

## $\eta'$ MULTIPLICITY AND WITTEN–VENEZIANO RELATION AT $T > 0^*$

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Recent RHIC results on  $\eta'$  multiplicity in heavy-ion collisions are of great importance because they clearly signal a partial restoration of  $U_A(1)$  symmetry at high temperatures  $T$ , and thus provide an unambiguous signature of the formation of a new state of matter. Prompted by these experimental results of STAR and PHENIX collaborations, we discuss and propose the minimal generalization of the Witten–Veneziano relation to finite  $T$ .

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### 1. Introduction

The most compelling signal for production of a new form of QCD matter at heavy-ion collider facilities like RHIC and LHC — *i.e.*, strongly coupled quark-gluon plasma (sQGP) — would be a restoration, in hot and/or dense matter, of the symmetries of the QCD Lagrangian which are broken in the vacuum. One of them is the  $[SU_A(N_f)$  flavor] chiral symmetry. Its dynamical breaking results in light, (almost-)Goldstone pseudoscalar ( $P$ ) mesons — namely the octet  $P = \pi^0, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta$  (for three light quark flavors,  $N_f = 3$ ). The second one is the  $U_A(1)$  symmetry. Its breaking by the non-Abelian axial Adler–Bell–Jackiw anomaly (‘gluon anomaly’ for short) makes the remaining pseudoscalar meson of the light-quark sector, the  $\eta'$ , much heavier, preventing its appearance as the ninth (almost-)Goldstone boson of dynamical chiral symmetry breaking (DChSB) in QCD.

The first experimental signature of a partial restoration of the  $U_A(1)$  symmetry seems to have been found in the  $\sqrt{s_{NN}} = 200$  GeV central Au+Au reactions at RHIC. Namely, Csörgő *et al.* [1] analyzed the combined PHENIX [2] and STAR [3] data very robustly, through six popular models for hadron multiplicities, and found that at 99.9% confidence level, the  $\eta'$  mass in the

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vacuum,  $M_{\eta'} = 958$  MeV, is reduced by at least 200 MeV inside the fireball. It is the sign of the disappearing contribution of the gluon axial anomaly to the  $\eta'$  mass, which would drop to a value readily understood together with the (flavor-symmetry-broken) octet of  $q\bar{q}'$  ( $q, q' = u, d, s$ ) pseudoscalar mesons. This is the “return of the prodigal Goldstone boson” predicted [4] as a signal of the  $U_A(1)$  symmetry restoration.

A related theoretical issue, which we want to address here, is the status, at  $T > 0$ , of the famous Witten–Veneziano relation (WVR) [5, 6]

$$M_{\eta'}^2 + M_{\eta}^2 - 2M_K^2 = \frac{6\chi_{\text{YM}}}{f_{\pi}^2} \quad (1)$$

between the  $\eta'$ ,  $\eta$  and  $K$ -meson masses  $M_{\eta',\eta,K}$ , pion decay constant  $f_{\pi}$ , and Yang–Mills (YM) topological susceptibility  $\chi_{\text{YM}}$ . It is well satisfied at  $T = 0$  for  $\chi_{\text{YM}}$  obtained by lattice calculations (*e.g.*, [7–10]). Nevertheless, the  $T$ -dependence of  $\chi_{\text{YM}}$  is such [11] that the straightforward extension of Eq. (1) to  $T > 0$  [11], *i.e.*, replacement of all quantities therein by their respective  $T$ -dependent versions  $M_{\eta'}(T)$ ,  $M_{\eta}(T)$ ,  $M_K(T)$ ,  $f_{\pi}(T)$  and  $\chi_{\text{YM}}(T)$ , leads to a conflict with experiment [1]. Nevertheless, this paper details a mechanism, proposed in Refs. [12, 13] which enables WVR to agree with experiment at  $T > 0$ .

## 2. The relations connecting two theories, QCD and YM

The dependence of WVR (1) on YM topological susceptibility  $\chi_{\text{YM}}$  implies  $T$ -dependence of  $\eta'$  mass in conflict with the recent experimental results [1]. Namely, WVR is very remarkable because it connects two different theories: QCD with quarks and its pure gauge, YM counterpart. The latter, however, has much higher characteristic temperatures than QCD with quarks: the “melting temperature”  $T_{\text{YM}}$  where  $\chi_{\text{YM}}(T)$  starts to decrease appreciably was found on lattice to be, for example,  $T_{\text{YM}} \approx 260$  MeV [14, 15] or even higher,  $T_{\text{YM}} \approx 300$  MeV [16]. In contrast, the pseudocritical temperatures for the chiral and deconfinement transitions in the full QCD are lower than  $T_{\text{YM}}$  by some 100 MeV or more (*e.g.*, see Ref. [17]) due to the presence of the quark degrees of freedom.

This difference in characteristic temperatures, in conjunction with  $\chi_{\text{YM}}(T)$  in WVR (1) would imply that the (partial) restoration of the  $U_A(1)$  symmetry (understood as the disappearance of the anomalous  $\eta_0/\eta'$  mass) should happen well after the restoration of the chiral symmetry. But, this contradicts the RHIC experimental observations of the reduced  $\eta'$  mass [1] *if* WVR (1) holds unchanged also close to the QCD chiral restoration temperature  $T_{\text{Ch}}$ , around which  $f_{\pi}(T)$  decreases still relatively steeply [11] for realistic explicit ChSB, thus leading to the increase of  $6\chi_{\text{YM}}(T)/f_{\pi}(T)^2$  and consequently also of  $M_{\eta'}$ .

There is still more to the relatively high resistance of  $\chi_{\text{YM}}(T)$  to temperature: not only does it start falling at rather high  $T_{\text{YM}}$ , but  $\chi_{\text{YM}}(T)$  found on the lattice is falling with  $T$  *relatively* slowly. In some of the applications in the past (*e.g.*, see Refs. [18, 19]), it was customary to simply rescale a temperature characterizing the pure gauge, YM sector to a value characterizing QCD with quarks. (For example, Refs. [18, 19] rescaled  $T_{\text{YM}} = 260$  MeV found by Ref. [14] to 150 MeV.) However, even if we rescale the critical temperature for melting of the topological susceptibility  $\chi_{\text{YM}}(T)$  from  $T_{\text{YM}}$  down to  $T_{\text{Ch}}$ , the value of  $6\chi_{\text{YM}}(T)/f_\pi(T)^2$  still increases a lot [11] for the pertinent temperature interval starting already below  $T_{\text{Ch}}$ . This happens because  $\chi_{\text{YM}}(T)$  falls with  $T$  more slowly than  $f_\pi(T)^2$ .

One must, therefore, conclude that either WVR breaks down as soon as  $T$  approaches  $T_{\text{Ch}}$ , or that the  $T$ -dependence of its anomalous contribution is different from the pure-gauge  $\chi_{\text{YM}}(T)$ . We will show that the latter alternative is possible, since WVR can be reconciled with experiment thanks to the existence of another relation which, similarly to WVR, connects the YM theory with full QCD. Namely, using large- $N_c$  arguments, Leutwyler and Smilga derived [20], at  $T = 0$ ,

$$\chi_{\text{YM}} = \chi \left( 1 + \chi \frac{N_f}{m \langle \bar{q}q \rangle_0} \right)^{-1} \quad (\equiv \tilde{\chi}), \tag{2}$$

the relation (in our notation) between the YM topological susceptibility  $\chi_{\text{YM}}$ , and the full-QCD topological susceptibility  $\chi$ , the *chiral-limit* quark condensate  $\langle \bar{q}q \rangle_0$ , and  $m$ , the harmonic average of  $N_f$  current quark masses  $m_q$ . That is,  $m$  is  $N_f$  times the reduced mass. In the present case of  $N_f = 3$ ,  $q = u, d, s$ , so that  $N_f/m = 1/m_u + 1/m_d + 1/m_s$ .

Equation (2) is a remarkable relation between the two pertinent theories. For example, in the limit of all very heavy quarks ( $m_q \rightarrow \infty$ ,  $q = u, d, s$ ), it correctly leads to the result that  $\chi_{\text{YM}}$  is equal to the value of the topological susceptibility in *quenched* QCD,  $\chi_{\text{YM}} = \chi(m_q = \infty)$ . This holds because  $\chi$  is by definition the vacuum expectation value of a gluonic operator, so that the absence of quark loops would leave only the pure-gauge, YM contribution. However, the Leutwyler–Smilga relation (2) also holds in the opposite (and presently pertinent) limit of light quarks. This limit still presents a problem for getting the full-QCD topological susceptibility  $\chi$  on the lattice [21], but we can use the light-quark-sector result [20, 22]

$$\chi = -\frac{m \langle \bar{q}q \rangle_0}{N_f} + \mathcal{C}_m, \tag{3}$$

where  $\mathcal{C}_m$  stands for corrections of higher orders in small  $m_q$ , and thus of small magnitude. The leading term is positive (as  $\langle \bar{q}q \rangle_0 < 0$ ), but  $\mathcal{C}_m$  is negative, since Eq. (2) shows that  $\chi \leq \min(-m \langle \bar{q}q \rangle_0 / N_f, \chi_{\text{YM}})$ .

Although small,  $\mathcal{C}_m$  should not be neglected, since  $\mathcal{C}_m = 0$  would imply  $\chi_{\text{YM}} = \infty$ , by Eq. (2). Instead, its value (at  $T = 0$ ) is fixed by Eq. (2)

$$\mathcal{C}_m = \mathcal{C}_m(0) = \frac{m \langle \bar{q}q \rangle_0}{N_f} \left( 1 - \chi_{\text{YM}} \frac{N_f}{m \langle \bar{q}q \rangle_0} \right)^{-1}. \quad (4)$$

All this, starting from Eq. (2), has so far been at  $T = 0$ . If the left- and right-hand side (RHS) of Eq. (2) are extended to  $T > 0$ , it is obvious that the equality cannot hold at arbitrary temperature  $T > 0$ . The relation (2) must break down somewhere close to the (pseudo)critical temperatures of full QCD ( $\sim T_{\text{Ch}}$ ) since the pure-gauge quantity  $\chi_{\text{YM}}$  is much more temperature-resistant than RHS, abbreviated as  $\tilde{\chi}$ . The quantity  $\tilde{\chi}$ , which may be called the effective susceptibility, consists of the full-QCD quantities  $\chi$  and  $\langle \bar{q}q \rangle_0$ , the quantities of full QCD with quarks, characterized by  $T_{\text{Ch}}$ , just as  $f_\pi(T)$ . As  $T \rightarrow T_{\text{Ch}}$ , the chiral quark condensate  $\langle \bar{q}q \rangle_0(T)$  drops faster than the other DChSB parameter in the present problem, namely  $f_\pi(T)$  for realistically small explicit ChSB. (See Fig. 1 in our Ref. [12] for the results of the dynamical model adopted here from Ref. [11], and, *e.g.*, Refs. [23, 24] for analogous results of different DS models.) Thus, the troublesome mismatch in  $T$ -dependences of  $f_\pi(T)$  and the pure-gauge quantity  $\chi_{\text{YM}}(T)$ , which causes the conflict of the temperature-extended WVR with experiment around  $T \geq T_{\text{Ch}}$ , is expected to disappear if  $\chi_{\text{YM}}(T)$  is replaced by  $\tilde{\chi}(T)$ , the temperature-extended effective susceptibility. The successful zero-temperature WVR (1) is, however, retained, since  $\chi_{\text{YM}} = \tilde{\chi}$  at  $T = 0$ .

Extending Eq. (3) to  $T > 0$  is something of a guesswork as there is no guidance from the lattice for  $\chi(T)$  [unlike  $\chi_{\text{YM}}(T)$ ]. Admittedly, the leading term is straightforward as it is plausible that its  $T$ -dependence will simply be that of  $\langle \bar{q}q \rangle_0(T)$ . Nevertheless, for the correction term  $\mathcal{C}_m$  such a plausible assumption about the form of  $T$ -dependence cannot be made and Eq. (4), which relates YM and QCD quantities, only gives its value at  $T = 0$ . We will, therefore, explore the  $T$ -dependence of the anomalous masses using the following Ansatz for the  $T \geq 0$  generalization of Eq. (3)

$$\chi(T) = -\frac{m \langle \bar{q}q \rangle_0(T)}{N_f} + \mathcal{C}_m(0) \left[ \frac{\langle \bar{q}q \rangle_0(T)}{\langle \bar{q}q \rangle_0(T=0)} \right]^\delta, \quad (5)$$

where the correction-term  $T$ -dependence is parametrized through the power  $\delta$  of the presently fastest-vanishing (as  $T \rightarrow T_{\text{Ch}}$ ) chiral order parameter  $\langle \bar{q}q \rangle_0(T)$ . In Eq. (6) below, it will become clear that  $\tilde{\chi}(T)$  blows up as  $T \rightarrow T_{\text{Ch}}$  if the correction term there vanishes faster than  $\langle \bar{q}q \rangle_0(T)$  squared. Thus, varying  $\delta$  between 0 and 2 covers the cases from the  $T$ -independent correction term, to (already experimentally excluded) enhanced anomalous masses for  $\delta$  noticeably above 1, to even sharper mass blow-ups for  $\delta \rightarrow 2$

when  $T \rightarrow T_{\text{Ch}}$ . On the other hand, it is not natural that the correction term vanishes faster than the fastest-vanishing order parameter  $\langle \bar{q}q \rangle_0(T)$ . This is why we depicted in Ref. [12] (in its Fig. 2) the  $\delta = 1$  case, and the  $\delta = 0$  ( $T$ -independent correction term) case, as two acceptable extremes. Since they turn out quite similar [12, 13] both qualitatively and quantitatively, there was no need to present any ‘in-between results’, for  $0 < \delta < 1$ .

To see how the above-mentioned results were obtained, note that Eq. (5) leads to the  $T \geq 0$  extension of  $\tilde{\chi}$  defined by Eq. (2)

$$\tilde{\chi}(T) = \frac{m \langle \bar{q}q \rangle_0(T)}{N_f} \left( 1 - \frac{m \langle \bar{q}q \rangle_0(T)}{N_f \mathcal{C}_m(0)} \left[ \frac{\langle \bar{q}q \rangle_0(T=0)}{\langle \bar{q}q \rangle_0(T)} \right]^\delta \right). \quad (6)$$

We now use  $\tilde{\chi}(T)$  in WVR instead of  $\chi_{\text{YM}}(T)$  used in Ref. [11]. Of course, at  $T = 0$ ,  $\tilde{\chi}(T) = \chi_{\text{YM}}(0)$ , which remains an excellent approximation even well beyond  $T = 0$ . Nevertheless, this changes drastically as  $T$  approaches  $T_{\text{Ch}}$ . For  $T \sim T_{\text{Ch}}$ , the behavior of  $\tilde{\chi}(T)$  is dominated by the  $T$ -dependence of the chiral condensate, tying the restoration of the  $U_A(1)$  symmetry to the chiral symmetry restoration.

As for the non-anomalous contributions to the meson masses, we use the same DS model (and parameter values) as in Ref. [11], since it includes both DChSB and correct QCD chiral behavior as well as realistic explicit ChSB. That is, all non-anomalous results ( $M_\pi, f_\pi, M_K, f_K$ , the chiral quark condensate  $\langle \bar{q}q \rangle_0$ , as well as  $M_{s\bar{s}}$  and  $f_{s\bar{s}}$ , the mass and the decay constant of the unphysical  $s\bar{s}$  pseudoscalar meson, and  $T$ -dependences thereof) in the present paper are, for all  $T$ , calculated in the model of Ref. [11]. For details of the non-anomalous sector, see Ref. [11], see Refs. [25, 26] for details on the construction of the  $\eta_0$ – $\eta_8$  complex, Ref. [12] for the original paper on the present topic, and Refs. [12, 13] for the detailed presentations of results.

### 3. Conclusion

Thanks to the Leutwyler–Smilga relation (2), the (partial) restoration of  $U_A(1)$  symmetry [*i.e.*, the disappearing contribution of the gluon anomaly to the  $\eta'$  ( $\eta_0$ ) mass] is naturally tied to the restoration of the  $SU_A(3)$  flavor chiral symmetry and to its characteristic temperature  $T_{\text{Ch}}$ , instead of  $T_{\text{YM}}$ .

In the both cases considered for the  $T$ -dependence of the topological susceptibility (5) [ $\delta = 0$ , *i.e.*, the constant correction term, and  $\delta = 1$ , *i.e.*, the strong  $T$ -dependence  $\propto \langle \bar{q}q \rangle_0(T)$  of both the leading and correction terms in  $\chi(T)$ ], we find [12, 13] that  $\eta'$  mass close to  $T_{\text{Ch}}$  suffers the drop of more than 200 MeV with respect to its vacuum value. This satisfies the minimal experimental requirement abundantly. That is, the results are consistent with the experimental findings on the decrease of the  $\eta'$  mass of Csörgő *et al.* [1], as announced in the end of Introduction.

We also note that our proposed mechanism, tying  $\tilde{\chi}$  to the chiral condensate  $\langle \bar{q}q \rangle_0$ , suggests that partial  $U_A(1)$ -symmetry restoration would also happen if, instead of temperature, matter density is increased sufficiently, so that the chiral symmetry restoration takes place and  $\langle \bar{q}q \rangle_0$  vanishes.

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