

STUDY OF THE TETRAQUARK SYSTEM FROM MESON–MESON SCATTERING WITH A COLOR FLIP–FLOP MODEL*

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In this work, we briefly review the lattice results for the two quark and two antiquarks system in the static limit, in particular the flux-tube recombination. Then, we first review the results obtained for a simple model developed to described tetraquarks. A flip–flop potential which takes into account the color structure is then developed. With this model, we study meson–meson scattering, for a system of two equal quarks and two equal antiquarks. By integrating out the internal degrees of freedom, we arrive at a coupled channel Schrödinger equation, from which we find bound states and resonances, corresponding to tetraquark states.

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1. Introduction

A long standing issue in strong interaction physics is the existence of hadronic particles, with a valence constitution different from $Q\bar{Q}$ and $QQQ\bar{Q}$, such as tetraquarks made of two quarks and two antiquarks. Experimentally, several particles have been advanced as candidates (for instance [1, 2]). Theoretically, the problem is a very interesting one for various reasons, for instance, it is a system of four particles interacting through a four-body potential and also because it is the simplest strong interacting system where more than one color singlet is possible.

2. $QQ\bar{Q}\bar{Q}$ system

Now, we consider a system of two quarks $Q_1 Q_2$ and two antiquarks $\bar{Q}_3 \bar{Q}_4$. The study of this kind of systems is important for the understanding of meson–meson scattering processes and the possible formation of tetraquarks particles made of two valence quarks and two valence antiquarks.

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In this system, we have two linearly independent color singlets. Those can be the two meson states: $|\mathcal{C}_I\rangle = \frac{1}{3}|Q_i Q_j \bar{Q}_i \bar{Q}_j\rangle$ and $|\mathcal{C}_{II}\rangle = \frac{1}{3}|Q_i Q_j \bar{Q}_j \bar{Q}_i\rangle$, or the anti-symmetric and symmetric color states: $|\mathcal{A}\rangle = \frac{\sqrt{3}}{2}(|\mathcal{C}_I\rangle - |\mathcal{C}_{II}\rangle)$ and $|\mathcal{S}\rangle = \sqrt{\frac{3}{8}}(|\mathcal{C}_I\rangle + |\mathcal{C}_{II}\rangle)$.

We know from lattice results [3, 4] that the ground state potential is given by $V_{FF} = \min(V_I, V_{II}, V_T)$, where V_I and V_{II} are the sum of the intra-meson potentials $V_I = V_M(r_{13})$ and $V_{II} = V_M(r_{24})$. This is well modeled by the Cornell potential: $V_M = K - \frac{\gamma}{r} + \sigma r$. V_T is the tetraquark potential which confines the four particles, being given by $V_T = 2K - \gamma \sum_{i<j} \frac{C_{ij}}{r_{ij}} + \sigma L_{\min}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$. Variational calculations in [5] indicate that this potential could bind a tetraquark. This kind of potentials [6] were first introduced as a way to prevent the long range non-physical behavior [7] present in the sum of two bodies Casimir scaled potentials $V_C = \sum_{i<j} C_{ij} V(r_{ij})$.

3. Chromo-fields results

Using the same operator as [3, 4] and then a variational basis, we were able to find color fields for a $QQ\bar{Q}\bar{Q}$ state [8, 9]. The results for the Lagrangian density $\mathcal{L}$ in the ground state is given in Figs. 1 and 2. There, we can see that at the ground state the system collapses in a two meson state, when the two $Q\bar{Q}$ pairs are far apart, as expected. However, when the two

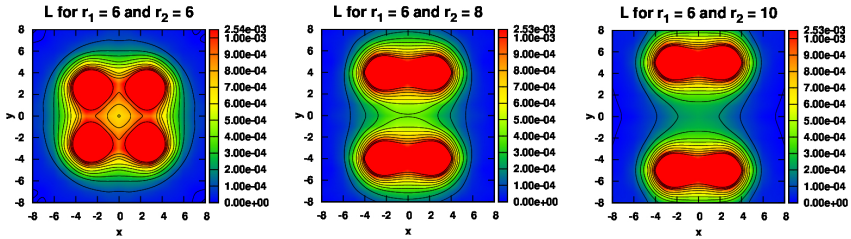


Fig. 1. Lagrangian density for the ground state of the antiparallel geometry, with the quarks at opposite corners of a rectangle.

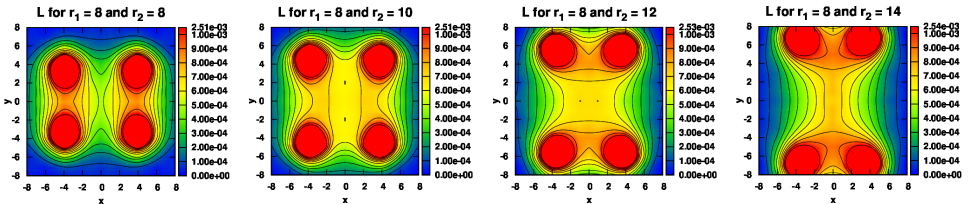


Fig. 2. Lagrangian density for the ground state of the parallel geometry, with both quarks at the bottom.

quarks are far from the two antiquarks, the stablest string configuration is one where the four particles are linked together, corresponding to the V_T sector of the flip–flop potential.

4. Simplified model

First, we use a simplified model to study the possible tetraquark bound states and resonances. To achieve this, we simply use the potential

$$V_{FF} = \sigma \min \left(2r, \sqrt{3}\rho + r \right). \quad (1)$$

We can find directly the bound states by diagonalizing the Hamiltonian. With this method we find a bound state for $l_r = 3$ (Fig. 3). However, we can also find resonances in this model. For this, we project the wavefunction in the asymptotic “meson” states $\psi_i(\boldsymbol{\rho}) = \int d^3\mathbf{r} \phi_i^*(\mathbf{r})\Psi(\mathbf{r}, \boldsymbol{\rho})$ with ϕ_i being the eigenfunctions of the “meson states” $-\frac{\hbar^2}{2m}\nabla_r^2\phi_i + 2\sigma r\phi_i = \epsilon_i\phi_i$. This way we arrive at a coupled channel Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi_i + V_{ij}\psi_j = (E - \epsilon_i)\psi_i. \quad (2)$$

By studying the asymptotic behavior of this equation, we can determine the phase shifts of the model.

As we can see in Fig. 3, we find resonances in channels with non-null angular momentum. So with this model, we predict that tetraquarks should have high orbital angular momentum. This is in accordance with previous works, for instance [10], where it was predicted that the existence of a centrifugal barrier between the diquarks would prevent the recombination of the system into two meson, this way retarding the system decay.

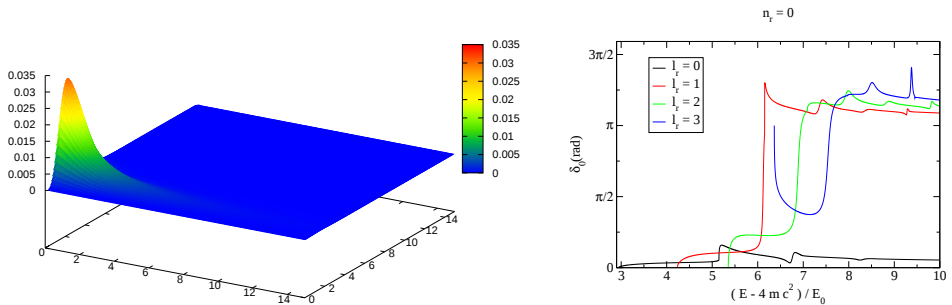


Fig. 3. Left: Bound state for $l_r = 3$. Right: Phase shifts for the lowest radial states of the simplified model. Note resonances for $l_r \neq 0$.

5. Potential

Several approximations were made in the previous session. One of them was that only one of the two meson–meson sectors was considered. Another one was the neglect of the internal color degrees of freedom of the system. As was said before, the system could form two linearly independent color singlets and as nothing forbids the transition between them, our potential could be represented by a 2×2 matrix. We know from lattice results that lowest eigenvalue is given by $v_0 = \min(V_I, V_{II}, V_T)$. To reconstruct the potential matrix, we need both the eigenvalues and the corresponding eigenvectors. The eigenvector of the ground state is already known. It will be $|\mathcal{C}_I\rangle$, $|\mathcal{C}_{II}\rangle$ and $|\mathcal{A}\rangle$, depending on which of the three branches of the potential the system is. Since the matrix has to be Hermitian, its eigenvectors have to be orthogonal. Thus, the second eigenvector has to be orthogonal to the first one. As for the second eigenvalue, we assume, for the transition to be as smooth as possible, the following hypothesis:

$$\begin{aligned} v_1 &= \min(V_{II}, V_T) & \text{when } v_0 = V_I, \\ v_1 &= \min(V_I, V_T) & \text{when } v_0 = V_{II}, \\ v_1 &= \min(V_I, V_{II}) & \text{when } v_0 = V_T. \end{aligned}$$

This hypothesis seems to be supported by some lattice results [11]. So with this, we can construct a flip–flop potential which is free from non-physical Casimir forces and which takes into account the color degrees of freedom of this system.

6. Meson–meson scattering

Now, we solve the problem of meson–meson scattering trying to find bound states and resonances which correspond to tetraquark states. Here, we first expand the wavefunction in the basis of two-meson states, that is $|\Psi\rangle = \Psi^A |\mathcal{C}_A\rangle$. Note that this basis is not orthogonal $\langle \mathcal{C}_I | \mathcal{C}_{II} \rangle \neq 0$. However, we can introduce a contravariant basis $|\mathcal{C}^A\rangle$, for which $\langle \mathcal{C}^A | \mathcal{C}_B \rangle = \delta_B^A$. This way, we choose the scattering kinetic operator as $\hat{T}_S = (\hat{T} + \hat{V}_I) |\mathcal{C}_I\rangle \langle \mathcal{C}^I| + (\hat{T} + \hat{V}_{II}) |\mathcal{C}_{II}\rangle \langle \mathcal{C}^{II}|$, corresponding to the kinetic energy of two free meson states in both color states of the basis. It follows then that the scattering potential is given by $\hat{V}_S = \hat{V} - \hat{V}_I |\mathcal{C}_I\rangle \langle \mathcal{C}^I| - \hat{V}_{II} |\mathcal{C}_{II}\rangle \langle \mathcal{C}^{II}|$. Now, if we expand the two color components of the wavefunction as $\Psi^A = \sum_i \phi_i^1(\boldsymbol{\rho}_1) \phi_i^2(\boldsymbol{\rho}_2) \psi_i^A(\mathbf{r}_A)$ and integrate the intra-mesonic degrees of freedom, we arrive at a coupled channel Schrödinger equation

$$-\frac{\hbar^2}{2\mu_\alpha} \nabla^2 \psi^\alpha(\mathbf{r}) + \int d^3\mathbf{r}' v_\beta^\alpha(\mathbf{r}, \mathbf{r}') \psi^\beta(\mathbf{r}') = (E - \epsilon_\alpha) \psi^\alpha(\mathbf{r}). \quad (3)$$

Here, the Greek letter indices represent both the color index A and the other quantum numbers i . This equation is similar to Eq. (2), except that the potential is non-local between two different color states (because the simplified model has only one color state).

In the following calculation, we will only consider systems made exclusively from heavy (c and b) quarks. This way, we will consider the kinematics of the meson–meson system as non-relativistic, although considering the masses of the mesons in the reduced masses. In the interaction potential, we will only consider the spin independent part, neglecting this way all the spin effects. We will also not consider quark–antiquark creation and destruction effects.

Here, it will be considered the case of an exotic tetraquark, constituted by two quarks of the same flavor and two similar antiquarks $QQ\bar{q}\bar{q}$, in particular, $b\bar{b}c\bar{c}$. In this case, we can see that the Hamiltonian has the form

$$\hat{H} = \begin{bmatrix} \hat{D} & \hat{A} \\ \hat{A} & \hat{D} \end{bmatrix}.$$

Therefore, its eigenfunctions can be given by $\Psi_\xi = \begin{bmatrix} u \\ \xi u \end{bmatrix}$, with $\xi = \pm 1$. We present in Figs. 4 and 5 results, respectively, for $L = 0$ and for $L = 1$ for both values of ξ . Here, the parameters $\sigma = 0.19 \text{ GeV}^2$ and $\alpha = 0.4$ are used for the potential parameters.

As can be seen, we are able to find a bound state for both $L = 0$ with $\xi = +1$. By calculating directly the eigenvalues of the Hamiltonian, we find it to have a binding energy of less than 1 MeV. For the other combinations of L and ξ , we do not find any bound state, but resonances are found for $L = 0$ with $\xi = -1$ and $L = 1$ with $\xi = +1$. Note however, that we only use two two-meson states for $L = 1$, while for $L = 0$ four states are used.

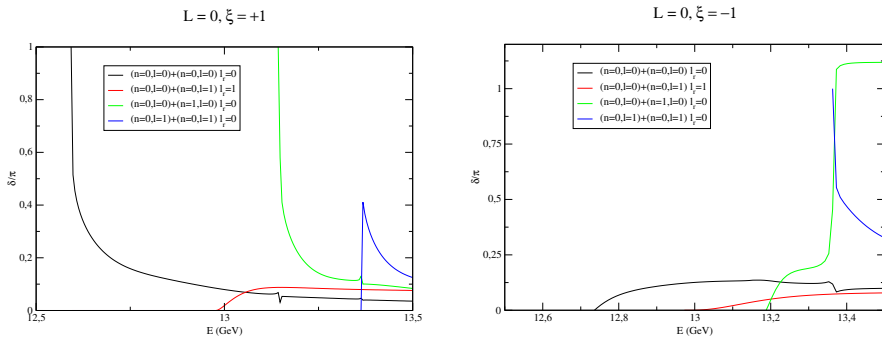


Fig. 4. Phase shifts for the $L = 0$ channel, for both values of ξ . Note the bound state for $\xi = +1$ and the resonance for $\xi = -1$.

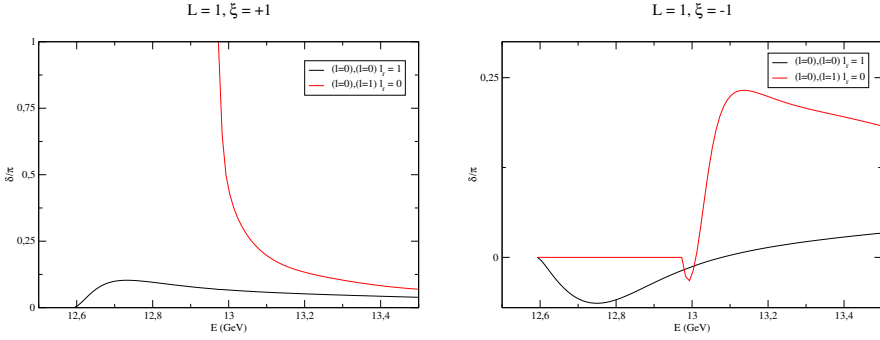


Fig. 5. Phase shifts for the $L = 1$ channel, for both values of ξ . Note the resonance for $\xi = +1$.

7. Conclusion

Here, an unitarized potential model for the computation of the meson–meson scattering was developed. This model was then applied to the heavy quark limit with spin effects neglected, where we found a very weak bound state (with binding energy with less than 1 MeV) and resonances for both $L = 0$ and $L = 1$. Refinement in this procedure can be done, by changing the potential to include other effects such as spin–spin interactions or to use other models of confinement. This procedure is also valid to complex energy and so it can, in principle, be used to find directly the poles of the T matrix, by using numerical methods such as the Newton’s method.

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