THERMODYNAMICS OF STRONG INTERACTION MATTER FROM LATTICE QCD AND THE HADRON RESONANCE GAS MODEL*

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We compare recent lattice QCD calculations of higher order cumulants of net-strangeness fluctuations with hadron resonance gas (HRG) model calculations. Up to the QCD transition temperature $T_{\rm c} = (154 \pm 9)$ MeV we find good agreement between QCD and HRG model calculations of second and fourth order cumulants, even when subtle aspects of net-baryon number, strangeness and electric charge fluctuations are probed. In particular, the fourth order cumulants indicate that also in the strangeness sector of QCD the failure of HRG model calculations sets in quite abruptly in the vicinity of the QCD transition temperature and is apparent in most observables for $T \gtrsim 160$ MeV.

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1. Introduction

A critical point, which is the endpoint of a line of first order phase transitions, has been postulated to exist in the QCD phase diagram at non-zero values of the baryon chemical potential μ_B [1]. Hints for the existence of such a critical point came from lattice QCD calculations that used a reweighting of Monte Carlo data generated at vanishing chemical potential to non-zero chemical potential [2]. The validity of these results, however, have been challenged [3]. The existence of a critical point for moderate values of $\mu_B/T \sim \mathcal{O}(1)$ has also been questioned on the basis of results obtained in lattice QCD calculations with imaginary values of the chemical potential [4]. Further information on the existence of a critical point and estimates for its

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location can be obtained from higher order cumulants calculated at vanishing baryon chemical potential. Cumulants are the expansion parameters of Taylor series for basic thermodynamic quantities, *e.g.* the pressure, baryon number density or susceptibilities [5, 6]. Estimates for the radius of convergence of these series and the location of the critical endpoint at non-zero baryon number density can be obtained from ratios of higher order cumulants of net-baryon number fluctuations, although it is not guaranteed that these estimators converge rapidly [7]. Nonetheless, some estimates for the location of the critical point have been obtained in this way from calculations with unimproved staggered fermion actions on coarse lattices [8].

Higher order cumulants of conserved charge fluctuations, *i.e.* fluctuations of net-baryon number, electric charge and strangeness, play a central role in the search for the critical endpoint and the exploration of the QCD phase diagram at vanishing and non-vanishing baryon chemical potential in general [9]. At non-vanishing baryon chemical potential quadratic fluctuations of net-baryon number will diverge in the vicinity of a critical point which makes them well suited also for the experimental search for the existence of such a prominent landmark in the QCD phase diagram [10].

Irrespective of the existence of a critical endpoint, ratios of higher order cumulants of charge fluctuations provide detailed information on properties of the different phases of strong interaction matter. They signal the change in relevant degrees of freedom in different phases [11] and they also are sensitive probes for the occurrence of a chiral phase transition at vanishing as

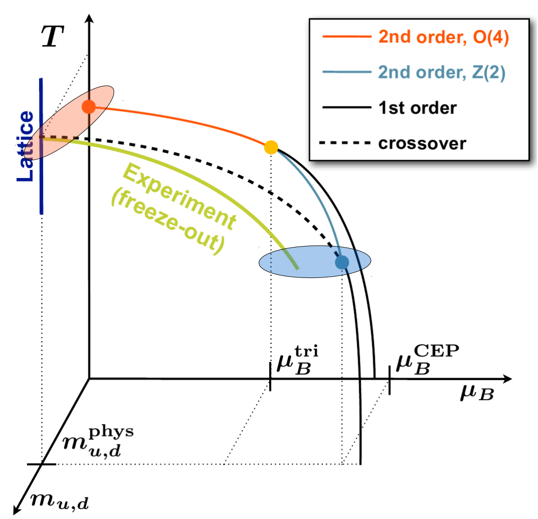

Fig. 1. Conjectured phase diagram of QCD in the space of temperature, baryon chemical potential and light-quark mass.

well as non-vanishing baryon chemical potential. While in the latter case third order cumulants will diverge in the chiral limit, at vanishing chemical potential only sixth order cumulants will for the first time show divergent behaviour [9]. The critical regions, eventually probed through the calculation of higher order cumulants, are illustrated in Fig. 1.

Below, the QCD transition temperature cumulants of net-charge fluctuations are known to agree quite well with hadron resonance gas (HRG) model calculations [11, 12]. This has been analyzed in detail for quadratic fluctuations of net charges [13, 14]. If this gets confirmed also for higher order cumulants, it puts severe constraints on estimates for the location of a critical point based on Taylor series expansions. In this conference contribution, we will focus on cumulants involving strangeness degrees of freedom [15]. We want to discuss net-strangeness fluctuations and correlations with net-baryon number fluctuations and will quantify to what extent higher order cumulants, involving strangeness fluctuations, agree with hadron resonance gas model calculations. We will show that strangeness fluctuations start deviating from HRG model calculations at or close to the QCD crossover temperature. Using also information on the modification of thermal strange meson correlation functions, we argue that this suggests the disappearance of strange hadronic bound states in the high temperature phase of QCD.

2. Confronting QCD results on higher order charge fluctuations with hadron resonance gas model calculations

Fluctuations of conserved charges and the correlation among moments of net-charge fluctuations can be derived from the logarithm of the QCD partition function, which defines the pressure, p,

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_B, \mu_S, \mu_Q) \,. \tag{1}$$

Taking derivatives with respect to chemical potentials for baryon number (μ_B), strangeness (μ_S) and electric charge (μ_Q) evaluated at $\vec{\mu} = (\mu_B, \mu_Q, \mu_S) = 0$, we obtain higher order cumulants of charge fluctuations (χ_n^X) and correlations (χ_{nm}^{XY}) among moments of these charge fluctuations

$$\chi_n^X = \left.\frac{\partial^n p/T^4}{\partial \hat{\mu}_X^n}\right|_{\vec{\mu}=0} , \qquad \chi_{nm}^{XY} = \left.\frac{\partial^{n+m} p/T^4}{\partial^n \hat{\mu}_X \partial^m \hat{\mu}_Y}\right|_{\vec{\mu}=0} . \tag{2}$$

Here, we use the notation $\hat{\mu}_X \equiv \mu_X/T$ and $X,\ Y = B,\ Q,\ S$.

We will compare results for fluctuations and correlations defined by Eq. (2) with hadron resonance gas model [16] calculations. The partition function of the HRG model can be split into mesonic and baryonic contributions

$$\frac{p^{\mathrm{HRG}}}{T^4} = \frac{1}{VT^3} \sum_{i \,\in\, \text{mesons}} \ln \mathcal{Z}^{M}_{M_i}(T,V,\mu_Q,\mu_S) + \frac{1}{VT^3} \sum_{i \,\in\, \text{baryons}} \ln \mathcal{Z}^{B}_{M_i}(T,V,\mu_B,\mu_Q,\mu_S)\,, \tag{3}$$

where the partition function for mesonic (M) or baryonic (B) particle species i with mass M_i is given by

$$\ln \mathcal{Z}^{M/B}_{M_i} = \frac{VT^3}{2\pi^2} d_i \left(\frac{M_i}{T}\right)^2 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{1}{k^2} K_2(kM_i/T) \times \exp\left(k(B_i\mu_B + Q_i\mu_Q + S_i\mu_S)/T\right)\,. \tag{4}$$

Here, upper signs correspond to mesons and lower signs to baryons. In the temperature range of interest to us the Boltzmann approximation, which amounts to restricting the sums in Eq. (4) to the $k = 1$ term only, is a good approximation for all particle species, except for pions. We will use this approximation in the following discussion.

In Fig. 2 we show results for some 4$^{\text{th}}$ order correlations of moments of net-baryon number and net-strangeness fluctuations, χ^{BS}_{nm}, with $n + m = 4$, $n > 0$. In a gas of uncorrelated hadrons, these correlations receive contributions from baryons in the three different strangeness sectors, $|S| = 1,\ 2$

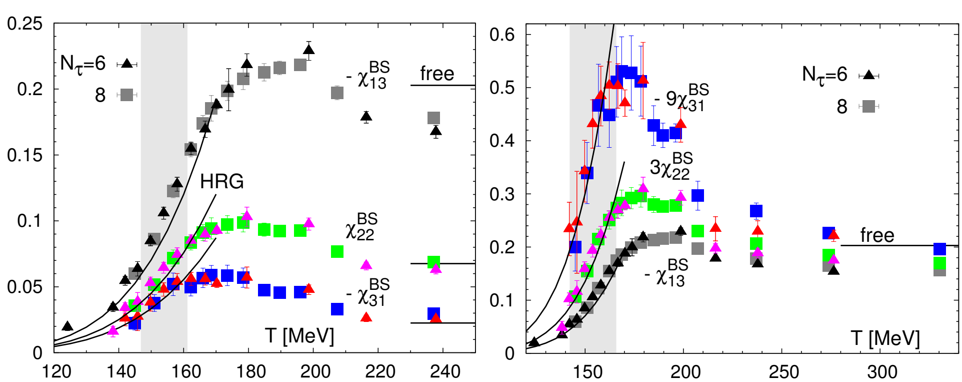

Fig. 2. Fourth order correlations of moments of net-baryon number and net-strangeness fluctuations. Curves on the low temperature side give HRG model results and the vertical lines on the high temperature side give the ideal gas results. The grey band corresponds to the crossover transition region $T_{\rm c} = (154 \pm 9)$ MeV [18] determined by the HotQCD Collaboration from the chiral susceptibility.

and 3. Within the HRG model approximation $\chi_{nm}^{\rm BS}$ can be represented by a weighted sum of partial baryonic pressure contributions, $P_{|S|=m,B}^{\rm HRG}$, arising from the different strangeness sectors [15]

$$\left(\chi_{nm}^{\rm BS}\right)_{\rm HRG} = P_{|S|=1,B}^{\rm HRG} + 2^m P_{|S|=2,B}^{\rm HRG} + 3^m P_{|S|=3,B}^{\rm HRG}\,, \qquad n>0\,. \tag{5}$$

With increasing power m of the strangeness moments, the correlations $\chi_{nm}^{\rm BS}$ thus give larger weight to multiple strange baryons, *i.e.* an ordering $\chi_{31}^{\rm BS} < \chi_{2}^{\rm BS} < \chi_{13}^{\rm BS}$ is naturally expected. As can be seen in the left-hand part of Fig. 2, this is indeed the case (even in the high temperature phase). Moreover, all three fourth order correlations $\chi_{nm}^{\rm BS}$ agree quite well with HRG model calculations up to temperatures close to the QCD transition temperature.

In the infinite temperature, ideal gas limit BS-correlations are expected to approach

$$\left(\chi_{nm}^{\rm BS}\right)_{\rm free} = (-1)^m \frac{1}{3^n} \begin{cases} 1\,, & n+m=2 \\ \frac{6}{\pi^2}\,, & n+m=4 \end{cases}.$$

This indeed seems to be a good approximation for temperatures $T \gtrsim 2T_{\rm c}$ as can be seen from the right-hand part of Fig. 2 where we show the fourth order BS-correlations rescaled such that their high temperature ideal gas limits coincide. At high temperature, the baryon–strangeness correlations thus suggest that the degrees of freedom carrying strangeness are weakly interacting quasi-particles with quark quantum numbers, $B = 1/3$, $S = -1$. However, in the temperature range $T \le T \lesssim 2T_{\rm c}$ such a picture of weakly interacting quasi-particle clearly does not apply (see also discussion in [15]).

The agreement with HRG model calculations at low temperature can be probed in more detail by comparing second and fourth order BS-correlations that have identical behaviour at low temperature but widely different ideal gas limits. Similar to the ratio of fourth and second order baryon number susceptibilities, $\chi_4^{\rm B}/\chi_2^{\rm B}$, which had been introduced to probe the baryon number carrying degrees of freedom at low and high temperature [11], one may consider $\chi_{31}^{\rm BS}/\chi_{11}^{\rm BS}$. This ratio is sensitive to the strangeness carrying, baryonic degrees of freedom [17]. Instead of this ratio, the corresponding difference $\chi_{31}^{\rm BS} - \chi_{11}^{\rm BS}$ has been presented in Ref. [15]. We show both variants of this observable in Fig. 3. We note that the differences $\chi_2^{\rm B} - \chi_4^{\rm B}$ and $v_1 \equiv \chi_{31}^{\rm BS} - \chi_{11}^{\rm BS}$ show similar behaviour and drop rapidly at or close to the chiral transition temperature. This has been confirmed in calculations using a different fermion discretization scheme [19]. The temperature dependence of these differences of second and fourth order cumulants is reminiscent of that of an order parameter. However, it should be clear that these observables are not 'order parameters' in the literal sense. Even in the chiral limit,

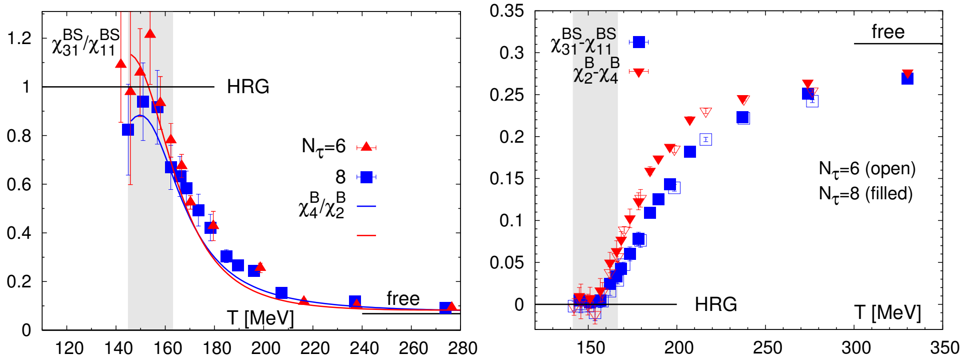

Fig. 3. Fourth order correlations of moments of net-baryon number and net-strangeness fluctuations. Curves on the low temperature side give HRG model results and the vertical lines on the high temperature side give the ideal gas results. The grey band corresponds to the crossover transition region $T_c = (154 \pm 9)$ MeV [18] determined by the HotQCD Collaboration from the chiral susceptibility. The curves in the left-hand figure show fits to the ratio of net-baryon number cumulants, χ_4^B/χ_2^B, which receives i contributions also from the non-strange baryon sector.

they would not vanish exactly below the QCD phase transition temperature, *i.e.* in the hadronic phase. In fact, in the vicinity of the chiral phase transition temperature the second and fourth order cumulants receive different non-analytic contributions that are proportional to universal $O(4)$-scaling functions [20]. This enforces deviations from simple HRG behaviour in the vicinity of the QCD phase transition.

The BS-correlations discussed above are all sensitive to the strange baryon sector of QCD. In order to become sensitive also to the strange meson sector, we construct observables that give the mesonic contribution to the pressure in an uncorrelated hadron resonance gas. We can do so by including also cumulants of strangeness fluctuation [15], χ_n^S, from which we subtract the baryon contribution by using suitable combinations of BS-correlations. This gives us quite some freedom. We introduce

$$M(c_1, c_2) = \chi_2^S - \chi_{22}^{BS} + c_1 v_1 + c_2 v_2 \,, \tag{6}$$

where c_1 and c_2 are free parameters, v_1 has been introduced above and $v_2 = \frac{1}{3}(\chi_2^S - \chi_4^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$. Similar to v_1 also v_2 is a combination of susceptibilities (independent from v_1) that vanishes in a gas of uncorrelated hadrons. We show in Fig. 4 this observable for three different choices of c_1 and c_2. As can be seen at low temperatures, $T \lesssim 160$ MeV, and irrespective of the choice of c_1, c_2 the mesonic observables $M(c_1, c_2)$ agree well with the strange meson contribution to the pressure of a HRG.

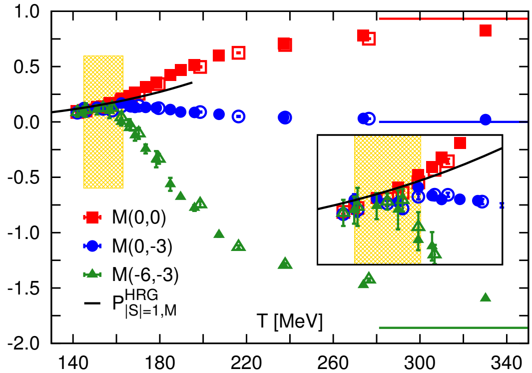

Fig. 4. Different combinations of net-strangeness fluctuations and correlations with net-baryon number fluctuations that yield the same strange meson contribution to the pressure in a gas of uncorrelated hadrons. The grey/yellow band indicates the temperature range of the QCD crossover transition, $T_c = (154 \pm 9)$ MeV. The black line gives the pressure of strange mesons in a hadron resonance gas model. The horizontal lines show the ideal gas well for the observables $M(c_1, c_2)$ introduced in Eq. (6).

We thus conclude that at or close to the QCD transition temperature the HRG model breaks down as a description of mesonic and baryonic degrees of freedom in strong interaction matter.

3. Strange screening masses

The analysis of various cumulants involving moments of net-strangeness fluctuations presented in the previous section shows that the agreement with HRG model calculations breaks down at temperatures close to the chiral transition temperature. Of course, deviations from the simple HRG results can also have different origin and may, for instance, result from thermal modifications of the hadron spectrum itself. First calculations of thermal meson spectral functions performed in a quark mass regime corresponding to $\bar{s}s$-meson states indeed suggest that such states may still exist in the QGP [21]. However, so far these calculations have only been performed in quenched QCD; the influence of screening due to dynamical quark degrees of freedom may well lead to an earlier melting of strange meson states. In order to gain further insight into this question we have analyzed spatial correlation functions of strange mesons [22]. These correlation functions have a representation in terms of finite temperature spectral functions, $\rho(\omega, p_z, T)$, although the contribution of resonance peaks is not directly evident in these correlators

$$C(z,T) = \int_0^\infty \frac{2d\omega}{\omega} \int_{-\infty}^\infty dp_z e^{ip_z z} \rho(\omega, p_z, T)\,. \qquad (7)$$

The large distance behaviour of these correlation functions yield temperature dependent screening masses, $M(T)$. In the infinite temperature limit $M(T)/T$ approaches twice the lowest Matsubara frequency reflecting the propagation of two uncorrelated quarks. In Fig. 5 we show the screening mass in pseudo-scalar and vector channels of strange quark–antiquark states. It is obvious that the screening masses show a strong temperature dependence already at temperatures close to the QCD transition temperature. In fact, already in the crossover region to the high temperature phase deviations from the zero temperature $\bar{s}s$-meson mass, m_0, are about 5%. In the case of charmonium states, which are known to melt at about 1.5 T_c [23] (or earlier), the finite temperature screening masses deviate from the zero temperature J/ψ or η_c masses only by about 2% [22]. This suggests that all strange meson and baryon states dissolve already at the QCD transition.

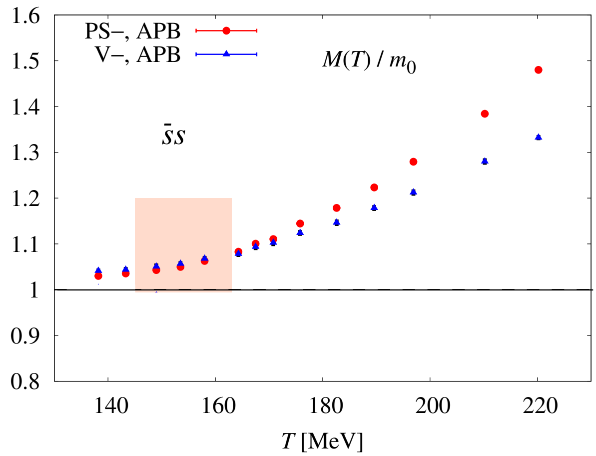

Fig. 5. Screening mass ($M(T)$) in units of the corresponding zero temperature meson masses (m_0) extracted from spatial strange meson correlation functions in the pseudo-scalar (PS) and vector (V) channels. Calculations are based on an analysis of gauge configuration generated by the HotQCD Collaboration in (2+1)-flavour QCD using the HISQ action [18]. The grey/yellow band indicates the temperature range of the QCD crossover transition, $T_c = (154 \pm 9)$ MeV.

4. Conclusions

The hadron resonance gas model provides a remarkably good description of the thermodynamics of strong interaction matter in the low temperature hadronic phase. Even subtle aspects of fourth order charge fluctuations like the relative contributions of different strangeness sectors to bulk thermodynamics are well described by the HRG model. Its rapid breakdown in the crossover region is apparent also in the strange hadron sector and suggests that strange mesons and baryons disappear at or close to the QCD transition.

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