

SOME INTENSIVE AND EXTENSIVE QUANTITIES IN HIGH-ENERGY COLLISIONS*

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We review the evolution of some statistical and thermodynamical quantities measured in different sizes of high-energy collisions at different energies. We differentiate between intensive and extensive quantities and discuss the importance of their distinguishability in characterizing possible critical phenomena of nuclear collisions at various energies with different initial conditions.

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1. Introduction

The terminology “intensive and extensive quantity” was introduced by Richard C. Tolman [1] in order to distinguish between different thermodynamical parameters, properties, variables, *etc.* Therefore, the defining of such quantities as intensive or extensive may depend on the way in which subsystems are arranged [1]. In order to characterize possible critical phenomena of the nuclear collisions, which likely become complex at ultra high energy, various signatures have been proposed [2]. It is obvious that the critical phenomena of intensive or extensive variables [3] should be differentiated. The extensive variables, like total charge multiplicity, obtain about equal contributions from the initial (due to fluctuations in spectators) and final stage (resonances). The intensive variables, like particle ratios, are well

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described by resonances at the freeze-out [4–7]. In the present work, we show how the distinguishability between extensive and intensive quantities behaves at various energies and with different initial conditions.

The implication of statistical-thermal models on high-energy physics dates back to about six decades [8]. Koppe introduced an almost-complete recipe for the statistical description of particle production [9]. The particle abundances in Fermi model [10] are treated by means of statistical weights. Furthermore, Fermi model [10] gives a generalization of the *statistical model*, in which one starts with a general cross-section formula and inserts into it a simplifying assumption about the matrix element of the process, which reflects many features of the high-energy reactions dominated by the density in phase space of the final states. In 1951, Pomeranchuk [11] came up with the conjecture that a finite hadron size would imply a critical density above which the hadronic matter cannot be in the compound state, known as hadrons. Using all tools of statistical physics, Hagedorn introduced in 1965 the mass spectrum to describe the abundant formation of resonances with increasing masses and rotational degrees of freedom [12] which relate the number of hadronic resonances to their masses as an exponential. Accordingly, Hagedorn formulated the concept of limiting temperature based on the statistical bootstrap model.

The statistical and thermodynamical variables, properties and parameters can be classified into intensive, extensive, normalized intensive and extensive, process and conjugate. There are physical properties which are neither intensive nor extensive, *e.g.* electric resistance, invariant mass and special relativity. The intensivity is apparently additive and, therefore, a state variable. The intensive (bulk) properties do not depend on the system size or the amount of existing material. Therefore, it is scale invariant. The extensivity is field and point variable but not additive. The extensive properties are additive for independent and non-interacting subsystems. They are directly proportional to the amount of existing material. Normalized intensive and extensive quantities are densities. They are not additive. The process depends on past history of the system. Therefore, they are differentiable, inexactly. The conjugates are intensive and extensive pairs, like temperature and entropy. For example, in grand canonical ensemble, strongly intensive quantities have been suggested as fluctuation measures not depending on the system volume and its fluctuations [13]. The charge distribution is inclusive, while isotropically resolved particle observation is an exclusive property. We review the evolution of some statistical and thermodynamical quantities measured in different sizes of the high-energy collisions at different energies.

The present paper is organized as follows. The intensivity and extensivity of statistical properties are shortly reviewed in Sec. 2. The dissipative properties are elaborated in Sec. 3. The energy dependence of temperature shall be estimated in Sec. 4. Section 5 is devoted to the conclusions and outlook.

2. Statistical properties: multiplicities and particle ratios

Only two independent intensive variables are needed in order to fully specify the entire state of the system of interest. Other intensive properties can be derived from these known ones. An exclusive property implies that energy and momentum, for instance, of all products are measured. The intensivity means that some quantities of the products are left unmeasured.

An extensive comparison between the particle multiplicity $dN_{\text{ch}}/d\eta$ per participating nucleon at mid-rapidity in central heavy-ion collisions [14–24] and the corresponding results from $p + p(\bar{p})$ [25–33] and $p(d) + A$ collisions [14, 34, 35] is presented in Fig. 1. It is obvious that the energy dependence

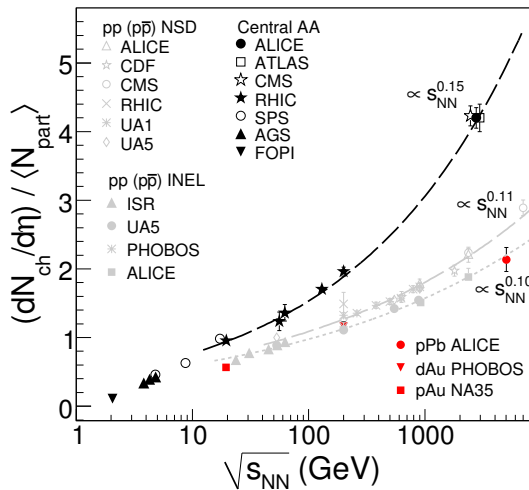


Fig. 1. A comparison of $dN_{\text{ch}}/d\eta$ per participating nucleon at mid-rapidity in central heavy-ion collisions to corresponding results from $p + p(\bar{p})$ and $p(d) + A$ collisions. The quantities are given in physical units. The graph taken from Ref. [36].

of the total multiplicity is distinguishable. In other words, the initial state plays an essential role. The extensivity can be related to canonical ensemble

$$Z(N, T, V) = \text{Tr}_N \exp \left(-\frac{H}{T} \right), \quad (1)$$

where H is the Hamiltonian, while grand canonical ensemble is related to intensivity

$$Z(\mu, T, V) = \text{Tr}_N \exp \left(-\frac{H - \mu N}{T} \right), \quad (2)$$

where N stands for the degrees of freedom. With Dirac delta function and when the chemical potential μ is Wick rotated, then extensivity can be related to intensivity

$$Z(N, T, V) = \frac{1}{2\pi} \int_0^{2\pi} Z(iT\theta, T, V) \exp(-iN\theta) d\theta. \quad (3)$$

In Fig. 2, the results of $\bar{p}/p$ calculated in HRG are represented by solid line, which seems to be a kind of a universal curve. In heavy-ion collisions, the proton ratio varies strongly with the center-of-mass energy $\sqrt{s}$. The HRG models describes very well the heavy-ion results. Also, ALICE pp results are reproduced by means of HRG model. The ratios from pp - and AA -collisions run very close to unity implying almost vanishing matter–antimatter asymmetry. On the other hand, it can also be concluded that the

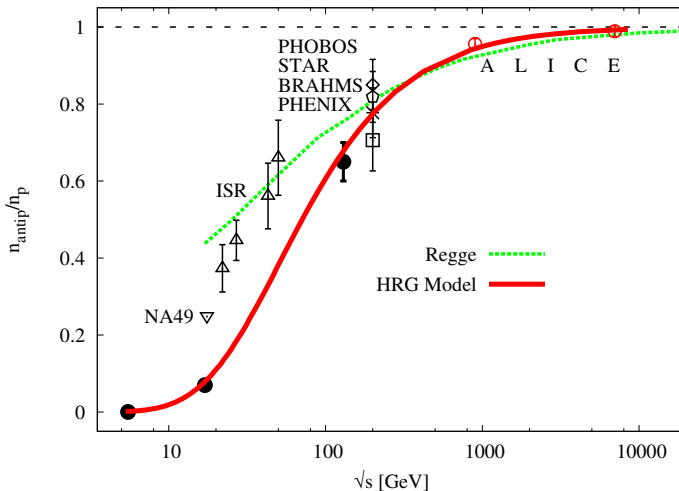


Fig. 2. $n_{\bar{p}}/n_p$ ratios depicted in whole available range of $\sqrt{s}$. Open symbols stand for the results from various pp experiments (labeled). The solid symbols give the heavy-ion results from AGS, SPS and RHIC, respectively. The fitting of pp results according to Regge model is given by the dashed curve [37]. The solid curve is the HRG results. Contrary to the dashed curve, the solid line is not a fitting to experimental data. The graph taken from Ref. [38].

statistical-thermal models including HRG seem to excellently describe the hadronization at very large energies and the condition deriving the chemical freeze-out at the final state of hadronization, the constant degrees of freedom or $S(\sqrt{s}, T) = 7(4/\pi^2)VT^3$, seems to be valid at all center-of-mass energies spanning between AGS and LHC. So far, we conclude that the distinguishability between proton ratios in pp -collisions and that in AA -collisions disappears with increasing $\sqrt{s}$.

3. Dissipative properties: elliptic flow

The azimuthal distribution with respect to the reaction plane reads

$$\frac{dN}{d(\phi_i - \Psi_n)} \sim 1 + 2 \sum_{n=1} v_n \cos[n(\phi_i - \Psi_n)] . \quad (4)$$

The reaction plane angle Ψ_n is not directly measurable, but can be determined from particle azimuthal distributions. There are various possible sources of azimuthal correlations, like jet formation, resonances, which do not depend on the reaction plane (non-flow correlations). The Fourier coefficient v_n , which refers to the correlation in n particle emission with respect to the reaction plane, is given by

$$v_n = \langle \cos[n(\phi_i - \Psi_n)] \rangle . \quad (5)$$

Figure 3 shows data collected over about four decades spanning from the GSI, AGS, SPS, RHIC to LHC facilities. The integrated elliptic flow measured in relative central heavy-ion collisions (20–30%) is given in dependence on nucleus–nucleus center-of-mass energy in Fig. 3. For the comparison, the integrated elliptic flow is corrected for p_t cutoff of 0.2 GeV/c. The estimated magnitude of this correction is $12 \pm 5\%$ based on calculations with *Therminator*. The figure shows that there is a continuous increase in the magnitude of elliptic flow for this centrality region from RHIC to LHC energies. In comparison to the elliptic flow measurements in Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV, we observe about a 30% increase in the magnitude of v_2 at $\sqrt{s_{NN}} = 2.76$ TeV. The rapid decrease of v_2 at very low energy, FOPI data, refers to *bounce-off*. Increasing $\sqrt{s_{NN}}$, a *squeeze-out* will set on. At larger energies, the behavior can be described by *in-plane elliptic flow* due to pressure gradient.

The elliptic flow shows a rich structure; a transition from in-plane to out-of-plane and back to in-plane emission. Apparently, it is sensitive to the properties of the medium created in heavy-ion collisions. There are evidences that the elliptic flow of charged and identified particles indicates a strong rise of the expansion velocity of the medium (radial flow) at the RHIC *vs.* LHC.

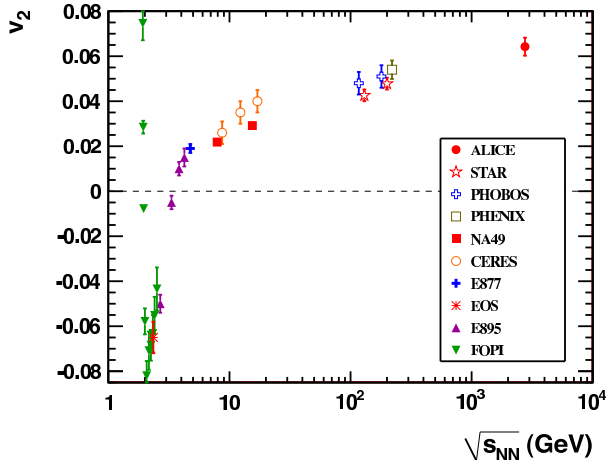


Fig. 3. Integrated elliptic flow measured in central heavy-ion collisions (20–30%) is given in dependence on nucleus–nucleus center-of-mass energy. The graph taken from Ref. [39].

On the other hand, it was assumed that there are no correlations due to elliptic flow in pp collisions at the RHIC energy [40]. The methods of measuring elliptic flow can hardly be employed with the currently available number of recorded pp interactions of ALICE at the LHC. Furthermore, none of the available microscopic Monte Carlo (MC) models describes the development of anisotropic flow in elementary hadron–hadron interactions yet [40]. The particular non-perturbative approach was suggested as a mechanism of anisotropic flow that might be a leading one in hadron collisions, since those have smaller geometrical extension and the probability of hydrodynamical generation of elliptic flow is lower compared to the collisions of nuclei [41].

pp collisions simulated by Pythia, Phojet and Epos at 900 and 7000 GeV are analyzed by two-particle correlation methods. The integrated v_2 coefficients reconstructed by the methods are found to vary from 10%–15%. These values are attributed solely to the non-flow correlations [40].

4. Hagedorn temperature: energy and system size dependence

The transverse mass spectra of well-identified particles have been studied at various energies, for instance in [45]. Accordingly, Stefan–Boltzmann approximation results is

$$\frac{1}{m_T} \frac{dN}{dm_T dy} = a \exp\left(-\frac{m_T}{T}\right), \quad (6)$$

where $m_T = \sqrt{p_T^2 + m^2}$ is the dispersion relation and a is a fitting parameter. Figure 4 presents the energy dependence of the inverse slope parameter T of the transverse mass spectra of K^+ (left panel) and K^- mesons (right panel) produced in central Pb+Pb and Au+Au collisions. There is a plateau at SPS energies [45] which is preceded by a steep rise of T measured at the AGS [42] and followed by an indication of a further increase of the RHIC data [43]. Although the scatter of data points is large, T appears to increase smoothly in $p + p(\bar{p})$ collisions [44], left panel of Fig. 4. The dependence of T on the system size is obvious. For completeness, we recall that the direct thermal photons have been used to estimate the Hagedorn temperature

$$T = \begin{cases} 304 \pm 51 \text{ MeV} & \text{ALICE} & [46] \\ 221 \pm 19 \text{ MeV} & \text{PHENIX} & [47] \end{cases} . \quad (7)$$

The dependence on system size is illustrated in the left panel of Fig. 4. The Hagedorn temperature in pp collisions seems to be smaller than that in AA collisions. Its variation with the center-of-mass energy is apparently weaker than the variation in AA collisions. A much more systematic measurement would help in proving or disproving such a conclusion.

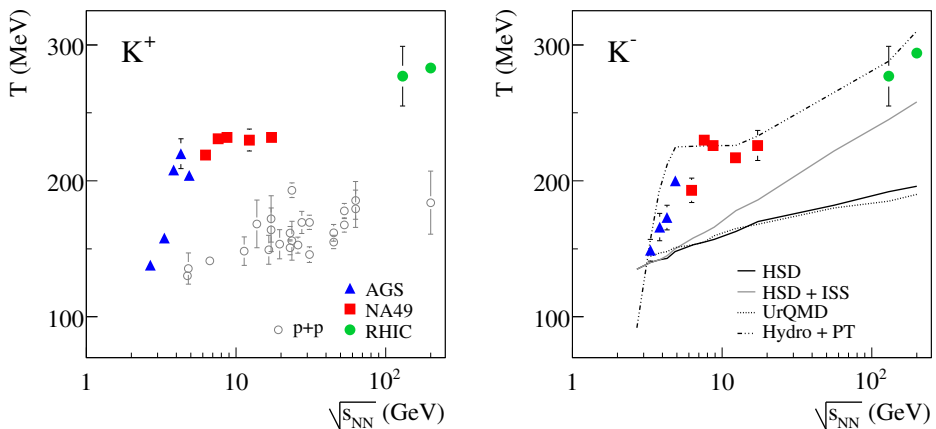


Fig. 4. Energy dependence of T related to the transverse mass spectra of K^+ (left panel) and K^- mesons (right panel) produced in central Pb+Pb and Au+Au collisions. The graphs taken from Ref. [45].

5. Conclusions and outlook

The ultimate goal of the physics program of high-energy collisions is the study of properties of strongly interacting matter under extreme conditions of temperature and/or compression. The particle multiplicities and their fluctuations and correlations are experimental tools to analyse the nature, composition, and size of the medium, from which they are originating. Of particular interest is the extent to which the measured particle yields are showing equilibration. Based on analysing the particle abundances or momentum spectra, the degree of equilibrium of the produced particles can be estimated. The particle abundances can help to establish the chemical composition of the system. The momentum spectra can give additional information on the dynamical evolution and the collective flow.

In order to characterize possible critical phenomena, signatures based on particle multiplicities and their fluctuations and correlations have been proposed. Intensive or extensive quantities should be separated, systematically. Extensivity obtains about equal contributions from the initial and final stage. Intensivity is well described by produced particles in final state. The present work introduces the importance of distinguishability between extensive and intensive quantities at various energies and in different system sizes.

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REFERENCES

- [1] O. Redlich, *J. Chem. Edu.* **47**, 154 (1970).
- [2] S.A. Bass, M. Gyulassy, H. Stoecker, W. Greiner, *J. Phys. G* **25**, R1 (1999).
- [3] E.V. Shuryak, [arXiv:hep-ph/0011208](#).
- [4] A. Tawfik, *Phys. Rev.* **C88**, 035203 (2013).
- [5] A. Tawfik, *Nucl. Phys.* **A922**, 225 (2014) [[arXiv:1306.1025 \[hep-ph\]](#)].
- [6] A. Tawfik, H. Magdy, E. Gamal, *Ukr. J. Phys.* **58**, 933 (2013) [[arXiv:1301.1828 \[nucl-th\]](#)].
- [7] A. Tawfik, *Nucl. Phys.* **A764**, 387 (2006).
- [8] H. Koppe, *Z. Naturforschg.* **3a**, 251 (1948); *Ann. Phys. (Berlin)* **437**, 103 (1948).
- [9] A. Tawfik, Review on Equilibrium Statistical-Thermal Models in High-Energy Physics, in preparation.
- [10] E. Fermi, *Prog. Theor. Phys.* **5**, 570 (1950).
- [11] I.Ya. Pomeranchuk, *Dokl. Akad. Nauk. SSSR* **78**, 889 (1951).

- [12] R. Hagedorn, *Suppl. Nuovo Cimento* **III**, 147 (1965); *Nuovo Cimento* **35**, 395 (1965).
- [13] M. Gazdzicki, M.I. Gorenstein, M. Mackowiak-Pawlowska, *Phys. Rev.* **C88**, 024907 (2013).
- [14] B.B. Back *et al.* [PHOBOS Coll.], *Phys. Rev. Lett.* **93**, 082301 (2004).
- [15] M.C. Abreu *et al.* [NA50 Coll.], *Phys. Lett.* **B530**, 33 (2002).
- [16] C. Adler *et al.* [STAR Coll.], *Phys. Rev. Lett.* **87**, 112303 (2001).
- [17] I.G. Bearden *et al.* [BRAHMS Coll.], *Phys. Lett.* **B523**, 227 (2001).
- [18] I.G. Bearden *et al.* [BRAHMS Coll.], *Phys. Rev. Lett.* **88**, 202301 (2002).
- [19] K. Adcox *et al.* [PHENIX Coll.], *Phys. Rev. Lett.* **86**, 3500 (2001).
- [20] B.B. Back *et al.* [PHOBOS Coll.], *Phys. Rev. Lett.* **85**, 3100 (2000).
- [21] B.B. Back *et al.*, *Phys. Rev. Lett.* **91**, 052303 (2003).
- [22] K. Aamodt *et al.* [ALICE Coll.], *Phys. Rev. Lett.* **105**, 252301 (2010).
- [23] G. Aad *et al.* [ATLAS Coll.], *Phys. Lett.* **B710**, 363 (2012).
- [24] S. Chatrchyan *et al.* [CMS Coll.], *J. High Energy Phys.* **1108**, 141 (2011).
- [25] C. Albajar *et al.* [UA1 Coll.], *Nucl. Phys.* **B335**, 261 (1990).
- [26] K. Alpgard *et al.* [UA5 Coll.], *Phys. Lett.* **B121**, 209 (1983); G. Alner *et al.* [UA5 Coll.], *Z. Phys.* **C33**, 1 (1986).
- [27] B.I. Abelev *et al.* [STAR Coll.], *Phys. Rev.* **C79**, 034909 (2009).
- [28] F. Abe *et al.* [CDF Coll.], *Phys. Rev.* **D41**, 2330 (1990).
- [29] K. Aamodt *et al.* [ALICE Coll.], *Eur. Phys. J.* **C68**, 345 (2010).
- [30] V. Khachatryan *et al.* [CMS Coll.], *J. High Energy Phys.* **1002**, 041 (2010).
- [31] W. Thome *et al.*, *Nucl. Phys.* **B129**, 1 (1986).
- [32] K. Aamodt *et al.* [ALICE Coll.], *Eur. Phys. J.* **C68**, 89 (2010).
- [33] B. Alver *et al.* [PHOBOS Coll.], *Phys. Rev.* **C83**, 024913 (2011).
- [34] B. Abelev *et al.* [ALICE Coll.], *Phys. Rev. Lett.* **110**, 032301 (2013).
- [35] T. Alber *et al.* [NA35 Coll.], *Eur. Phys. J.* **C2**, 643 (1998).
- [36] R. Singh, L. Kumar, P.K. Netrakanti, B. Mohanty, *Adv. High Energy Phys.* **2013**, 761474 (2013) [1304.2969 [nucl-ex]].
- [37] K. Aamodt *et al.* [ALICE Coll.], *Phys. Rev. Lett.* **105**, 072002 (2010).
- [38] A. Tawfik, *Nucl. Phys.* **A859**, 63 (2011).
- [39] K. Aamodt *et al.* [ALICE Coll.], *Phys. Rev. Lett.* **105**, 252302 (2010).
- [40] G. Eyyubova [ALICE Coll.], *J. Phys. Conf. Ser.* **270**, 012047 (2011).
- [41] S.M. Troshin, N.E. Tyurin, *Mod. Phys. Lett.* **A26**, 1095 (2011).
- [42] D. Pelte *et al.* [FOPI Coll.], *Z. Phys.* **A357**, 215 (1997); L. Ahle *et al.* [E802 Coll.], *Phys. Rev.* **C57**, 466 (1998); L. Ahle *et al.* [E802 Coll.], *Phys. Rev.* **C58**, 3523 (1998); L. Ahle *et al.* [E802 Coll.], *Phys. Rev.* **C60**, 044904 (1999); L. Ahle *et al.* [E802 Coll.], *Phys. Rev.* **C60**, 064901 (1999); L. Ahle *et al.* [E866 and E917 colls.], *Phys. Lett.* **B476**, 1 (2000); L. Ahle *et al.*

- [E866 and E917 colls.], *Phys. Lett.* **B490**, 53 (2000); J. Barrette *et al.* [E877 Coll.], *Phys. Rev.* **C62**, 024901 (2000); J. Klay *et al.* [E895 Coll.], *Phys. Rev. Lett.* **88**, 102301 (2002); B.B. Back *et al.* [E917 Coll.], *Phys. Rev.* **C66**, 054901 (2002); J. Klay *et al.* [E895 Coll.], *Phys. Rev.* **C68**, 054905 (2003).
- [43] I.G. Bearden *et al.* [BRAHMS Coll.], *Phys. Rev. Lett.* **88**, 202301 (2002); B.B. Back *et al.* [PHOBOS Coll.], [arXiv:nuc1-ex/0301017](#); M. Velkovsky [PHENIX Coll.], *J. Phys. G* **30**, S187 (2004); C. Adler *et al.* [STAR Coll.], *Phys. Lett.* **B595**, 143 (2004); S.S. Adler *et al.* [PHENIX Coll.], *Phys. Rev.* **C69**, 034909 (2004); K. Adcox *et al.* [PHENIX Coll.], *Phys. Rev.* **C69**, 024904 (2004); J. Adams *et al.* [STAR Coll.], *Phys. Rev. Lett.* **92**, 112301 (2004); I.G. Bearden *et al.* [BRAHMS Coll.], *Phys. Rev. Lett.* **94**, 162301 (2005); I. Arsene *et al.* [BRAHMS Coll.], *Phys. Rev.* **C72**, 014908 (2005).
- [44] M. Kliemant, B. Lungwitz, M. Gazdzicki, *Phys. Rev.* **C69**, 044903 (2004).
- [45] C. Alt *et al.* [NA49 Coll.], *Phys. Rev.* **C77**, 024903 (2008).
- [46] M. Wilde, *Nucl. Phys.* **A904-905**, 573c (2013).
- [47] A. Adare *et al.* [PHENIX Coll.], *Phys. Rev. Lett.* **104**, 132301 (2010).