

YANG–MILLS THERMODYNAMICS: AN EFFECTIVE THEORY APPROACH*

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We derive the Polyakov-loop thermodynamic potential in the perturbative approach to pure SU(3) Yang–Mills theory. The potential expressed in terms of the Polyakov loop in the fundamental representation corresponds to that of the strong-coupling expansion, of which the relevant coefficients of the gluon energy distribution are specified by characters of the SU(3) group. At high temperatures, the derived gluon potential exhibits the correct asymptotic behavior, whereas at low temperatures, it disfavors gluons as appropriate dynamical degrees of freedom. In order to quantify the Yang–Mills thermodynamics in a confined phase, we propose a hybrid approach which matches the effective gluon potential to the one of glueballs constrained by the QCD trace anomaly in terms of a dilaton. We also discuss the interplay between the chromomagnetic and chromoelectric gluon dynamics.

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1. Introduction

The structure of the QCD phase diagram and thermodynamics at finite baryon density is of crucial importance in heavy-ion phenomenology. Due to the sign problem in lattice calculations, a major approach to a finite density QCD is based on effective Lagrangians possessing the same global symmetries as the underlying QCD. The SU(N_c) Yang–Mills theory has a global Z(N_c) symmetry which is dynamically broken at high temperature. This is characterized by the Polyakov loop that plays a role of an order parameter of the Z(N_c) symmetry [1]. Effective models for the Polyakov loop

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were suggested as a macroscopic approach to the pure gauge theory [2, 3]. Their thermodynamics is qualitatively in agreement with that obtained in lattice gauge theories [4]. Alternative approaches are based on the quasi-particle picture of thermal gluons [5]. When gluons propagating in a constant gluon background are considered, the quasi-particle models naturally merge with the Polyakov loops, that appear in the partition function, as characters of the color gauge group [6–10].

In this contribution, we show that the SU(3) gluon thermodynamic potential derived from the Yang–Mills Lagrangian is expressed in terms of the Polyakov loops in the fundamental representation. We summarize its properties and argue that at high temperatures, it exhibits the correct asymptotic behavior, whereas at low temperatures, it disfavors gluons [11]. We, therefore, suggest a hybrid approach to Yang–Mills thermodynamics, which combines the effective gluon potential with glueballs implemented as dilaton fields.

We propose also an effective theory of SU(3) gluonic matter [12]. The theory is constructed based on the center and scale symmetries and their dynamical breaking, so that the interplay between color–electric and color–magnetic gluons is included coherently. We suggest that the magnetic gluon condensate changes its thermal behavior qualitatively above the critical temperature, as a consequence of matching to the dimensionally-reduced magnetic theories.

2. Thermodynamics of hot gluons

We start from the partition function of the pure Yang–Mills theory

$$Z = \int \mathcal{D}A_\mu \mathcal{D}C \mathcal{D}\bar{C} \exp \left[i \int d^4x \mathcal{L}_{\text{YM}} \right], \quad (1)$$

with gluon A_μ and ghost C fields. Following [3, 13], we employ the background field method to evaluate the functional integral. The gluon field is decomposed into the background \bar{A}_μ and the quantum \tilde{A}_μ fields

$$A_\mu = \bar{A}_\mu + g\tilde{A}_\mu. \quad (2)$$

The partition function is arranged as

$$\ln Z = V \int \frac{d^3p}{(2\pi)^3} \ln \det \left(1 - \hat{L}_A e^{-|\vec{p}|/T} \right) + \ln M(\phi_1, \phi_2), \quad (3)$$

where \hat{L}_A is the Polyakov loop matrix in the adjoint representation and the two angular variables, ϕ_1 and ϕ_2 , represent the rank of the SU(3) group.

The $M(\phi_1, \phi_2)$ is the Haar measure

$$M(\phi_1, \phi_2) = \frac{8}{9\pi^2} \sin^2 \left(\frac{\phi_1 - \phi_2}{2} \right) \sin^2 \left(\frac{2\phi_1 + \phi_2}{2} \right) \sin^2 \left(\frac{\phi_1 + 2\phi_2}{2} \right) \quad (4)$$

for a fixed volume V , which is normalized such that

$$\int_0^{2\pi} \int_0^{2\pi} d\phi_1 d\phi_2 M(\phi_1, \phi_2) = 1. \quad (5)$$

The first term in Eq. (3) yields the gluon thermodynamic potential

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \text{tr} \ln \left(1 - \hat{L}_A e^{-E_g/T} \right), \quad (6)$$

where $E_g = \sqrt{|\vec{p}|^2 + M_g^2}$ is the quasi-gluon energy and the effective gluon mass M_g is introduced from phenomenological reasons.

We define the gauge invariant quantities from the Polyakov loop matrix in the fundamental representation \hat{L}_F , as

$$\Phi = \frac{1}{3} \text{tr} \hat{L}_F, \quad \bar{\Phi} = \frac{1}{3} \text{tr} \hat{L}_F^\dagger. \quad (7)$$

Performing the trace over colors and expressing it in terms of Φ and its conjugate $\bar{\Phi}$, one arrives at

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 + \sum_{n=1}^8 C_n e^{-nE_g/T} \right), \quad (8)$$

with the coefficients C_n

$$\begin{aligned} C_8 &= 1, \\ C_1 &= C_7 = 1 - 9\bar{\Phi}\Phi, \\ C_2 &= C_6 = 1 - 27\bar{\Phi}\Phi + 27(\bar{\Phi}^3 + \Phi^3), \\ C_3 &= C_5 = -2 + 27\bar{\Phi}\Phi - 81(\bar{\Phi}\Phi)^2, \\ C_4 &= 2 \left[-1 + 9\bar{\Phi}\Phi - 27(\bar{\Phi}^3 + \Phi^3) + 81(\bar{\Phi}\Phi)^2 \right]. \end{aligned} \quad (9)$$

Thus, the gluon energy distribution is identified solely by the characters of the fundamental and the conjugate representations of the $SU(3)$ gauge group.

We introduce an effective thermodynamic potential in the large volume limit from Eq. (3) as follows

$$\Omega = \Omega_g + \Omega_\Phi + c_0, \quad (10)$$

where Ω_g is given by Eq. (8) and the Haar measure part is found as

$$\Omega_\Phi = -a_0 T \ln \left[1 - 6\bar{\Phi}\Phi + 4(\Phi^3 + \bar{\Phi}^3) - 3(\bar{\Phi}\Phi)^2 \right]. \quad (11)$$

The potential (10) has, in general, three free parameters; a_0 , c_0 and the gluon mass M_g . They can be chosen *e.g.* to reproduce the equation of state obtained in lattice gauge theories. It is straightforward to see that the result of a non-interacting boson gas is recovered at asymptotically high temperature. Indeed, taking $\Phi, \bar{\Phi} \rightarrow 1$, one finds

$$\Omega_g(\Phi = \bar{\Phi} = 1) = 16T \int \frac{d^3p}{(2\pi)^3} \ln(1 - e^{-E_g/T}). \quad (12)$$

On the other hand, for a sufficiently large M_g/T , as expected near the phase transition, one can approximate the potential as

$$\Omega_g \simeq \frac{T^2 M_g^2}{\pi^2} \sum_{n=1}^8 \frac{C_n}{n} K_2(n\beta M_g) \quad (13)$$

with the Bessel function $K_2(x)$. In the quasi-particle approach, the above result can also be considered as a strong-coupling expansion, regarding the relation $M_g(T) = g(T)T$ with an effective gauge coupling $g(T)$.

The effective action to the next-to-leading order of the strong coupling expansion is obtained in terms of group characters as [10]

$$S_{\text{eff}}^{(\text{SC})} = \lambda_{10} S_{10} + \lambda_{20} S_{20} + \lambda_{11} S_{11} + \lambda_{21} S_{21} \quad (14)$$

with products of characters S_{pq} , specified by two integers p and q counting the numbers of fundamental and conjugate representations, and couplings λ_{pq} being real functions of temperature. Making the character expansion of Eq. (13), one readily finds the correspondence between S_{pq} and C_n as

$$C_{1,7} = S_{10}, \quad C_{2,6} = S_{21}, \quad C_{3,5} = S_{11}, \quad C_4 = S_{20}. \quad (15)$$

On the other hand, taking the leading contribution, $\exp[-M_g/T]$ in the expansion, the “minimal model” is deduced with

$$\Omega_g \simeq -\mathcal{F}(T, M_g) \bar{\Phi}\Phi, \quad (16)$$

where the negative sign is required for a first-order transition [10]. The function \mathcal{F} can be extracted from Eq. (10) and the resulting potential is of the form widely used in the PNJL model [14–17]. See also [18].

3. A hybrid approach

Although the potential (10) describes quite well thermodynamics in deconfined phase, it totally fails in the confined phase. In the confined phase, $\langle \Phi \rangle = 0$ is dynamically favored by the ground state, thus the $C_1 = 1$ term remains as the main contribution. Consequently,

$$\Omega_g (\Phi = \bar{\Phi} = 0) \simeq 2T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 + e^{-E_g/T} \right). \quad (17)$$

One clearly sees that Ω_g does not possess the correct sign in front of $\exp[-E_g/T]$, expected from the Bose-Einstein statistics. This implies that the entropy and the energy densities are *negative*. On the other hand, if one uses the approximated form (16), the pressure vanishes at any temperature below T_c . Obviously, this is an unphysical behavior since there exist color-singlet states, *i.e.* glueballs, contributing to thermodynamics and they must generate a non-vanishing pressure.

This aspect is in a striking contrast to the quark sector. The thermodynamic potential for quarks and anti-quarks with N_f flavors is obtained as [14, 19]

$$\begin{aligned} \Omega_{q+\bar{q}} = & -2N_f T \int \frac{d^3p}{(2\pi)^3} \ln \left[1 + N_c \left(\Phi + \bar{\Phi} e^{-E^+/T} \right) e^{-E^+/T} + e^{-3E^+/T} \right] \\ & + (\mu \rightarrow -\mu), \end{aligned} \quad (18)$$

with $E^\pm = E_q \mp \mu$ being the energy of a quark or anti-quark. In the limit, $\Phi, \bar{\Phi} \rightarrow 0$, the one- and two-quark states are suppressed and only the three-quark (“baryonic”) states, $\sim \exp(-3E^{(\pm)}/T)$, survives. This, on a qualitative level, is similar to confinement properties in QCD thermodynamics [16]. One should, however, keep in mind that such quark models yield only colored quarks being *statistically* suppressed at low temperatures. On the other hand, unphysical thermodynamics below T_c described by the gluon sector (10) apparently indicates that gluons are *physically* forbidden. Interestingly, this property is not spoiled by the presence of quarks. Indeed, in this case and at $T < T_c$, the thermodynamic potential is approximated as

$$\Omega_g + \Omega_{q+\bar{q}} \simeq \frac{T^2}{\pi^2} \left[M_g^2 K_2 \left(\frac{M_g}{T} \right) - \frac{2N_f}{3} M_q^2 K_2 \left(\frac{3M_q}{T} \right) \right]. \quad (19)$$

Assuming that glueballs and nucleons are made from two weakly-interacting massive gluons and three massive quarks respectively and putting empirical numbers, $M_{\text{glueball}} = 1.7$ GeV and $M_{\text{nucleon}} = 0.94$ GeV, one finds that $M_g = 0.85$ GeV and $M_q = 0.31$ GeV. Substituting these mass values in Eq. (19), one still gets the negative entropy density at any temperature and for either $N_f = 2$ or $N_f = 3$, as found in the pure Yang-Mills theory.

The unphysical equation of state (EoS) in confined phase can be avoided, when gluon degrees of freedom are replaced with glueballs. A glueball is introduced as a dilaton field χ representing the gluon composite $\langle A_{\mu\nu} A^{\mu\nu} \rangle$, which is responsible for the QCD trace anomaly [20]. The Lagrangian is of the standard form

$$\mathcal{L}_\chi = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V_\chi, \quad V_\chi = \frac{B}{4} \left(\frac{\chi}{\chi_0} \right)^4 \left[\ln \left(\frac{\chi}{\chi_0} \right)^4 - 1 \right], \quad (20)$$

with the bag constant B and a dimensionful quantity χ_0 to be fixed from the vacuum energy density and the glueball mass. One readily finds the thermodynamic potential of the glueballs as

$$\begin{aligned} \Omega &= \Omega_\chi + V_\chi + \frac{B}{4}, & \Omega_\chi &= T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 - e^{-E_\chi/T} \right), \\ E_\chi &= \sqrt{|\vec{p}|^2 + M_\chi^2}, & M_\chi^2 &= \frac{\partial^2 V_\chi}{\partial \chi^2}, \end{aligned} \quad (21)$$

where a constant $B/4$ is added so that $\Omega = 0$ at zero temperature.

We propose the following hybrid approach which accounts for gluons and glueballs degrees of freedom by combining Eqs. (10) and (21)

$$\Omega = \Theta(T_c - T) \Omega(\chi) + \Theta(T - T_c) \Omega(\Phi). \quad (22)$$

For a given M_g , the model parameters, a_0 and c_0 , are fixed by requiring that $\Omega(\Phi)$ yields a first-order phase transition at $T_c = 270$ MeV and that $\Omega(\chi)$ and $\Omega(\Phi)$ match at T_c . The resulting EoS follows general trends seen in lattice data [11]. The model can be improved further by introducing a thermal gluon mass, $M_g(T) \sim g(T)T$, as carried out *e.g.* in [8].

4. Magnetic confinement

Asymptotic properties of non-Abelian gauge theories at finite temperature are successfully captured in the quasi-particle description, which can be consistently calculated in the leading-order perturbation theory [21]. However, a naive perturbative treatment in the weak coupling g is spoiled since the magnetic screening mass is dynamically generated as an ultra-soft scale $g^2 T$ [13, 22]. The magnetic sector remains non-perturbative in the high temperature phase, and consequently, the spatial string tension is non-vanishing for all temperatures [23, 24], indicating certain confining properties.

This residual interaction brings apparent deviations in equations of state (EoS) from their Stefan–Boltzmann limit at high temperature. In particular, the interaction measure $I(T)$ is the best observable to examine dynamical

breaking of scale invariance of the Yang–Mills (YM) Lagrangian. In lattice simulations of pure SU(3) YM theory the $I(T)/T^2T_c^2$, with the deconfinement critical temperature T_c , is nearly constant in the range $T_c < T < 5T_c$. This observation strongly suggests non-trivial dynamical effects [4, 25–30]. Beyond this temperature range, the lattice data follow the results from the Hard Thermal Loop (HTL) resummed perturbation theory. Thus, a non-perturbative part in the lattice data is extracted by subtracting the HTL contribution [25]. The resultant non-perturbative part in $I(T)/T^2T_c^2$ is *monotonically* decreasing, whereas the HTL result is monotonically increasing with T . A plateau that arises in intermediate temperatures in $I(T)/T^2T_c^2$ can be therefore understood as resulting from the summation of those two contributions.

In [12], we formulate an effective theory of SU(3) gluonic matter, which accounts for two dynamically different contributions, the chromomagnetic and chromoelectric gluons. In general, the dilaton couples also to the Polyakov loop which is the order parameter of confinement-deconfinement phase transition and belongs to the color-electric sector. Thus, the dilaton captures the thermodynamic properties around the critical point T_c , which are related with both, the color-electric and color-magnetic gluons.

Thermal behavior of the magnetic gluon condensate at high temperature is found, using the three-dimensional YM theories [31–34], as [35]

$$\langle H \rangle = c_H (g^2(T)T)^4 \quad (23)$$

with

$$c_H = \frac{6}{\pi} c_\sigma^2 c_m^2. \quad (24)$$

The constants c_σ and c_m appear in σ_s and in the magnetic gluon mass as

$$\sqrt{\sigma_s(T)} = c_\sigma g^2(T)T, \quad m_g(T) = c_m g^2(T)T. \quad (25)$$

For SU(3) YM theory $c_\sigma = 0.566$ [4] and $c_m = 0.491$ [36].

The potential that mixes the dilaton field and the Polyakov loop should be manifestly invariant under $Z(N_c)$ and scale transformation. For $N_c = 3$, its most general form is as the following [37]

$$V_{\text{mix}} = \chi^4 \left(G_1 \bar{\Phi} \Phi + G_2 (\bar{\Phi}^3 + \Phi^3) + G_3 (\bar{\Phi} \Phi)^2 + \dots \right), \quad (26)$$

with unknown coefficients G_i . In the following, we take only the first term.

At high temperature, due to the dimensional reduction, the theory in four dimensions should match the three-dimensional YM theory. We postulate the following matching condition

$$\frac{\langle \chi \rangle}{\chi_0} = \left(\frac{\langle H \rangle}{H_0} \right)^{1/4}. \quad (27)$$

This transmutation from $\langle\chi\rangle \sim \text{const.}$ to $\langle H\rangle^{1/4} \sim g^2 T$ leads to an additional contribution to the interaction measure

$$\delta I = -B \frac{\langle H\rangle}{H_0} + \left(2b_0 + \frac{b_1}{b_0} \frac{1}{\ln(T/\Lambda_\sigma)} \right) \frac{\langle H\rangle}{g^4(T)H_0}. \quad (28)$$

The interaction measure normalized by $T^2 T_c^2$ is monotonically decreasing even at high temperature when no matching to the 3-dim YM is made. The magnetic contribution generates a T^2 dependence. The sum of those two contributions forms a plateau-like behavior in $I/T^2 T_c^2$ at moderate temperature, $T/T_c \sim 2\text{--}4$. This property appears due to the residual chromomagnetic interaction encoded in the dilaton, $\chi^4 \sim H$. The resulting behavior of $I/T^2 T_c^2$ with temperature qualitatively agrees with the latest high-precision lattice data [25]. We note that a smooth switching from the dilaton to the magnetic condensate must happen dynamically, so that thermodynamic quantities, such as the specific heat, do not experience any irregular behavior above T_c .

5. Summary

We have derived the thermodynamic potential in the SU(3) Yang–Mills theory in the presence of a uniform gluon background field. The potential accounts for quantum statistics and reproduces an ideal gas limit at high temperature. Within the character expansion, the one-to-one correspondence to the effective action in the strong-coupling expansion is obtained. Different effective potentials used so far appear as limiting cases of our result.

The phenomenological consequence is that gluons are disfavored as appropriate degrees of freedom in confined phase. This property is in remarkable contrast to the description of “confinement” within a class of chiral models with Polyakov loops [14, 17], where colored quarks are activated at any temperature.

We have also presented an effective theory implementing the major global symmetries, the center and scale symmetries, and their dynamical breaking. This naturally allows a mixing between the Polyakov loop and the dilaton field. Consequently, the magnetic confinement is effectively embedded and results in deviations of the EoS from their Stefan–Boltzmann limit at high temperature. Through a matching to the 3-dimensional YM theory, the gluon condensate increases with temperature in deconfined phase. Contrary, in the conventional treatment of the dilaton condensate, there is a weak thermal behavior of the composite gluon in a wide range of temperature. This suggests, that at some temperature above T_c , the gluon condensate exhibits a distinct behavior on T . In the present theory, this temperature is roughly

estimated as $\sim 2.4 T_c$, compatible with $\sim 2 T_c$ extracted from the spatial string tension [35]. Applying this idea to the interaction measure, the role of the magnetic gluon turns out to be alternative to the HTL contribution.

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