

NUCLEAR FISSION WITHIN THE LUBLIN–STRASBOURG DROP MODEL*

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A review of applications of the nuclear Lublin–Strasbourg Drop (LSD) model to evaluation of masses and fission barrier heights is presented. Significant differences between the binding energies of neutron rich isotopes close to the neutron drip line obtained within the LSD and the Thomas–Fermi model of Świątecki and co-workers were found.

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1. Introduction

The nuclear liquid drop (LD) model proposed first in 1935 by Weizsäcker has explained quantitatively the systematics of nuclear binding energies known at that time [1]. This model was extended in 1939 by Meitner and Frisch by adding the deformation dependence to the surface and Coulomb terms in the LD mass formula [2]. This extension was needed to explain new phenomenon, nuclear fission, discovered in fall 1938 by Hahn and Strassman who have discovered barium in the natural uranium radiated with neutrons [3].

First quantitative calculation related to the fission process with the use of deformed liquid drop was performed by Bohr and Wheeler who have expanded the surface of a fissioning nucleus in the spherical harmonics series [4]. They have got a reasonable estimate of the fission barrier height and kinetic energies of the fission fragments.

Next important step, which we would like to remind, was done by Myers and Świątecki who have added the shell and pairing corrections to the liquid drop binding energy [5]. This new macroscopic–microscopic model became

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very successful in reproduction of nuclear data (masses, quadrupole moments, fission barrier heights, *etc.*) when the shell corrections were more precisely evaluated using a method proposed by Strutinsky [7, 8] and the pairing correction obtained within the Bardeen–Cooper–Schrieffer (BCS) theory [6, 8].

In 70s and later, it was shown that the standard liquid drop model failed in reproduction of measured fission barrier heights and some extensions of the LD like the droplet model (DM) [9] or the finite-range droplet model (FRDM) [12] were proposed.

Parallel self-consistent theories of the Hartree–Fock plus BCS or the Hartree–Fock–Bogolubov types with the effective nucleon–nucleon Skyrme or Gogny interaction as well as the relativistic mean-field theory were successfully elaborated and implemented to the description of the nuclear binding energies and the fission barriers (for review, see [10]). Nevertheless, the macroscopic–microscopic model is still very popular as it gives the most accurate predictions of global nuclear properties.

2. LSD model

Using the leptodermous expansion of the nuclear energy functional [10] obtained *e.g.* within the self-consistent models, one can easily obtain all terms (except the Coulomb one) which appear in the nuclear liquid-drop mass formula. An example of such an expansion made for ^{132}Sn using SkM* interaction is shown in Fig. 1. Different terms of the LD formula: volume, surface, 1st and 2nd order curvature terms are plotted as functions of the radius constant r_0 , where $R = r_0 A^{1/3}$ corresponds to a somehow arbitrary chosen nuclear sharp surface around which the leptodermous expansion is performed. It is seen in Fig. 1 that the magnitude of the surface and both curvature energies depends significantly on the choice of the expansion radius while the volume term by definition remains constant.

Using this idea, we have enriched the original Myers and Świątecki LD model [5] by the first and second order curvature terms and performed a least square fit to 2766 experimental binding energies taken from Ref. [11]. The microscopic (shell, pairing and deformation) corrections tabulated in Ref. [12] were added to our liquid drop energy when performing the fit. The values of the volume, surface, first and second curvature coefficients as well the r.m.s. deviation of the fit are plotted in Fig. 2 as a function of the charge radius constant r_{ch} . Qualitatively the results presented in Figs. 1 and 2 are similar, especially when one notices that growing charge radius leads to smaller Coulomb energy which should be recompensed by a correspondingly smaller surface energy, *i.e.* by a smaller surface radius. In other words, in the LD model the changes of the charge and surface radii

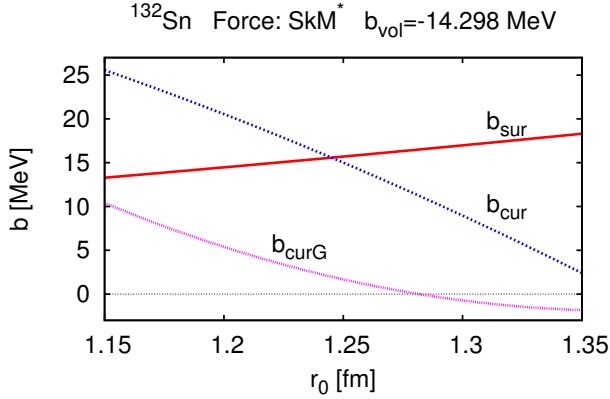


Fig. 1. Surface (b_{sur}), the 1st (b_{cur}) and 2nd (Gaussian) curvature (b_{curG}) terms as a function of the radius constant r_0 which defines the nuclear sharp surface radius.

constants are opposite in phase. One can also notice in Fig. 2 that the r.m.s. deviation of the theoretical and experimental masses does not change significantly with r_{ch} .

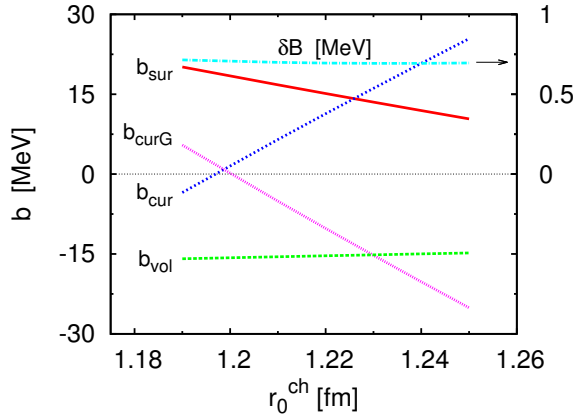


Fig. 2. Volume (b_{vol}), surface (b_{sur}), 1st (b_{cur}) and 2nd (Gaussian) curvature (b_{curG}) coefficients as a function of the charge radius constant r_{ch} . The value of the r.m.s. deviation of theoretical and measured masses (δB) is marked on the right-hand side scale.

The resulting coefficients of this Lublin–Strasbourg–Drop (LSD) mass-formula [13]

$$M(Z, N; \text{def}) = ZM_{\text{H}} + NM_{\text{n}} - b_{\text{elec}}Z^{2.39} + b_{\text{vol}}(1 - \kappa_{\text{vol}}I^2)A + b_{\text{surf}}(1 - \kappa_{\text{surf}}I^2)A^{\frac{2}{3}}B_{\text{surf}}(\text{def})$$

$$\begin{aligned}
& +b_{\text{cur}}(1-\kappa_{\text{cur}}I^2)A^{\frac{1}{3}}B_{\text{cur}}(\text{def}) + \frac{3}{5}\frac{e^2Z^2}{r_0^{\text{ch}}A^{1/3}}B_{\text{Coul}}(\text{def}) \\
& -C_4\frac{Z^2}{A} + E_{\text{micr}}(Z, N; \text{def}) + E_{\text{cong}}(Z, N)
\end{aligned} \tag{1}$$

with Z and N being the proton and neutron numbers, $I = (N - Z)/A$, are following:

$$\begin{aligned}
b_{\text{vol}} &= -15.4920, & \kappa_{\text{vol}} &= 1.8601, & b_{\text{surf}} &= 16.9707, & \kappa_{\text{surf}} &= 2.2938, \\
b_{\text{cur}} &= 3.8602, & \kappa_{\text{cur}} &= -2.3764, & r_0 &= 1.21725, & C_4 &= 0.9181.
\end{aligned}$$

The rest of the coefficients in Eq. (1) is the same as in the mass table of Moller *et al.* [12]: $M_{\text{H}} = 7.289034 \text{ MeV}$, $M_{\text{n}} = 8.071431 \text{ MeV}$, $b_{\text{elec}} = 1.433 \text{ eV}$. The congruence (Wigner) energy $E_{\text{cong}} = -10 \exp(-4.2|I|) \text{ MeV}$ which plays in (1) a role of the Gauss curvature term is also taken from Ref. [12]. Surprisingly, the quality of reproduction of the masses of the investigated 2766 isotopes with $Z, N > 8$ was better than that obtained within other more complex models like TF or FRDM [12].

3. Fission barrier heights

The LSD formula was used in Ref. [13] to evaluate the fission barrier heights. The topographical theorem of Myers and Świątecki, which says that in fissioning nuclei the microscopic energy correction at saddle points is negligible [14], is used to obtain the fission barrier heights. The quality of the LSD estimates as well as of the topographical theorem can be seen in Fig. 3, where the LSD saddle-point masses M_{sadd} of even-even isotopes in the actinide region are compared to the experimental data. The r.m.s. deviation of the both masses in the saddle point is equal 310 keV, only.

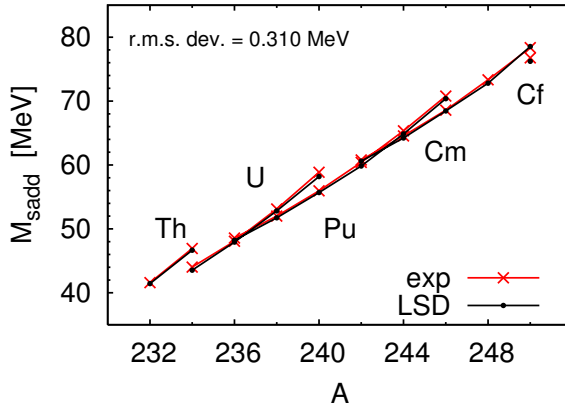


Fig. 3. Comparison of the experimental and LSD saddle-point masses of actinide nuclei as function of the mass number.

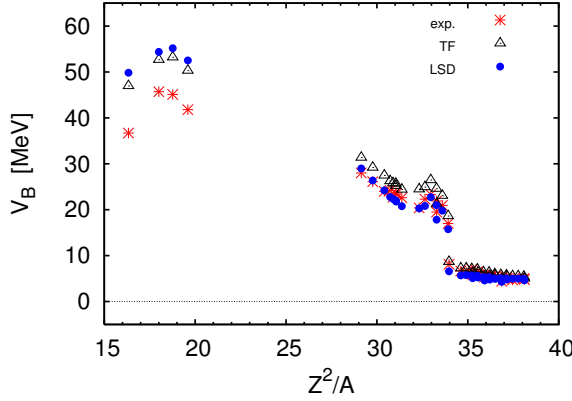


Fig. 4. Comparison of the LSD and TF fission barrier heights with the experimental data as function of the fissility parameter Z^2/A .

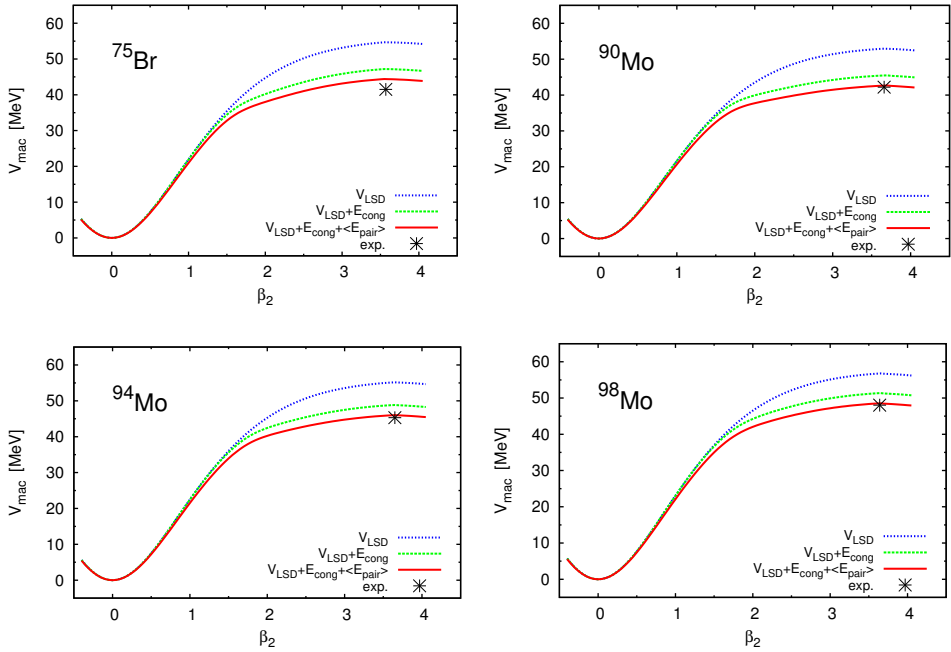


Fig. 5. LSD fission barrier corrected by the deformation dependent congruence and average pairing energies as function of the relative elongation of nucleus. The experimental barrier heights are marked by crosses.

Encouraged by this fact, we have evaluated the barrier heights as

$$V_B = M_{\text{sadd}}^{\text{mac}} - M_{\text{g.s.}}^{\text{exp}} \quad (2)$$

for nuclei with the mass number $75 \leq A \leq 252$ using the shape parametrisation independent model developed by Strutinsky and co-workers [15, 16]. The LSD estimates [13] are compared in Fig. 4 with the barrier heights obtained by Myers and Świątecki with the Thomas–Fermi (TF) model and the experimental data [14]. It is seen in Fig. 4 that except of four-light isotopes (^{75}Br , $^{90,94,98}\text{Mo}$) the LSD estimates are closer to the data than the TF results. It was shown in Ref. [17] that this discrepancy in the light isotopes can be removed when one took into account the deformation dependence of the congruence and average pairing energies. The corrected by the above two effects fission barriers V_{mac} for ^{75}Br and $^{90,94,98}\text{Mo}$ isotopes are shown in Fig. 5.

4. Discussion and conclusions

The obtained good agreement of the fission barrier heights proves that the LSD model with the parameters fitted to the ground-state binding energies only (not to the barrier heights!) can serve as a reliable and simple macroscopic model which is able to predict different properties of nuclei.

It is worth to mention here that in the last decade some other liquid drop models were successfully used to reproduce the ground state masses of nuclei but none of them was able to predict correctly the fission barrier heights in such a broad mass region as it is the case in the LSD model [18].

The last problem which we would like to mention are different predictions of the Thomas–Fermi [12] and the LSD [13] models when one would like to evaluate the masses of nuclei far from stability. In Fig. 6 the difference of the TF and LSD mass estimates for β -stable nuclei as well as for isotopes along the proton and neutron drip lines are plotted as functions of of the

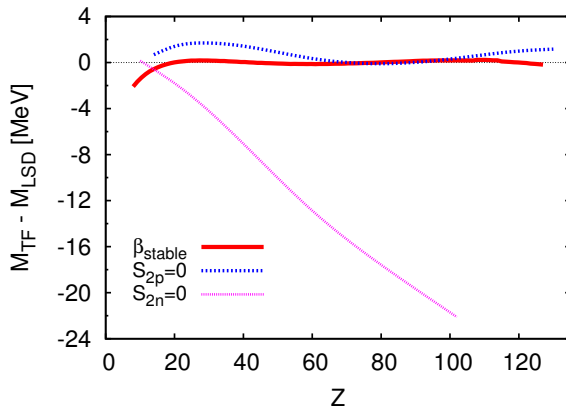


Fig. 6. Difference of the TF and LSD mass estimates along the β -stability, proton and neutron drip lines.

charge number. It is seen that the both estimates are close to each other for the β -stable and neutron deficient nuclei, while the difference between the LSD and TF results reaches even 24 MeV for heavy nuclei along the neutron drip-line. The proper prediction of masses of neutron rich isotopes is very important for astrophysics, especially for the description of the rapid neutron capture processes. A continuous progress in the radioactive beam technique gives a hope for verification which model estimates are closer to reality.

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