NUCLEON EXCITED STATES ON THE LATTICE*

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We study the pion–nucleon system in s-wave in the framework of lattice QCD in order to gain new information on the nucleon excited states. We perform simulations for $n_f = 2$ mass degenerate light quarks at a pion mass of 266 MeV. The results show that including the two-particle states drastically changes the energy levels. The variational analysis and the distillation approach play an important role in the extraction of the energy levels. The phase shift analysis allows to extract information on the resonance nature of the observed states.

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1. Introduction

Almost all the hadrons that constitute the QCD spectrum are unstable under strong interactions. Lattice QCD calculations have been traditionally treating these states as stable, without taking into account their resonant nature. Only recent studies have made exploratory steps in this direction, successfully studying mesonic resonances [1–8].

We study, for the first time the coupled pion–nucleon system explicitly including the two particles in our simulations [9]. This work is motivated by the fact that the lattice hadron spectroscopy does not satisfactory reproduce the negative parity sector of the nucleon states. The physical spectrum consists of two resonances $N^*(1535)$ and $N^*(1650)$. So far, lattice simulations [10–14] have measured in this channel two low-lying states that are assigned to the two resonances, even though the lower measured state lies below the physical value of $N^*(1535)$ [15]. All these simulations considered only 3-quark interpolators that should, in principle, couple to meson-baryon states via dynamical quark loops. However, this coupling seems to be weak and meson-baryon interpolators have to be explicitly included in the set of operators in order to achieve a complete study of these resonances.

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The negative parity resonances of the nucleon couple to $N\pi$ in s-wave, but this is not the only decay channel: $N^*(1535) \to N\eta$, $N^*(1650) \to N\eta, \Lambda K$ [16]. However, we work at an unphysical pion mass ($m_\pi = 266$ MeV) that prevents this channel from being in the influence region of the two resonances. We, therefore, simulate on our lattice the coupled channel of a 3-quark nucleon together with the $(4+1)$-quark $N\pi$ system in the rest frame. The results presented here have already been published in [9].

2. Tools and setup

2.1. Variational analysis

The energy levels of the nucleon and the $N\pi$ system are determined using the variational method [17–19]. We measure the Euclidean cross-correlation matrix $C(t)$ between different interpolators $O_i(t)$

$$C_{ij}(t) = \left\langle O_i(t)\, \bar{O}_j(0) \right\rangle = \sum_n \langle O_i(t)|n\rangle e^{-E_n t} \left\langle n|\bar{O}_j(0)\right\rangle , \tag{1}$$

and then solve the generalized eigenvalue problem

$$C(t)\vec{u}_n(t) = \lambda_n(t) C(t_0)\vec{u}_n(t) \tag{2}$$

to disentangle the eigenstates with the eigenvalues $\lambda_n(t,t_0) \sim e^{-E_n(t-t_0)}$. The energy values of the eigenstates are determined by exponential fits. The fit range is indicated by a plateau-like behavior of the effective masses $E_n(t) = \log[\lambda_n(t)/\lambda_n(t+1)]$.

2.2. Distillation

The evaluation of the correlation matrix in the case of $(4+1)$ quarks turns out to be hardly accessible to traditional techniques, due to the large amount of different diagrams involved. The distillation method [20] allows to evaluate partially disconnected diagrams within an affordable amount of computer time. The quark sources are smeared using a truncated expansion of the 3D Laplacian operator

$$q(x) \to S\left(x, x^{'}\right) q\left(x^{'}\right) = \sum_{i=1}^{N_v} v^i(x) v^{i^\dagger}\left(x^{'}\right) q\left(x^{'}\right) . \tag{3}$$

The correlation function for the 3-quark nucleon operator reads

$$C\left(t_{\rm snk}, t_{\rm src}\right) = \phi_{t_{\rm snk}}(i,j,k)\;\; \tau\left(i,i'\right) \tau\left(j,j'\right) \tau\left(k,k'\right)\;\; \phi^\dagger_{t_{\rm src}}\left(i',j',k'\right) , \tag{4}$$

where the perambulators $\tau(n,m)$ are quark propagators from source eigenvector v^m to sink v^n. The functions ϕ include all the information on the Dirac structure of the specific interpolator

$$\phi_{t_{\rm snk}}(i,j,k) = \sum_{\vec{x}} \epsilon_{abc} D\, v_a^i(\vec{x})\, v_b^j(\vec{x})\, v_c^k(\vec{x}) \,, \tag{5}$$

where D carries all the Dirac indices.

2.3. Interpolators

The set of interpolators has to be as complete as possible in order to reliably extract the spectrum. We use the nucleon interpolator

$$N_\pm^{(i)}(\vec{p}=0) = \sum_{\vec{x}} \epsilon_{abc}\, P_\pm\, \Gamma_1^{(i)}\, u_a(\vec{x})\, u_b^T(\vec{x})\, \Gamma_2^{(i)}\, d_c(\vec{x})\,, \tag{6}$$

where $(\Gamma_1, \Gamma_2) = (\mathbb{1}, C\gamma_5), (\gamma_5, C), (i\mathbb{1}, C\gamma_t\gamma_5)$ and each quark source is smeared combining $N_v = 32$ and 64 eigenvectors. For the $N\pi$ system, we use

$$N\pi(\vec{p}=0) = \gamma_5 N_+(\vec{p}=0)\, \pi(\vec{p}=0) \,, \tag{7}$$

and we project to isospin 1/2: $O_{N\pi} = p\pi^0 + \sqrt{2}\, n\pi^+$ with

$$\begin{aligned} \pi^0\left(\vec{0}\right) &= \frac{1}{\sqrt{2}} \sum_{\vec{x}} \left\{ \bar{u}_a(\vec{x})\, \gamma_5 u_a(\vec{x}) - \bar{d}_a(\vec{x})\, \gamma_5 d_a(\vec{x}) \right\}, \\ \pi^+\left(\vec{0}\right) &= \sum_{\vec{x}} \bar{d}_a(\vec{x})\, \gamma_5 u_a(\vec{x}) \,. \end{aligned} \tag{8}$$

2.4. Interpretation of the energy levels

Once the energy levels are computed, one can relate the measured spectrum of the coupled system to the physical resonances. In the elastic region, Lüscher's formula gives a relation between the discrete energy levels and the phase shift in the continuum [18, 22] (for the rest frame system; for a discussion of meson-baryon systems in moving frames, see [21])

$$\tan\delta(q) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)} \,, \tag{9}$$

where the generalized zeta function Z_{lm} is given in [22] and

$$q = p^* \frac{L}{2\pi} \,, \qquad p^{*2} = \frac{\left[s - (m_N + m_\pi)^2\right]\left[s - (m_N - m_\pi)^2\right]}{4s} \tag{10}$$

with $s = (E_n)^2$. Given a phase shift model, Eq. (9) can be numerically inverted to obtain the expected energy levels for the two interacting particle system (Fig. 2, r.h.s.). For a different method to predict the expected energy levels in finite volume, see *e.g.*, [23].

3. Results

We use 280 configurations generated for two flavors of mass-degenerate light quarks and a tree level improved Wilson–Clover action. $m_\pi = 266$ MeV, $a = 0.12$ fm, $V = 16^3 \times 32$ [24].

3.1. One particle sector

Using a set of 3-quark interpolators, we reproduce the usual observed spectrum [15]. In the positive sector, we observe the nucleon ground state at $m_N = 1068(6)$ MeV and another state that lies far above the physical Roper. In the negative sector, we observe two nearby levels, the lowest lying below $N^*(1535)$ (Fig. 1, l.h.s.).

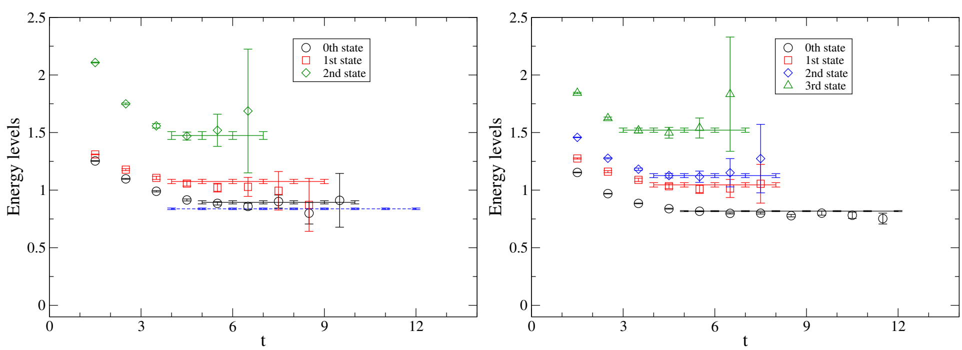

Fig. 1. Effective mass values for N_-. Left: Results for 3-quark interpolators. The dashed (blue) line denotes the non-interacting $N\pi$ state. Right: Results for the N_- and $N_+\pi$ coupled system.

3.2. Coupled N and $N\pi$ system

First, we compute the energy level for the two particles propagating independently (*i.e.*, the threshold) and we find that it is overlapping with the first of the two levels measured in the single-particle approach (Fig. 1, l.h.s.). When $O_{N\pi}$ is included, a new energy level appears and the effective energy levels of the $N\pi$ system show less fluctuations compared to the 3-quark case. The lowest level now lies slightly below the $N\pi$ threshold, a feature typical for attractive s-wave and a finite volume artifact. The next-higher two levels now lie approximately 130 MeV above the physical resonance positions of $N^*(1535)$ and $N^*(1650)$, similar to the shift of the nucleon ground state for this value of m_π.

3.3. Phase shift analysis

A comparison with the expected energy levels obtained inverting the Lüscher formula (9) and assuming a single elastic resonance parameterization shows an excellent agreement (Fig. 2, r.h.s.). Assuming a Breit–Wigner shape for the first resonance, we can also extract the resonance mass: $m_{\mathrm{R}} = 1.678(99)$ GeV.

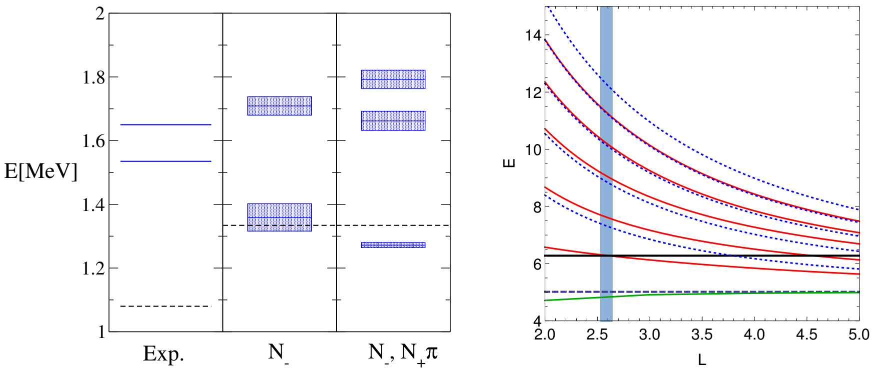

Fig. 2. Left: Comparison between the experimental masses of the negative parity nucleon resonances, the energy spectrum obtained from the single particle analysis and the results from the coupled N and $N\pi$ systems. Right: Energy levels expected for the interacting $N\pi$ system obtained inverting the Lüscher formula and assuming a Breit–Wigner parametrization for $N(1530)$.

4. Conclusions

This study is intended to shed some light on the excited energy levels of the nucleon spectrum, which still represents an outstanding challenge for lattice QCD. We find that meson-baryon interpolators are indeed needed for a reliable picture of the N_- spectrum. The study of two particle systems on the lattice improves our understanding of LQCD and this work is a first step into that direction.

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