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BULK VISCOSITY OF THE GLUON PLASMA IN A HOLOGRAPHIC APPROACH*

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A gravity-scalar model in 5-dimensional Riemann space is adjusted to the thermodynamics of SU(3) gauge field theory in the temperature range of 1–10 T/T_{c} to calculate holographically the bulk viscosity in 4-dimensional Minkowski space. Various settings are compared, and it is argued that, upon an adjustment of the scalar potential to reproduce exactly the lattice data within a restricted temperature interval above T_{c}, rather robust values of the bulk viscosity to entropy density ratio are obtained.

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1. Introduction

The duality of $\mathcal{N} = 4$ supersymmetric Yang–Mills theory in 4-dimensional Minkowki space with type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ [1] has initiated a wealth of investigations aimed at exploiting the AdS/CFT correspondence to relate mutually properties of the gravitation sector (which is anti-de Sitter (AdS) in 5-dimensional Riemann space) with conformal field theories (CFT). Such techniques look particularly useful for 4-dimensional strongly coupled theories, where real-time processes are difficult to access. This, in turn, applies especially to strongly interacting systems, as subjects to QCD, created in the course of relativistic heavy-ion collisions, *i.e.* the quark–gluon plasma (QGP). Here, holographic techniques, based on the AdS/CFT correspondence, allow to calculate from suitable gravity duals the wanted observables quantifying properties of the QGP. Among the important

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quantities is the bulk viscosity which has a potentially strong impact on the analysis of the flow pattern in relativistic heavy-ion collisions [2] and may help to solve the photon-v_2 puzzle [3].

The gravity dual of QCD, even in the pure Yang–Mills sector, is not known. Moreover, QCD is not a CFT since, due to dimensional transmutation, an inherent energy scale is emergent which steers the running coupling. In such a situation and with a lacking top–down approach from string theory, it looks promising to utilize a bottom–up approach which incorporates a selected set of properties one is going to calculate after an appropriate adjustment of the 5-dimensional Einstein gravity theory which emerges, strictly speaking, only in the large-N_{c} limit and at large 't Hooft coupling. A famous example is the gravity-scalar set-up, where a real scalar field is consistently coupled to gravity. The scalar ϕ, dual to an operator $\mathcal{O}_\phi$, breaks conformal invariance of AdS space, simulating the corresponding breaking in Yang–Mills theory, the latter being expressed by the trace anomaly relation $T^\mu_\mu = \beta(\alpha)/(8\pi\alpha^2)\mathrm{Tr}\, F^2$ of the Yang–Mills energy-momentum tensor $T_{\mu\nu}$, β function, running coupling α and trace of the field strength tensor squared $\mathrm{Tr}\, F^2$. Being interested in thermodynamic properties of the gluon plasma, one embeds in the asymptotically AdS space a black brane which introduces a temperature via Hawking temperature and an entropy via Bekenstein–Hawking entropy. Besides the equilibrium thermodynamics, encoded in the gravity metric as dual of the gauge theory energy-momentum tensor, near-to-equilibrium quantities are accessible as correlators based on the energy-momentum tensor. For a medium without conserved charges, these are the shear and bulk viscosities as first-order transport coefficients in a gradient expansion.

2. Gravity-scalar holographic models

The class of gravity-scalar duals is defined by the action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + \mathcal{L}_{\mathrm{GH}}\,, \tag{1}$$

where $\mathcal{L}_{\mathrm{GH}}$ is the Gibbons–Hawking surface term, irrelevant for our purposes, and G_5 denotes the 5-dimensional gravity constant. The "potential" $V(\phi)$ determines the self-interaction of the scalar ϕ; it contains the constant term $V_0 = -12/L^2$ ensuring asymptotic AdS behavior with L being the curvature scale set by the negative cosmological constant. The Riemann space is accordingly specified by extending the conformally flat 4-dimensional space-time by the bulk variable u resulting in the ansatz for the infinitesimal line element squared

$$ds^2 = \exp\{2A(u)\}\left(d\vec{x}^{\,2} - f(u)dt^2 + \frac{1}{f(u)}du^2\right), \tag{2}$$

where (in conformal coordinates) $\lim_{u\to 0} f(u) = 1$ and $\lim_{u\to 0} A = \log(L/u)$ ensure the AdS property at the boundary $u \to 0$ and the simple zero of $f(u_{\rm H})$ defines the horizon at $u_{\rm H} > 0$.

The scalar is supposed to have a radial profile $\phi(u)$ which, for potentials such as $V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \dots$, is constrained by the equation of motion to $\phi(u) = \phi_{(4-\Delta)}u^{4-\Delta} + \phi_\Delta u^\Delta + \dots$ near the boundary of AdS, where $\phi_{(4-\Delta)}$ implies an additional term $\propto \int d^4x \phi_{(4-\Delta)}\mathcal{O}_\phi$ as deformation of the original CFT and $\langle \mathcal{O}_\phi \rangle \propto \phi_\Delta$, *i.e.* ϕ is holographically dual to the operator $\mathcal{O}_\phi$ with conformal dimension Δ_ϕ. For $\Delta_\phi = 4$, the dual operator is exactly marginal and the scalar field is massless, while for $\Delta_\phi \neq 4$ the source $\phi_{(4-\Delta)}$ introduces a mass scale $\Lambda = \phi_{(4-\Delta)}^{1/(4-\Delta)}$ which explicitly breaks conformal invariance. The mass m and the conformal dimension Δ_ϕ are related by $m^2L^2 = \Delta_\phi(\Delta_\phi - 4)$, which must satisfy $m^2L^2 \geq -4$ to fulfill the Breitenlohner–Freedman bound. Renormalizability on the gauge theory side requires $\Delta_\phi \leq 4$, *i.e.* $m^2L^2 \leq 0$. While an extension to $1 \leq \Delta_\phi \leq 2$ is possible [4], we restrict our attention to the upper branch of the mass-dimension relation and relevant operators, *i.e.* $2 < \Delta < 4$. This is already a special setting which follows, *e.g.*, [5, 7, 8] and serves as outline of our analysis below. The improved holographic QCD (IHQCD) model [12], in contrast, is based on different potential asymptotics $V(\phi) - V_0 \propto e^\phi + \dots$ which encodes the running 't Hooft coupling $\lambda \propto e^\phi$ close to the boundary (here at $\phi \to -\infty$) and results in the marginal case $\Delta_\phi = 4$, while, for large 't Hooft coupling, $V(\phi)$ is constructed to accomodate confinement and a linear glueball spectrum, *cf.* [5, 9].

3. Thermodynamics

The two basic AdS/CFT thermodynamic relations

$$T = -\frac{1}{4\pi}\frac{df}{du}\bigg|_{u_{\rm H}}, \qquad s = \frac{1}{4G_5}\exp\{3A\}|_{u_{\rm H}} \tag{3}$$

determine the thermodynamics, *e.g.* by $s(T)/T^3$ for parametrically given temperature $T(u_{\rm H})$ and entropy density $s(u_{\rm H})$. Here, $u_{\rm H}$ is the horizon position in the bulk. Einstein's equations determine, via the above conditions at the boundary, the metric coefficients at $u_{\rm H}$. To be specific, we utilize

$$V(\phi)L^2 = -12\cosh\gamma\phi + b\phi^2 + \sum_{n=2}^{5} c_{2n}\phi^{2n} \tag{4}$$

with $b = 6\gamma^2 + \Delta(\Delta - 4)/2$ from [7, 8], but use solely the matching condition to lattice data of the SU(3) Yang–Mills equation of state in a finite temperature interval above $T_{\rm c}$. That is we ignore an *a priori* scale setting at a certain energy and leave thus $2 < \Delta < 4$, γ and c_{2n} as free parameters.

Without further integration constant, the velocity of sound squared, $v_\mathrm{s}^2 = d \log s / d \log T$, is given, while the pressure $p = p_\mathrm{c} + \int_{T_\mathrm{c}}^{T} dT' \, s(T')$, energy density $e = -p + sT$ and interaction measure $I = e - 3p$ need one additional constant. A possibility is to employ the lattice input with $p_\mathrm{c} = p(T_\mathrm{c})$, which needs a definition of T_c. The IHQCD model has a clear definition of T_c; other options could be to choose $T_\mathrm{c} = T_\mathrm{min}$, where T_min is the minimum of the temperature T as a function of u_H or s/T^3; in the latter case, the inflection point T_ip can be utilized to define T_c in cases where T as a function of s/T^3 does not have a minimum. If one refrains to catch Yang–Mills features at zero temperature (*e.g.* a linear glue ball spectrum w.r.t. a radial quantum number) and the latent heat in the deconfinement phase transition as in IHQCD [12], one can adjust the value of T_c arbitrarily; also, G_5 can be chosen without other constraints than the optimum reproduction of a given data set in a restricted temperature interval above T_c. Here, we choose $LT_\mathrm{c} = (LT_\mathrm{min}, LT_\mathrm{ip})$ and adjust γ, $\varDelta$, c_{2n} and G_5/L^3 by minimizing

$$\chi^2_{s/T^3} = \log \left(\frac{1}{N} \sum_{i=1}^{N} \Big[\sigma(x_i) - y(x_i T_\mathrm{c} L) \Big]^2 \right) , \tag{5}$$

where $\sigma \equiv s(T)/T^3$ refers to the lattice data at N mesh points $x_i \equiv T_i/T_\mathrm{c}$ and $y \equiv G_5 s(TL)/(TL)^3$ to the holographically calculated scaled entropy density.

4. Bulk viscosity

The class of gravity-scalar models considered here belongs to the so-called two-derivative models which provide the normalized shear viscosity $\eta/s = 1/(4\pi)$, irrespectively of a specific form of $V(\phi)$, at variance with the asymptotic behavior of weakly coupled QCD [13] and the expected minimum near T_c. Higher-order gravity models [10] abandon such a temperature independence. Nevertheless, in the strongly coupled region, $\eta/s = 1/(4\pi)$ represents an intriguingly important result which got popular since the analysis of flow observables in relativistic heavy-ion collisions at the RHIC and LHC appeared consistent with that.

The bulk viscosity (ζ) follows within the present set-up from

$$\frac{\zeta}{\eta} = \left(\frac{d \log V}{d\phi} \right)^2 |p_{11}|^2 \Big|_{\phi_\mathrm{H}} , \tag{6}$$

where (using the profile of the scalar field as bulk coordinate) the horizon value of the perturbation p_{11} of the $x_1 x_1$-metric component is determined by solving a linearized Einstein equation [11].

4.1. Optimum adjustment to lattice data

As shown in [15], a perfect matching to lattice data is accomplished by the potential (4) for $\Delta = 3.7650$ and $\gamma = 0.6580$ when including the polynomial distortions c_{2n}; omitting the latter ones (with $\Delta = 3.5976$ and $\gamma = 0.6938$) the match is near-perfect, see left panel in Fig. 1. The bulk to shear viscosity ratio (*cf.* right panel in Fig. 1) displays a linear section, where $\zeta/\eta = \pi C \Delta v_{\mathrm{s}}^2$ with $C \approx 1.2$, thus fulfilling the Buchel bound $\zeta/\eta \geq 2\Delta v_{\mathrm{s}}^2$ [16]. Such a linear relation $\zeta/\eta \propto \Delta v_{\mathrm{s}}^2 = 1/3 - v_{\mathrm{s}}^2$ is considered in [6] as interesting but as unclear whether it is a generic result of *Dp* brane gauge theories. With the results of the next subsection, we argue that it is generic for the gravity-scalar set-up only for perfect matching to SU(3) Yang–Mills theory. We emphasize that a quasi-particle model [17] obeys quantitatively a similar proportionality in the strong coupling regime, also with the perfect matching of SU(3) Yang–Mills thermodynamics as a prerequisite.

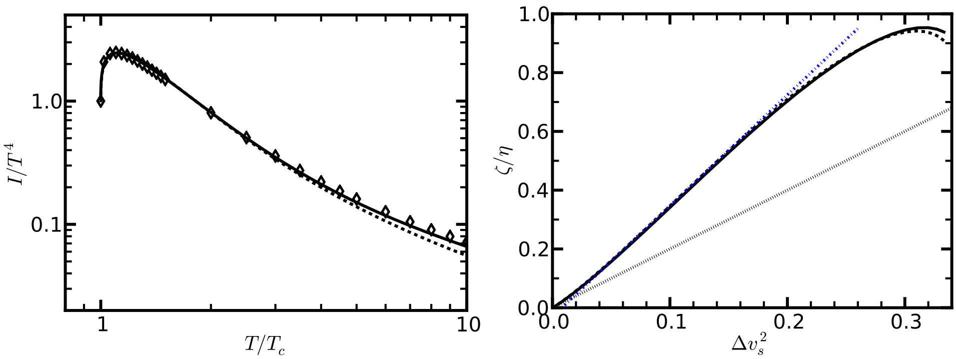

Fig. 1. Left: Scaled interaction measure as a function of T/T_{c}. The solid (dashed) curve is for the potential (4) with (without) the polynomial distortions $c_{2n}\phi^{2n}$. Other thermodynamic quantities (*e.g.* v_{s}^2, e/T^4, p/T^4 and s/T^3) agree perfectly (*cf.* [15]) with the lattice data (symbols, from [14]). Right: Bulk to shear viscosity ratio as a function of the non-conformality measure. The dot-dashed/blue line is a linear fit $\zeta/\eta = 1.2\pi\Delta v_{\mathrm{s}}^2 - 0.03$, while the dotted/black line depicts the Buchel bound $\zeta/\eta = 2\Delta v_{\mathrm{s}}^2$ [16].

4.2. Dependence of bulk viscosity on potential parameters

We demonstrate now the sensitivity of the bulk viscosity on the parameters of the potential (4) with $c_{2n} = 0$. The analysis is restricted to $3 \leq \Delta \leq 3.9$. The crosses in Fig. 2 indicate selected loci at which we calculate the equation of state and the bulk viscosity exhibited in Fig. 3 below.

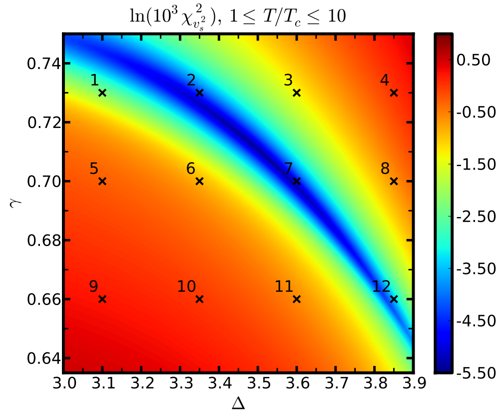

Fig. 2. The $\chi^2_{v_s^2}$ landscape over the γ *vs.* Δ plane. The crosses with numbers indicate loci of selected parameter choices to be analyzed.

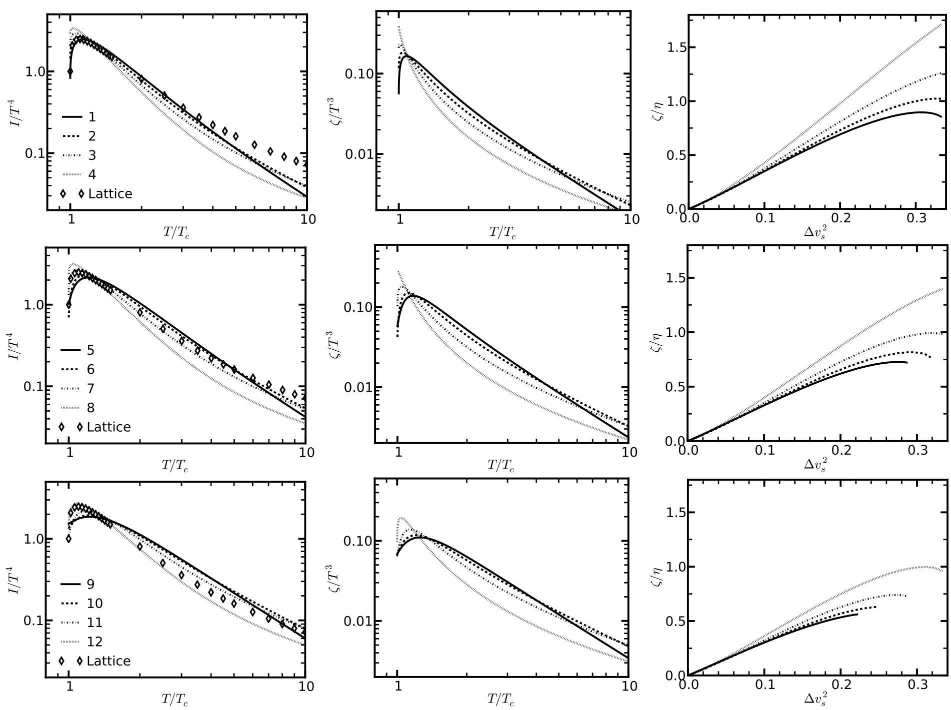

Fig. 3. Equation of state I/T^4 as a function of temperature (left column), scaled bulk viscosity ζ/T^3 as a function of temperature (middle column) and bulk to shear viscosity ratio as a function of non-conformality measure (right column). The numbers in the left panels refer to the loci in the γ *vs.* Δ plane in Fig. 2.

The deviation measure $\chi^2_{v_s^2} = \frac{1}{N}\sum_{i=1}^{N}[v_s^2(x_i) - v_{s,L}^2(x_i T_c L)]^2$ indicates already the (in)accuracy of matching the velocity of sound squared, v_s^2, from lattice QCD. Hereby, v_s^2 and $v_{s,L}^2$ are obtained from the holographic calculation and the lattice data; x_i and LT_c are as in (5). We emphasize the corridor, in which the points 2, 7 and 12 are localized, which deliver an equally good, though not perfect, reproduction of the lattice data (*cf.* left column of Fig. 3), due to the individual adjustments of G_5. The values of ζ/T^3 spread out by a factor of three for $T > T_c$ when comparing the results for all considered loci 1–12 (*cf.* middle column of Fig. 3). In contrast, ζ/η as a function of the non-conformality measure Δv_s^2 looks very much the same for loci 2, 7 and 12, while for the other loci significant variations of ζ/η can be observed, in particular for $\Delta v_s^2 \to 1/3$, *i.e.* for $T \to T_c$. This observation lets us argue that a perfect matching of the equation of state may lead to a robust result for ζ/η.

5. Summary

Despite of a lacking gravity dual to thermal SU(3) gauge theory, a gravity-scalar model with an appropriate ansatz for the potential allows for perfectly matching of thermodynamics in the temperature region (1–10) T_c. Note that no additional constraints are required, *e.g.* on scale settings or on the confined low-temperature phase or on the asymptotic behavior. The matching condition forces the bulk to shear viscosity ratio to $\zeta/\eta = C\pi\Delta v_s^2$ with $C \approx 1.2$ for $\Delta v_s^2 < 0.25$, in agreement with a previously employed quasi-particle model [17] and the IHQCD model [12]. Without matching, the considered class of potentials exhibits significant variations of both s/T^3 and ζ/η; deviations from the linear relation $\zeta/\eta \propto \Delta v_s^2$ may occur over a larger range of Δv_s^2. The increase of ζ/η as a function of the temperature toward T_c, however, seems to be a generic feature. It is always less pronounced than the behavior found in [18].

Our considerations ignore potentially strong curvature effects beyond the classical gravity scenario, the reference to large 't Hooft coupling as well as a direct link to the QCD β function. In so far, we present an exploratory study of a restricted set of observables in a special bottom-up set-up leaving a systematic relation to the *ad hoc* employed AdS/CFT correspondence with controlled deformation to accommodate the non-conformality for further studies.

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