Vol. 7 (2014) *Acta Physica Polonica B Proceedings Supplement* No 1

RELAXATION AND COUPLING COEFFICIENTS IN THIRD ORDER RELATIVISTIC FLUID DYNAMICS*

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(Received March 13, 2014)

From the third order entropy four-current expression for a non-equilibrium single component system, we calculate its third order relaxation and coupling coefficients at ultrarelativistic (high temperature) limit and also give the results for the non-relativistic case. The ultrarelativistic limit is more interesting as it is in this region where one could expect the formation of quark–gluon plasma (QGP) in heavy ion collisions. The coefficients arise as a result of the generalization of the second order theory of relativistic fluid dynamics to the third order theory.

DOI:10.5506/APhysPolBSupp.7.197
PACS numbers: 47.75.+f, 05.60.–k, 5.75.Nq, 51.20.+d

1. Introduction

Transport coefficients, such as viscosities, diffusivities, and conductivities characterize the dynamics of fluctuations in a system [1]. Transport phenomenon in relativistic fluids are of great interest in connection with the study of astrophysical conditions such as those in neutron stars, the study of the cosmological conditions such as those in the early universe, as well as the study of the relativistic nuclear collisions such as those at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC).

Knowledge of transport coefficients and associated lengths and time scales is important in comparing observables with theoretical predictions. In the study of relativistic nuclear collisions, knowledge of transport coefficients will greatly advance our current efforts and interests in the use of relativistic dissipative fluid dynamics in describing the observables [1, 2]. To get information about the transport coefficients and associated relaxation

* Lecture presented at the XXXI Max Born Symposium and HIC for FAIR Workshop "Three Days of Critical Behaviour in Hot and Dense QCD", Wrocław, Poland, June 14–16, 2013.

times and relaxation lengths, one needs to know the values of relaxation and coupling coefficients at non-relativistic and ultrarelativistic limit.

In this work, we present the third order relaxation and coupling coefficients for single component system at ultrarelativistic limits. The second order coefficients were already discussed in [3, 4]. These coefficients are also studied as functions of temperature [5].

2. Entropy 4-current up to third order deviation from equilibrium

The expression for entropy 4-current is found using Grad's 14 moment method of expanding the distribution function around equilibrium up to third order. The third order entropy four-current expression is found to be [5]

$$\begin{aligned} S^\mu =& S^\mu_{\rm eq} + \frac{q^\mu}{T} \\ &-\frac{1}{2}\beta u^\mu \left\{\mathcal{S}^2_1 \mathit{\Pi}^2 - \mathcal{S}^2_2 q^\alpha q_\alpha - \mathcal{S}^2_3 \pi^{\nu\alpha}\pi_{\nu\alpha}\right\} - \beta\left\{\mathcal{S}^2_4 q^\mu \mathit{\Pi} - \mathcal{S}^2_5 q_\alpha \pi^{\mu\alpha}\right\} \\ &+\frac{1}{6}\beta u^\mu \left\{\mathcal{S}^3_1 \mathit{\Pi}^3 + \mathcal{S}^3_2 \mathit{\Pi} q^\alpha q_\alpha + \mathcal{S}^3_3 \mathit{\Pi}\pi^{\nu\alpha}\pi_{\nu\alpha} + \mathcal{S}^3_4 q_\nu q_\alpha \pi^{\nu\alpha} + \mathcal{S}^3_5 \pi_{\nu\alpha}\pi^\nu_\beta \pi^{\alpha\beta}\right\} \\ &-\frac{1}{6}\beta q^\mu \left\{\mathcal{S}^3_6 \mathit{\Pi}^2 + \mathcal{S}^3_7 q^\alpha q_\alpha - \mathcal{S}^3_8 \pi^{\nu\alpha}\pi_{\nu\alpha}\right\} - \beta\mathcal{S}^3_9 \mathit{\Pi} q_\alpha \pi^{\mu\alpha} \\ &+\frac{1}{2}\beta\mathcal{S}^3_{10} q_\alpha \pi^{\nu\alpha}\pi^\mu_\nu\,, \end{aligned} \tag{2.1}$$

where the relaxation and coupling coefficients $\mathcal{S}^j_i$, have been defined in [5]. In this coefficients the superscript denotes the order and the subscript labels the coefficient number in that order. In this article, we present the third order coefficients $\mathcal{S}^3_1$ to $\mathcal{S}^3_{10}$.

The third order coefficients come out as [5]

$$\begin{aligned} \mathcal{S}^3_1 =& \frac{1}{\beta}\left[\mathcal{I}_{10}\mathcal{A}^3_0 - 3\mathcal{I}_{20}\mathcal{A}^2_0\mathcal{A}_1 + 9\mathcal{I}_{30}\mathcal{A}^2_0\mathcal{A}_2\right. \\ &-\mathcal{I}_{40}\mathcal{A}_1\left(\mathcal{A}^2_1 + 18\mathcal{A}_0\mathcal{A}_2\right) + 9\mathcal{I}_{50}\mathcal{A}_2\left(3\mathcal{A}_0\mathcal{A}_2 - \mathcal{A}^2_1\right) \\ &\left.-27\mathcal{I}_{60}\mathcal{A}_1\mathcal{A}^2_2 + 27\mathcal{I}_{70}\mathcal{A}^3_2\right]\,, \end{aligned} \tag{2.2}$$

$$\begin{aligned} \mathcal{S}^3_2 =& -\frac{1}{\beta}\left[3\mathcal{I}_{41}\mathcal{B}_0\left(2\mathcal{A}_0\mathcal{B}_1 + \mathcal{A}_1\mathcal{B}_0\right) + 3\mathcal{I}_{51}\left(2\mathcal{A}_1\mathcal{B}_0\mathcal{B}_1 - \mathcal{A}_0\mathcal{B}^2_1 + 3\mathcal{A}_2\mathcal{B}^2_0\right)\right. \\ &\left.+3\mathcal{I}_{61}\left(6\mathcal{A}_2\mathcal{B}_0\mathcal{B}_1 + \mathcal{A}_1\mathcal{B}^2_1\right) - 9\mathcal{I}_{71}\mathcal{A}_2\mathcal{B}^2_1\right]\,, \end{aligned} \tag{2.3}$$

$$\mathcal{S}^3_3 = \frac{1}{\beta}\left[6\mathcal{I}_{52}\mathcal{A}_0\mathcal{C}^2_0 - 6\mathcal{I}_{62}\mathcal{A}_1\mathcal{C}^2_0 + 18\mathcal{I}_{72}\mathcal{A}_2\mathcal{C}^2_0\right]\,, \tag{2.4}$$

$$\mathcal{S}^3_4 = \frac{3\mathcal{C}_0}{\beta}\left[\mathcal{I}_{52}\mathcal{B}^2_0 - 2\mathcal{I}_{62}\mathcal{B}_0\mathcal{B}_1 + \mathcal{I}_{72}\mathcal{B}^2_1\right]\,, \tag{2.5}$$

$$\mathcal{S}_5^3 = \frac{2\mathcal{I}_{73}\mathcal{C}_0^3}{\beta}\,, \tag{2.6}$$

$$\begin{aligned}\mathcal{S}_6^3 = \frac{3}{\beta}\Big[&\mathcal{I}_{21}\mathcal{A}_0^2\mathcal{B}_0 + \mathcal{I}_{31}\mathcal{A}_0^2\mathcal{B}_1 + \mathcal{I}_{41}(\mathcal{A}_1^2\mathcal{B}_0 + 2\mathcal{A}_0\mathcal{A}_1\mathcal{B}_1 + 6\mathcal{A}_0\mathcal{A}_2\mathcal{B}_1)\\ &+\mathcal{I}_{51}\left(\mathcal{A}_1^2\mathcal{B}_1 - 6\mathcal{A}_0\mathcal{A}_2\mathcal{B}_1 + 6\mathcal{A}_1\mathcal{A}_2\mathcal{B}_0\right) + 3\mathcal{I}_{61}\mathcal{A}_2\left(2\mathcal{A}_1\mathcal{B}_1 + 3\mathcal{A}_2\mathcal{B}_0\right)\\ &-9\mathcal{I}_{71}\mathcal{A}_2^2\mathcal{B}_1\Big]\,,\end{aligned} \tag{2.7}$$

$$\mathcal{S}_7^3 = -\frac{1}{\beta}\left[\mathcal{I}_{42}\mathcal{B}_0^3 - 3\mathcal{I}_{52}\mathcal{B}_0^2\mathcal{B}_1 + 3\mathcal{I}_{62}\mathcal{B}_0\mathcal{B}_1^2 - \mathcal{I}_{72}\mathcal{B}_0^3\right]\,, \tag{2.8}$$

$$\mathcal{S}_8^3 = \frac{3}{\beta}\left[\mathcal{I}_{73}\mathcal{B}_1\mathcal{C}_0^2 - \mathcal{I}_{63}\mathcal{B}_0\mathcal{C}_0^2\right]\,, \tag{2.9}$$

$$\begin{aligned}\mathcal{S}_9^3 = -\frac{1}{\beta}\big[&\mathcal{I}_{42}\mathcal{A}_0\mathcal{B}_0\mathcal{C}_0 - \mathcal{I}_{52}\mathcal{A}_1\mathcal{B}_0\mathcal{C}_0\mathcal{I}_{52}\mathcal{A}_1\mathcal{B}_1\mathcal{C}_0 + \mathcal{I}_{62}\mathcal{A}_1\mathcal{B}_1\mathcal{C}_0\\ &+3\mathcal{I}_{62}\mathcal{A}_2\mathcal{B}_0\mathcal{C}_0 - 3\mathcal{I}_{62}\mathcal{A}_2\mathcal{B}_1\mathcal{C}_0\big]\,,\end{aligned} \tag{2.10}$$

$$\mathcal{S}_{10}^3 = \frac{1}{\beta}\left[\mathcal{I}_{73}\mathcal{B}_1\mathcal{C}_0^2 - \mathcal{I}_{63}\mathcal{B}_0\mathcal{C}_0^2\right]\,, \tag{2.11}$$

where $\mathcal{I}_{nk}$ are the equilibrium thermodynamic functions presented in [4] and the coefficients $\mathcal{A}_i$, $\mathcal{B}_i$, and $\mathcal{C}_i$ are the thermodynamical functions given by [4]

$$\mathcal{A}_2 = \frac{1}{4\mathcal{I}_{42}\Omega}\,, \qquad \mathcal{B}_1 = \frac{1}{\Lambda\mathcal{I}_{21}}\,, \qquad \mathcal{C}_0 = \frac{1}{2\mathcal{I}_{42}}\,, \tag{2.12}$$

$$\mathcal{A}_1 = 3\mathcal{A}_2 D_{20}^{-1}\left[4\left(\mathcal{I}_{10}\mathcal{I}_{41} - \mathcal{I}_{20}\mathcal{I}_{31}\right)\right]\,, \qquad \mathcal{B}_0 = \mathcal{B}_1\frac{\mathcal{I}_{41}}{\mathcal{I}_{31}}\,, \tag{2.13}$$

$$\mathcal{A}_0 = 3\mathcal{A}_2 D_{20}^{-1}\left(D_{30} + \mathcal{I}_{41}\mathcal{I}_{20} - \mathcal{I}_{30}\mathcal{I}_{31}\right)\,, \tag{2.14}$$

with

$$\Lambda = \frac{D_{31}}{\mathcal{I}_{21}^2}\,, \tag{2.15}$$

$$\Omega = -\frac{\mathcal{I}_{10}}{D_{20}\,\mathcal{I}_{31}}\left[\mathcal{I}_{30}\left(\mathcal{I}_{30} - \frac{\mathcal{I}_{31}}{\mathcal{I}_{21}}\mathcal{I}_{20}\right) - \mathcal{I}_{40}\left(\mathcal{I}_{20} - \frac{\mathcal{I}_{31}}{\mathcal{I}_{21}}\mathcal{I}_{10}\right)\right] + \beta\frac{\mathcal{I}_{41}}{\mathcal{I}_{31}}\,, \tag{2.16}$$

$$D_{nk} = \mathcal{I}_{n+1,k}\mathcal{I}_{n-1,k} - (\mathcal{I}_{nk})^2\,. \tag{2.17}$$

3. Third order relaxation and coupling coefficients in the ultrarelativistic limit

Ultrarelativistic particles are characterized by vanishing rest mass *i.e.* $z = \frac{m}{T} \approx 0$. In this case, the moment integrals, $\mathcal{I}_{nk}$ takes the simple form [4]

$$\mathcal{I}_{nk} = \frac{4\pi A_0}{(2k+1)!!} T^{n+2} \int_0^\infty dx x^{n+1} \frac{1}{e^{x-\phi}} \,. \tag{3.1}$$

For our studies, the chemical potential is considered to be zero *i.e.* $\phi = 0$. Then above integral becomes

$$\mathcal{I}_{nk} = \frac{4\pi A_0}{(2k+1)!!} T^{n+2} \int_0^\infty dx x^{n+1} e^{-x} \,. \tag{3.2}$$

But

$$\int_0^\infty dx x^{n+1} e^{-x} = (n+1)! \,, \tag{3.3}$$

then we have

$$\mathcal{I}_{nk} = \frac{4\pi A_0}{(2k+1)!!} T^{n+2} (n+1)! \,. \tag{3.4}$$

In the ultrarelativistic limit, the integrals $\mathcal{I}_{nk}$ can be expressed in terms of the pressure p by the comparing the integrals with the thermodynamic integral for pressure. Then these thermodynamic integrals help us to calculate various quantities needed for calculating the relaxation and coupling coefficients in the ultrarelativistic limit. The quantities Ω that come in the bulk viscosity and Λ that appears in the thermal conductivity coefficient become, in the ultrarelativistic limit,

$$\Omega = 0 \,, \qquad \Lambda = 4\,T^2 \,, \tag{3.5}$$

while the coefficients $\mathcal{A}_i$ and $\mathcal{B}_i$ become

$$\begin{aligned} \mathcal{A}_2 &= \infty \,, & \mathcal{A}_0 &= \infty \,, & \mathcal{A}_1 &= \infty \,, \\ \mathcal{B}_1 &= \frac{1}{4\,p\,T^2} \,, & \mathcal{B}_0 &= \frac{5}{4\,p\,T} \,, & \mathcal{C}_0 &= \frac{1}{8\,p\,T^2} \,, \end{aligned} \tag{3.6}$$

where

$$p = \mathcal{I}_{21} = \frac{4\pi A_0}{3} T^4 3! \,. \tag{3.7}$$

Using the above results, the third order non-classical coefficients in the ultrarelativistic limit are found to be

$$\mathcal{S}_1^3 = \infty\,, \qquad \mathcal{S}_2^3 = \infty\,, \qquad \mathcal{S}_3^3 = \infty\,, \qquad \mathcal{S}_6^3 = \infty\,, \qquad \mathcal{S}_9^3 = \infty\,, \tag{3.8}$$

$$\mathcal{S}_4^3 \approx 6p^{-2}\,, \qquad \mathcal{S}_7^3 \approx 2p^{-2} = 2\mathcal{S}_2^2 p^{-1}\,, \tag{3.9}$$

$$\mathcal{S}_5^3 \approx \tfrac{3}{4}p^{-2} = \mathcal{S}_3^2 p^{-1}\,, \qquad \mathcal{S}_8^3 \approx \tfrac{27}{32}p^{-2}\,, \qquad \mathcal{S}_{10}^3 \approx \tfrac{9}{32}p^{-2}\,. \tag{3.10}$$

One can show using the thermodynamic integrals from [4] that in the non-relativistic limit, the coefficients become

$$\mathcal{S}_4^3 \approx \tfrac{8}{5}zp^{-2}\,, \qquad \mathcal{S}_7^3 \approx \tfrac{6}{5}zp^{-2} = 3\mathcal{S}_2^2 p^{-1}\,, \tag{3.11}$$

$$\mathcal{S}_5^3 \approx \tfrac{1}{3}p^{-2} = \tfrac{1}{2}\mathcal{S}_3^2 p^{-1}\,, \qquad \mathcal{S}_8^3 \approx \tfrac{2}{3}p^{-2}\,, \qquad \mathcal{S}_{10}^3 \approx \tfrac{1}{4}p^{-2}\,. \tag{3.12}$$

4. Conclusion

From third order entropy 4-current expression, we presented the third order relaxation and coupling coefficients at ultrarelativistic limit and also those of non-relativistic case. An interesting point provided by this analysis is the relationship between the third order coefficients and those at second order. A numerical comparison is also in agreement with the results obtained in [5].

This work was supported by NRF/SA-Poland bilateral grant. I would like to acknowledge the hospitality of the University of Venda where this work was written.

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