ON THE LANDAU GAUGE MATTER-GLUON VERTEX IN SCALAR QCD IN A FUNCTIONAL APPROACH*

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(Received July 10, 2013)

Recently, the quark-gluon vertex has been investigated in Landau gauge using a combined Dyson–Schwinger and nPI effective action approach. We present here a numerical analysis of a simpler system where the quarks have been replaced by charged scalar fields. We solve the coupled system of the Dyson–Schwinger equations for the scalar propagator, the scalar-gluon vertex and the Yang–Mills propagators in a truncation related to earlier studies. The calculations have been performed for scalars both in the fundamental and the adjoint representation. A clear suppression of the Abelian diagram is found in both cases. Thus, within the used truncation the suppression of the Abelian diagram predominantly happens dynamically and is to a high degree independent of the colour structure. The numerical techniques developed here can directly be applied to the fermionic case.

DOI:10.5506/APhysPolBSupp.6.929
PACS numbers: 11.15.–q, 11.30.Rd, 12.38.Aw

1. Motivation

Confinement and dynamical chiral symmetry breaking (DχSB) are prominent features of QCD but still not satisfactorily understood. There is evidence that the quark-gluon vertex may provide a key position in the pursuit of finding mechanisms behind these phenomena, see, *e.g.*, Ref. [1] and references therein. Since one is interested in the non-perturbative behaviour of this Green function, appropriate methods have to be implemented. The Dyson–Schwinger equations (DSEs), suitably combined with other functional methods as, *e.g.*, Functional Renormalization Group Equations or nPI Effective Actions and cross-checked with corresponding lattice results where available, provide a tool to investigate these low-energy phenomena, see, *e.g.*, Ref. [2] and references therein.

* Presented at the Workshop "Excited QCD 2013", Bjelašnica Mountain, Sarajevo, Bosnia–Herzegovina, February 3–9, 2013.

It turns out that the quark-gluon vertex is a quite complex multi-tensor object due to its Dirac structure. Only very recently attempts towards a full self-consistent solution including all possible tensor structures have been made [3], where, however, up to now only the non-Abelian diagram has been taken into account. However, despite the indications that the Abelian counterpart might be subleading, only a full self-consistent treatment can prove this assumption. Herein, we dwell on another possibility to shed light on this issue. One can substitute the quarks by charged scalar fields residing in the fundamental representation of SU(3). Details and applications of scalar QCD within this context can be found, *e.g.*, in Refs. [4, 5] as well as references therein. Thus, one can try to mimic certain aspects of QCD while on the other hand, can work with a much simpler system due to the absence of Dirac structure. We will follow this strategy throughout the following presentation.

1.1. Scalar QCD

Scalar QCD provides an excellent playground to test, implement or improve numerical techniques and routines which can subsequently be applied in the fermionic system. Despite its simplicity, however, there are some drawbacks as can be seen from the DSE for the scalar propagator depicted in Fig. 1. Compared to the fermionic case, one obtains a much richer diagrammatic structure due to four-scalar and two-scalar-two-gluon vertices not present in QCD[1]. Additional diagrams also appear in the DSEs for the gluon propagator and the scalar-gluon vertex, see Ref. [4] for details. But

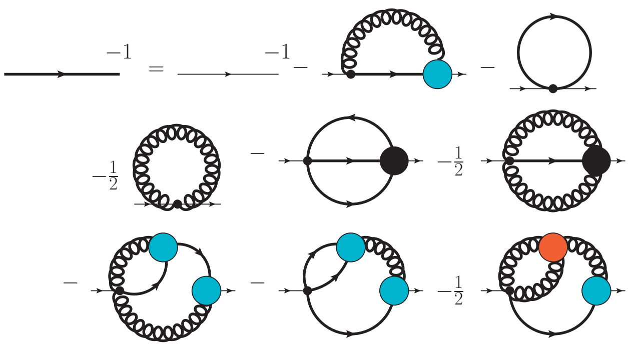

Fig. 1. The DSE for the scalar propagator [4]. Full blobs denote dressed vertices, all internal propagators are dressed.

[1] The constant contribution from the tadpole terms can, in principle, be absorbed within the renormalization procedure but, nevertheless, the remaining two-loop terms very likely contribute at least in the mid-momentum regime, *cf.*, Ref. [6].

since our aim is to apply the techniques to the fermionic case, a truncation which meets the requirements of Ref. [3] is appropriate. In Fig. 2 and Fig. 3, the corresponding matter sector as well as the Yang–Mills system is shown.

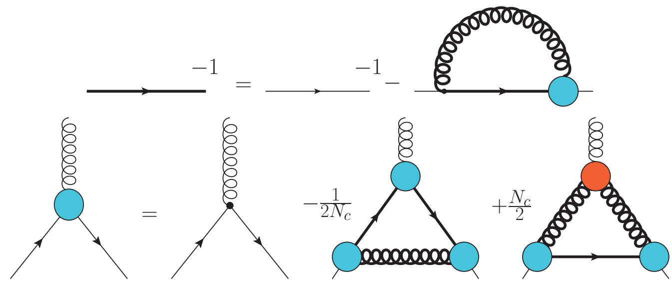

Fig. 2. The scalar propagator and the scalar-gluon vertex DSE in a diagrammatically equivalent truncation to the system treated in Ref. [3], *cf.* also Ref. [1]. All internal propagators are dressed. Full blobs denote dressed vertices. The three-gluon vertex has been modelled according to Ref. [7].

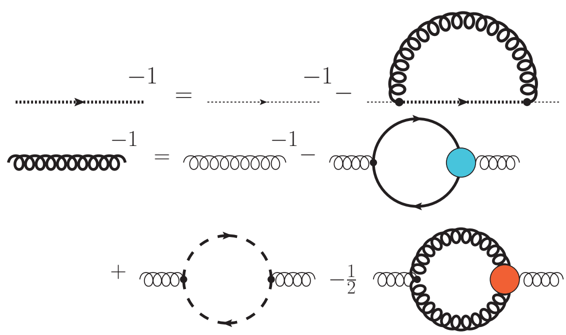

Fig. 3. The Yang–Mills system in a truncation corresponding to Ref. [7] except that in our work a scalar loop has been employed. The ghost vertices are left bare.

We note that in Ref. [3] a DSE-like equation for the quark-gluon vertex has been employed, adopted from the nPI-guided ansatz proposed in Ref. [1]. Thus, all vertices in the scalar-gluon vertex DSE are dressed[2]. One gets a closed system if the three-gluon vertex is specified for both systems. Here the model proposed in Ref. [7] has been employed. The last two diagrams on the right-hand side of the vertex equation are referred to as the *Abelian* and the *non-Abelian* diagram. The colour traces yield prefactors of $-1/2N_c$ and $N_c/2$, respectively. We will show that this natural colour suppression of the Abelian diagram by N_c^2 is additionally amplified dynamically. Indeed,

[2] For details, we refer the Reader to Refs. [1, 8].

in our setup depicted in Fig. 2 it is mostly driven by the dynamics of the system as it will be shown that this suppression is also apparent for adjoint scalars, *i.e.* if both diagrams own the same colour trace.

To dress the vertex, we use the decomposition

$$\Gamma^\nu(p,q;k) = \tilde{A}\,p^\nu + \tilde{B}\,q^\nu = \left(\tilde{A}+\tilde{B}\right)\,q^\nu + \tilde{A}\,k^\nu = \left(\tilde{A}+\tilde{B}\right)\,p^\nu - \tilde{B}\,k^\nu\,, \quad (1)$$

where p^ν and q^ν are the in- and outgoing momenta and $k^\nu = p^\nu - q^\nu$ is their relative momentum. The dressing functions $\tilde{A}$ and $\tilde{B}$ depend on the three invariants p^2, q^2 and $p\cdot q$. Using the transversal projectors $P^{\mu\nu}(k)$ appearing in the equations, one obtains

$$P^{\mu\nu}(k)\Gamma^\nu(p,q;k) = A\,P^{\mu\nu}(k)\,q^\nu = A\,P^{\mu\nu}(k)\,p^\nu\,, \quad (2)$$

i.e., a single dressing function $A(p^2,q^2,p\cdot q)$ is enough in the transverse Landau gauge to include the vertex. The abbreviations $x \equiv p^2$, $y \equiv q^2$ and $\zeta \equiv p\cdot q/|p||q|$ will be used in the following.

1.2. Numerical treatment

The coupled system depicted in Fig. 2 and Fig. 3 has been solved using standard techniques, where the calculations have been performed on Graphics Processing Units (GPUs). For the Yang–Mills part, the numerical routines proposed in Ref. [9] have been adopted as well as improved in order to allow for an arbitrary number of GPU devices. For the matter sector, a conventional integration grid has been employed using a non-linear mapping for the Gauss–Legendre nodes. Furthermore, an internal integration grid for the radial integrals is used which differs from the external one in order to avoid numerical instabilities. A further advantage of this step is the numerical performance gain since it allows to calculate the vertex on a rather coarse external grid, where subsequently a cubic spline interpolation routine interpolates smoothly between the calculated external grid points and generates the values needed for the finer internal grid. This step is performed within each iteration and can be done on GPUs in a very efficient way due to the high degree of parallelism. With this simple method, one can reduce the amount of computing time without loss of accuracy resulting in running times of only a few minutes for the whole system. Within the Abelian diagram, further interpolation steps have to be done. Here, a bi/trilinear interpolation routine between the spline values has been implemented. While for the scalar case this seems to over-shoot the problem, our aim is to adopt the methods to the fermionic case where it is possible that the Abelian diagram behaves non-trivially. Furthermore, the renormalization procedure of the vertex system has been adopted from Ref. [3].

2. Results for the vertex dressing function

The system described in Sec. 1.1 has been solved self-consistently including both the Abelian and the non-Abelian diagram. In the following, we present our results for the vertex dressing function A introduced in Eq. (2). The calculations have been performed for scalars both in the fundamental as well as in the adjoint representation. Figure 4 (upper left) shows the con-

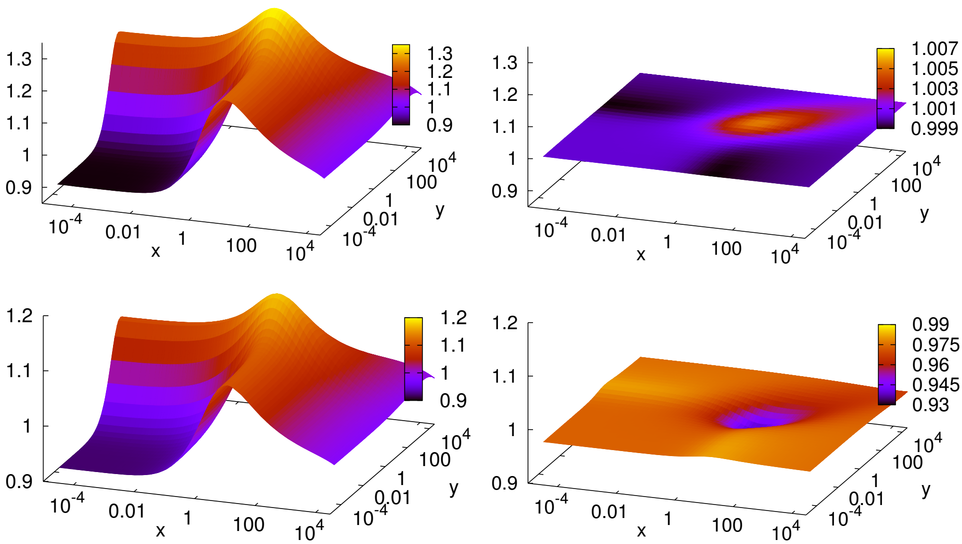

Fig. 4. The vertex dressing function $A(x,y,\zeta)$ plotted for $\zeta \approx -1$. The upper panel shows results for SU(3) with a scalar field in the fundamental representation. Non-Abelian contributions (left panel) are dominant. The results for the adjoint representation of SU(2) (lower panel) reveal that the colour structure has a minor influence. We also checked that this behaviour is hardly affected when varying the scalar masses. Here, no feedback from the scalar loop has been taken into account.

tribution of the non-Abelian diagram for the fundamental representation. As can be seen from Fig. 4 (upper right), the Abelian diagram contributes only very little. Besides its natural colour suppression, it is also suppressed dynamically. This is evident when one puts the scalar field into the adjoint representation. Here, the prefactors of the diagrams become the same, *i.e.*, the colour trace for the Abelian diagram changes from $-1/2N_c$ to $N_c/2$. However, as can be seen from Fig. 4 (lower left) and Fig. 4 (lower right) also in this case, the Abelian diagram is suppressed and contributes only marginally. We note that unquenching effects result in a global suppression of both diagrams. Here, the non-Abelian diagram is more affected due to its *a priori* larger contribution. Furthermore, in the adjoint representation the colour factor for the scalar loop changes from $\frac{1}{2}$ to N_c, and the system is more sensitive to flavour changes. A more detailed investigation of all these and related effects is deferred to an upcoming study.

3. Conclusions and outlook

Scalar QCD in the Landau gauge has been investigated using a Dyson–Schwinger/nPI-Action approach. The coupled equations for the scalar and Yang–Mills propagators as well as the scalar-gluon vertex have been solved self-consistently in a truncation adapted to the fermionic system treated in Ref. [3]. The calculations have been performed for scalar fields in both, fundamental and adjoint, representations. A strong dynamical suppression of the Abelian diagram in the scalar-gluon vertex DSE has been found in both cases. Therefore, this effect is dynamical and independent of the colour structure to a high degree. The numerical techniques can be transferred to the more complicated fermionic case [3] in a direct way.

We thank Mario Mitter for helpful discussions. M.H. acknowledges support from the Doktoratskolleg "Hadrons in Vacuum, Nuclei and Stars" of the Austrian Science Fund, FWF DK W1203-N16.

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