QCD-LIKE THEORIES UNDER EXTREME CONDITIONS*

Tomáš Brauner

Faculty of Physics, University of Bielefeld, 33615 Bielefeld, Germany
and
Nuclear Physics Institute ASCR, 25068 Řež, Czech Republic

(Received July 12, 2013)

I review the current status of the phase diagram of QCD-like theories as a function of temperature and/or chemical potential. The main focus is on two-color QCD with two flavors of fundamental quarks where most lattice data and model calculations exist. However, I also discuss theories with a different number of flavors, different representation of quarks, or even different gauge group to start with.

DOI:10.5506/APhysPolBSupp.6.941
PACS numbers: 12.38.Aw, 12.38.Gc, 11.10.Wx

1. Introduction

To understand the state of matter when compressed to extremely high density or heated up to extremely high temperature is a longstanding problem of contemporary physics. As in such environments, all fundamental forces but the strong nuclear one become negligible, this question translates into one about the structure of the phase diagram of the theory of the strong nuclear force, the Quantum ChromoDynamics (QCD).

At present, the phase diagram is being investigated using two main *ab initio* tools, namely experiments with relativistic heavy ions and numerical Monte Carlo simulations on the lattice. Unfortunately, none of these approaches is able to reach the regime of low temperature and densities well above the nuclear saturation density. In the case of lattice simulations, this is because of the infamous sign problem. As a consequence, the detailed structure of the phase diagram in this domain, and even its basic topology, remains highly conjectural (see [1] for a recent review).

* Presented at the Workshop "Excited QCD 2013", Bjelašnica Mountain, Sarajevo, Bosnia–Herzegovina, February 3–9, 2013.

With regards to the phase structure of QCD, we are obviously facing a very difficult problem. In such a situation, we have basically two options: *(i)* to change the methods to approach it; *(ii)* to change the problem. Lack of new ideas that would lead to substantial progress along the first direction has given rise to interest in the latter. From now on, I will thus consider *QCD-like theories*, that is, Yang–Mills theories with a different gauge group or quarks in a different representation thereof than in QCD (see [2] for a review). In particular, I will concentrate on theories with quarks in a (pseudo)real representation of the gauge group. The reason for this restriction is that the (pseudo)reality of the representation ensures the existence of an antiunitary discrete symmetry of the Dirac operator, which is, in turn, responsible for positivity, that is, absence of the sign problem.

Before getting into details, let us mention some distinguished examples of QCD-like theories. The simplest and most thoroughly studied theory in the whole class is undoubtedly *two-color QCD* (2cQCD), that is, SU(2) Yang–Mills theory coupled to quarks in the fundamental representation [3]. It is a prototype of a theory with quarks in a pseudoreal representation, dubbed generally as type-II [4]. Theories of type-I, in which the quark representation is strictly real, include, foremost, *adjoint QCD* (aQCD), that is, $\mathrm{SU}(N_c)$ Yang–Mills theory with any number of colors N_c and quarks in the adjoint representation [5]. Among other, more exotic examples, the G_2-*QCD*, that is, a Yang–Mills theory with fundamental quarks based on the exceptional G_2 gauge group [6], has recently attracted considerable attention.

2. Flavor symmetry

Due to the (pseudo)reality of the quark representation, a QCD-like theory is invariant under the exchange of quarks and antiquarks. Concretely, the right-handed quark field q_{R} can be replaced with its charge conjugate $\mathcal{P}q_{\mathrm{R}}^{\mathcal{C}}$ without affecting the Lagrangian of the theory. Here, $\mathcal{P}$ is the similarity transformation that establishes the equivalence of the quark representation with its complex conjugate [4].

A QCD-like theory with N flavors of Dirac quarks contains, as a consequence of this invariance, effectively $2N$ flavors of degenerate left-handed Weyl quarks. In the limit of zero quark mass, the usual $\mathrm{SU}(N)_{\mathrm{L}}\times\mathrm{SU}(N)_{\mathrm{R}}\times\mathrm{U}(1)_{\mathrm{B}}$ chiral symmetry is thus embedded into an extended global $\mathrm{SU}(2N)$ symmetry. The low-energy spectrum of the theory is determined by spontaneous breaking of this symmetry in the ground state. For type-I theories, $\mathcal{P}$ is a symmetric matrix and a two-quark color singlet such as the chiral condensate is thus symmetric under the exchange of color states of the two quarks. Due to its zero spin and the Pauli principle, it must then transform as a symmetric rank-two tensor of the flavor $\mathrm{SU}(2N)$ group, breaking the symmetry down to SO($2N$). Similarly, for type-II theories $\mathcal{P}$ is an antisymmetric matrix, leading to the SU($2N$) $\to$ Sp($2N$) symmetry-breaking pattern in the vacuum. The pattern of symmetry breaking for various combinations of parameters of the theory is summarized in Fig. 1.

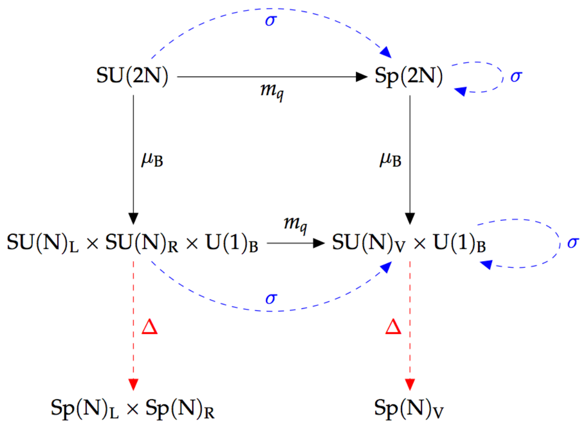

Fig. 1. Flavor symmetry of type-II QCD-like theories and its breaking depending of the parameters of the theory. Solid arrows indicate explicit breaking of the symmetry by either the quark mass m_q or the baryon chemical potential μ_B. Dashed arrows indicate, on the other hand, spontaneous breaking of the symmetry by either the chiral condensate σ or the diquark condensate Δ. The corresponding diagram for type-I theories can be obtained by replacing the Sp groups with SO ones. Here, N denotes the number of degenerate quark flavors in the theory.

Due to the extended symmetry, the (pseudo-)Nambu–Goldstone bosons stemming from its spontaneous breaking include not only the usual $N^2 - 1$ pseudoscalar mesons (pions), but also a set of diquarks. For type-I theories, there are altogether $N(N+1)/2$ diquark–antidiquark pairs, while for type-II theories, the number of these pairs is only $N(N-1)/2$. In the vacuum, their masses are equal to those of the pions. In the simplest nontrivial case of two flavors, this means one (anti)diquark for type-II theories but three (anti)diquarks for type-I theories. This again reflects the color symmetry of the wave function and the Pauli principle: diquarks in type-I theories are symmetric in flavor, that is, carry isospin one.

Let us emphasize that the presence of baryonic diquarks in the spectrum is a natural consequence of the invariance with respect to exchange of quarks and antiquarks. In particular for 2cQCD, a color singlet can only be constructed out of an even number of quarks so that, contrary to QCD, all baryons must be bosons. On the other hand, in theories such as aQCD or G_2-QCD, a color singlet can be made out of both two and three quarks so that the spectrum contains both bosonic and fermionic baryons [7].

3. Phase diagram

What are the qualitative anticipations for the phase diagram of QCD-like theories? At zero chemical potential and increasing temperature, we expect two phase transitions, associated with the restoration of the global flavor symmetry and with deconfinement of quarks. In QCD, both of these are mere sharp crossovers and occur at roughly the same temperature. A great deal of effort has been invested in understanding of this coincidence. It is, therefore, worth mentioning that it is not common to all QCD-like theories. Thus, in aQCD, chiral symmetry is known to be restored at a temperature far above the deconfinement transition [8]. Interesting in this respect is G_2-QCD, which has no symmetry that could be associated with the (de)confinement transition, yet the pure Yang–Mills theory does feature a weak first-order phase transition, similar to QCD.

When non-zero baryon chemical potential is switched on, the masses of diquarks and antidiquarks split off those of pions. Once the chemical potential reaches the pion mass, the diquarks undergo the Bose–Einstein condensation (BEC). As some of the diquarks are pseudo-Nambu–Goldstone bosons, this transition will always dominate the phase diagram at low density, as long as the quark masses are close to the chiral limit. Consequently, the phase diagram of QCD-like theories at non-zero temperature and density can, unlike in QCD, be described by chiral perturbation theory [9]. This gives us the opportunity to study the phase diagram in a model-independent manner.

At very high chemical potential, the mutual interactions of quarks become weak due to asymptotic freedom. The equilibrium state of the theory is then given by a Fermi sea of quarks, slightly deformed by Cooper pairing in the vicinity of the Fermi surface. Very dense quark matter in QCD-like theories, therefore, behaves as a weakly-coupled Bardeen–Cooper–Schrieffer (BCS) superfluid. The order parameters for BEC (diquark condensate) and for the BCS pairing (expectation value of a color-singlet diquark operator) carry the same quantum numbers and thus break the global symmetry of the theory in the same way. Therefore, these two regimes are expected to be smoothly connected by the so-called BCS–BEC crossover [10].

As the above features of the phase diagram are based solely on symmetry, it is not surprising that in a certain limit, namely that of a large number of colors N_c, the phase diagrams of a large class of QCD-like theories are strictly identical [11]. This has been predicted based on orbifold equivalence of the large-N_c theories and later supported by an explicit diagrammatic argument. The same argument can be used to infer equivalence of QCD with two quark flavors and its phase-quenched version, that is, QCD with isospin chemical potential, yet only in a limited part of the phase diagram, outside of the BEC region [12].

4. Two-color QCD

As an illustration of the above general arguments, I show in Fig. 2 the phase diagram of 2cQCD obtained in a model calculation [13]. An analogous diagram was obtained previously within a model, not taking into account the confining aspects of the quark–quark interaction [14]. A phase diagram with a very similar structure was found in lattice simulations [15]. This shows that the qualitative behavior of dense quark matter in 2cQCD is now well understood. However, there is a number of details in which the lattice and model calculations do not agree and which still need to be clarified.

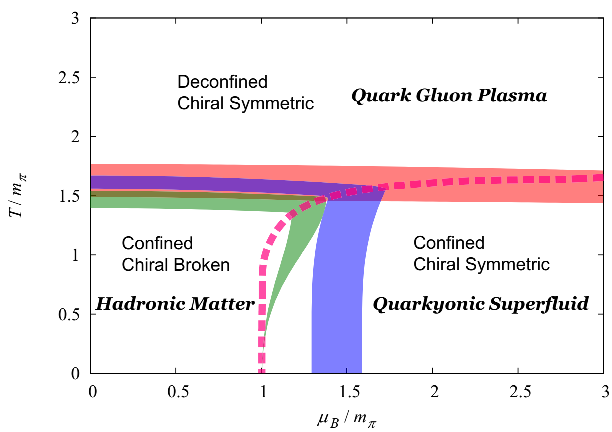

Fig. 2. Phase diagram of 2cQCD with two degenerate quark flavors, in the plane of temperature and baryon chemical potential. The dashed line denotes the diquark BEC transition. The gray (red), light gray (green) and dark gray (blue) bands indicate crossovers defined by the expectation value of the Polyakov loop, baryon number density and the chiral condensate, respectively. This figure was published previously in [13].

First, the thermodynamics of diquark BEC has been a subject to some controversy until recently. Around the BEC transition, thermodynamical observables, normalized to their values for a free gas of quarks, should exhibit a peak that reflects the dominance of bosonic degrees of freedom in this regime. Such a peak was indeed observed in earlier lattice simulations [16], yet to reproduce its height simultaneously for the pressure, baryon density and energy density turned out to be impossible within any kind of effective model based on the flavor symmetry [17]. Fortunately, recent more careful lattice computations showed that this mysterious effect was a sheer consequence of lattice artifacts and a linear extrapolation of a source for the diquark operator to zero [15].

Second, lattice computations predict that the deconfinement transition temperature, defined using the expectation value of the Polyakov loop, drops rather steeply as a function of the chemical potential [15]. This is in contrast to model calculations which tend to give transition temperature almost independent of the chemical potential [13]. However, this is almost certainly an artifact of the approximations used in current models. Namely, in order to reproduce the behavior found on the lattice, the back-reaction of the quark medium into the gauge sector has to be taken into account properly.

To summarize, QCD-like theories offer us an alternative tool to study dense quark matter. Recent progress in lattice simulations of these theories at high baryon density brings us to the point where we can start using the lattice data to extract information, inaccessible to other methods.

The presented work was supported by the Sofja Kovalevskaja program of the Alexander von Humboldt Foundation.

REFERENCES

[1] K. Fukushima, T. Hatsuda, *Rep. Prog. Phys.* **74**, 014001 (2011).

[2] L. von Smekal, *Nucl. Phys. Proc. Suppl.* **228**, 179 (2012).

[3] J.B. Kogut, M.A. Stephanov, D. Toublan, *Phys. Lett.* **B464**, 183 (1999).

[4] T. Zhang, T. Brauner, D.H. Rischke, *J. High Energy Phys.* **06**, 064 (2010).

[5] J.B. Kogut *et al.*, *Nucl. Phys.* **B582**, 477 (2000).

[6] M. Pepe, U.-J. Wiese, *Nucl. Phys.* **B768**, 21 (2007).

[7] A. Maas, L. von Smekal, B. Wellegehausen, A. Wipf, *Phys. Rev.* **D86**, 111901 (2012).

[8] F. Karsch, M. Lütgemeier, *Nucl. Phys.* **B550**, 449 (1999).

[9] K. Splittorff, D. Toublan, J.J.M. Verbaarschot, *Nucl. Phys.* **B639**, 524 (2002).

[10] D.T. Son, M.A. Stephanov, *Phys. Rev. Lett.* **86**, 592 (2001).

[11] A. Cherman, M. Hanada, D. Robles-Llana, *Phys. Rev. Lett.* **106**, 091603 (2011).

[12] M. Hanada, N. Yamamoto, *J. High Energy Phys.* **02**, 138 (2012).

[13] T. Brauner, K. Fukushima, Y. Hidaka, *Phys. Rev.* **D80**, 074035 (2009).

[14] C. Ratti, W. Weise, *Phys. Rev.* **D70**, 054013 (2004).

[15] S. Cotter, P. Giudice, S. Hands, J.-I. Skullerud, *Phys. Rev.* **D87**, 034507 (2013).

[16] S. Hands, S. Kim, J.-I. Skullerud, *Phys. Rev.* **D81**, 091502 (2010).

[17] J.O. Andersen, T. Brauner, *Phys. Rev.* **D81**, 096004 (2010).