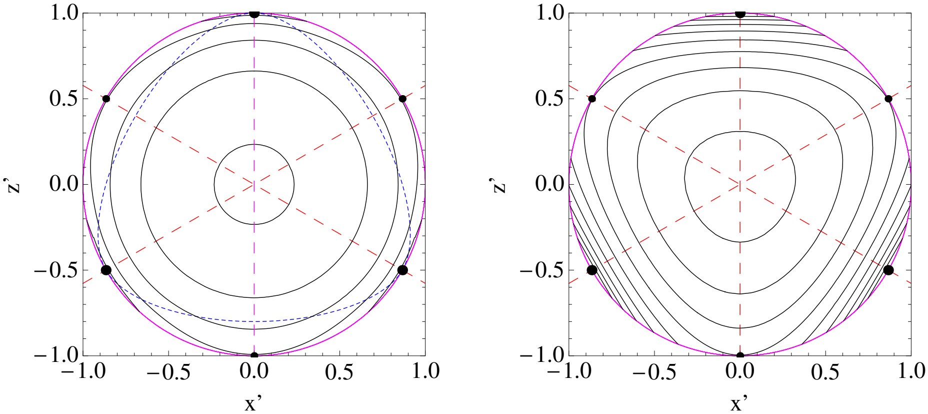

Fig. 1. Left: The equipotential contours for the central Y-string potential (solid), and the boundary between the central Y-string and two-string potentials (dashes). Right: The equipotential contour plot of the Δ-string potential as functions of $z^{'} = z = \cos 2\chi$ (vertical axis), and $x^{'} = x\sqrt{1 - z^2} = \cos\theta \sin 2\chi$ (horizontal axis). The three straight lines (long dashes) of reflection symmetry correspond to the three binary permutations, or "transpositions" S_2 subgroups of S_3. The rotations through $\phi = \pm\frac{2\pi}{3}$ correspond to two cyclic three-body permutations. The rotation symmetry of the Y-string potential (left panel) about the axis pointing out of the plane of the figure should be visible to the naked eye.

Of course, the sums of two-body potentials, such as the Δ-string potential, are invariant only under finite rotations through $\phi = \pm\frac{2\pi}{3}$, that correspond to cyclic permutations, as well as under reflections about the three symmetry axes. In that case, this generalized hyper-angular momentum G_3 is not an exact integral of motion, but an approximate one. The precise consequences in the energy spectra of systems with such a broken (approximate) symmetry will be shown below.

2. The O(4) algebraic method

The existence of an additional dynamical symmetry strongly suggests an algebraic approach, such as those used in Refs. [12–15]. A careful perusal of Refs. [12, 13] shows, however, that an O(2) group had been used as an enveloping structure for the (discrete) permutation group $S_3 \subset O(2)$, but was not interpreted as a (possible) dynamical symmetry. References [14, 15] did not use this symmetry, however. For the sake of technical simplicity, we confine ourselves to the two spatial dimensions here. In two dimensions (2D), the non-relativistic three-body kinetic energy is a quadratic form of the two Jacobi two-vector velocities, $\dot{\boldsymbol{\rho}}, \dot{\boldsymbol{\lambda}}$, so its "hyper-spherical symmetry" is O(4), and the residual dynamical symmetry of the Y-string potential is $O(2) \otimes O_L(2) \subset O(4)$, where $O_L(2)$ is the (orbital) angular momentum. As the $O(4)$ Lie group can be "factored" in two mutually commuting O(3) Lie groups: $O(4) \simeq O(3) \otimes O(3)$, one may use for our purposes many of the O(3) group results, such as the Clebsch–Gordan coefficients. The 3D case is more complicated than the 2D one; for reasons of simplicity, we limit ourselves to the two-dimensional case in this report.

We (re)formulate the problem in terms of $O(4)$ symmetric variables and then bring the potential into a form that can be (exactly) solved, *i.e.* we expand it in O(4) hyperspherical harmonics $\mathcal{Y}^{JJ}_{LM}$. The energy spectrum is a function of the O(4) hyperspherical expansion coefficients for the potential, and of the O(4) Clebsch–Gordan coefficients, that are products of the ordinary O(3) Clebsch–Gordan coefficients. As the potential is $O_L(2)$ rotation-symmetric, we have an additional constraint on the allowed hyperspherical harmonics and one finds that for values of $K \leq 3$ one needs only three terms: (1) the "hyper-spherical average", *i.e.* the $\mathcal{Y}^{00}_{00}$ matrix element, (2) the area-term containing the O(4) hyperspherical harmonic $\mathcal{Y}^{22}_{00}$ (which is related to the O(3) spherical harmonic $Y_{20}(\alpha, \phi)$ of the shape space (hyper)spherical angles (α, ϕ), *i.e.*, the V_4 term in the notation of Richard and Taxil [17]) that is present in both the two-body and the Y-string potentials; and (3) the O(2) symmetry-breaking term containing $\mathcal{Y}^{33}_{0\pm3} \simeq Y_{3\pm3}(\alpha, \phi)$, *i.e.*, the V_6 term in the notation of Richard and Taxil [17], that is important in the two-body potentials, and not at all in the Y-string potential Eq. (2).

3. Results

We have evaluated the $K = 2$, 3 bands' splittings in 2D, Ref. [16] and compare them with the 3D case, Ref. [17]:

(1) The only difference between the 2D and 3D $K = 2$ states' splittings is that the $[70, 0^+]$ and $[56, 2^+]$ states are degenerate in 2D, whereas in 3D they are split by one half of the energy difference between $[70, 2^+]$ and $[70, 0^+]$. This shows that the 2D case does relate fairly closely to the 3D one.

(2) We compare our 2D Y-string potential $K=3$ results with the 3D $K=3$ two-body potential results of Ref. [17] and find certain similarities, and a few distinctions. There are six SU(6) multiplets in the $K = 3$ sector (other than the hyper-radial excitation $[70, 1^-]''$ of the $K = 1$ state): $[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$, $[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$ in 3D. The main difference between the 2D and 3D is that the $[70, 2^-]$ state disappears in 2D.

In 3D two-body potential the energy splittings can be divided in two parts in Ref. [17]: (a) those due to the V_4 perturbation; and (b) due to the V_6 perturbation. This corresponds to our Y_{20} and $Y_{3\pm3}$ terms, respectively.

(a) In the $V_4 \neq 0$, $V_6 \to 0$ limit, the states can be (roughly) divided in two groups: the $[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$ which are pushed down, and the $[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$ which are pushed up by the V_4 perturbation. Two pairs of states are left degenerate: ($[20, 1^-]$, $[56, 1^-]$) in the lower set and ($[56, 3^-]$, $[20, 3^-]$) in the upper set. In this limit, in 2D we find complete degeneracy of all three members of the lower- ($[20, 1^-]$, $[56, 1^-]$, $[70, 3^-]$) and upper levels ($[56, 3^-]$, $[70, 2^-]$, $[20, 3^-]$), Fig. 2 (b).

(b) In the $V_4 \neq 0$, $V_6 \neq 0$ case, the remaining degeneracy of states is removed in 3D, Fig. 2 (a): the $[20, 1^-]$ and the $[56, 1^-]$ are split in the "lower set" and the $[56, 3^-]$ and the $[20, 3^-]$ in the "upper set". In 2D, we find the same pattern of splitting, and a similar ratio of strengths, Fig. 2 (b).

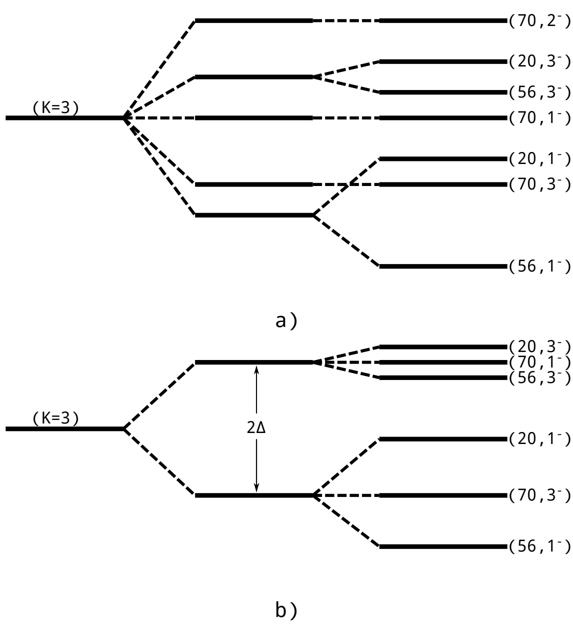

Fig. 2. Schematic representation of the $K = 3$ band in the energy spectrum of the Δ-string potential in (a) three dimensions, following Ref. [17]; and (b) two dimensions (present calculation). The sizes of the two splittings (the v^{Δ}_{20}-induced Δ and the subsequent $v^{\Delta}_{3\pm3}$-induced splitting) are not on the same scale, the latter having been increased, so as to be clearly visible. The Δ here is the same as the Δ in the $K = 2$ band.

So, in the $K = 2$, 3 bands, one sees similarities of dynamical symmetry-breaking patterns in 2D and 3D. This lends credence to the belief that this similarity may persist at higher values of K, where there are not known 3D results, at present.

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