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HADRONIC CORRELATORS AND SYMMETRIES*

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The nature of the QCD chiral transition depends crucially on the symmetries being restored at the transition temperature. These symmetries are reflected in the properties of correlations of meson operators as well as in the eigenmodes of the Dirac operator of the light quarks. The paper gives an account of our current results for these quantities which are based on lattice simulations with two light and a strange quark in improved discretization schemes.

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1. Introduction

The Lagrangian of Quantum Chromodynamics (QCD), the theory of the strong interactions, for $N_{\rm F}$ massless flavours is globally symmetric under $\mathrm{SU_L}(N_{\rm F}) \times \mathrm{SU_R}(N_{\rm F}) \times \mathrm{U_A}(1) \times \mathrm{U_V}(1)$ transformations. This symmetry is explicitely broken by non-vanishing quark masses. However, at the physical spectrum with two light (u, d) and the strange (s) quark, $m_u \lesssim m_d \ll m_s \ll m_{\rm proton}$, the effects of the explicit breaking are small, especially in the light quark sector, and can be treated as perturbation. It is well established that in the vacuum the $\mathrm{SU_L}(N_{\rm F}) \times \mathrm{SU_R}(N_{\rm F})$ symmetry is spontaneously broken to $\mathrm{SU_V}(N_{\rm F})$, signalled by a non-vanishing value for the chiral condensate $\langle q\bar{q}\rangle$ and leading to the appearance of pseudoscalar Goldstone mesons, the pions, as well as to the lifting of degeneracies between chiral partners, *e.g.* the $J^{\rm PC} = 1^{--}$ ρ and the 1^{++} a_1 meson. Moreover, the $\mathrm{U_A}(1)$ symmetry is

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explicitely broken by quantum effects, the so-called triangle anomaly, leading to a mass of the pseudoscalar flavour singlet, predominantly the η' meson, much larger than those of the other pseudoscalars, the pions and kaons.

At high temperatures, the chiral $\mathrm{SU_L}(N_\mathrm{F}) \times \mathrm{SU_R}(N_\mathrm{F})$ symmetry is restored. In the chiral, massless quark limit the restoration is signalled by the vanishing of the corresponding order parameter, the chiral condensate at the critical temperature T_c. Moreover, as is argued from universality the nature of the transition depends crucially on the number of flavours: for $N_\mathrm{F} = 3$ massless quarks the transition is of first order [1], while for $N_\mathrm{F} = 2$ the order may depend on the effect of the $\mathrm{U_A}(1)$ anomaly at the transition temperature [1, 2]. In the latter case, if the $\mathrm{U_A}(1)$ breaking is still effective, the transition is expected to be of second order with $O(4)$ critical behaviour. Relatedly, at non-vanishing quark masses the nature of the transition depends crucially on the quark mass values. Lattice QCD analyses with $2+1$ staggered quarks at physical masses have strongly indicated that the transition is a crossover [3]. Studies at a variety of light quark masses down to values even smaller than realized in nature exhibit compatibility of the quark mass dependence of the light quark chiral condensate with $O(4)$ scaling up to and including the physical point [4], an observation that has been corroborated in simulations within a different discretization scheme at smaller lattice spacings [5].

The realization of chiral symmetries is reflected in the spectrum of hadronic excitations. As the $\mathrm{SU_L}(N_\mathrm{F}) \times \mathrm{SU_R}(N_\mathrm{F})$ symmetry gets restored at high temperature, the vector and the axialvector spectral distributions are expected to become identical in the chiral limit. For two light quarks, the scalar–pseudoscalar sector is more complicated: the pion is to become degenerate with the scalar isoscalar $f_0(\sigma)$, whereas the degeneracy with the scalar isovector $a_0(\delta)$ depends on the effective restoration of the $\mathrm{U_A}(1)$ [6]. In the following section, we therefore discuss current results for meson correlators at temperatures in the vicinity of T_c and above. In section 3 this is supplemented by preliminary results from an analysis of the eigenmodes of the Dirac operator which are also sensitive to chiral symmetry, through the Banks–Casher relation, and to the effective restoration of $\mathrm{U_A}(1)$.

2. Hadron screening masses

At finite temperature, the extent of the thermal system in the temporal direction is limited by the inverse temperature. Therefore, many lattice studies of hadronic excitations investigate correlation functions of hadronic operators H in one of the spatial directions

$$G_H(z) = \sum_{x,y,t} \langle H(x,y,z,t) H(0) \rangle \xrightarrow[z \text{ large}]{} \exp\left(-m_H^{\mathrm{scr}} z\right) + \text{p.b.c.} \qquad (1)$$

which decay exponentially with the screening mass $m_H^{\rm scr}$ of the lowest contribution to the correlator at large separation. In the heatbath, where full Lorentz symmetry is not realized anymore, the screening masses generically are different from the masses obtained from temporal correlations. Yet, the spatial correlations originate from the same spectral density σ and are equally sensitive to the restoration of symmetries on the operator level

$$G(z) = \int\limits_{-\infty}^{+\infty} \frac{dp}{2\pi}\, e^{ipz} \int\limits_{-\infty}^{+\infty} dp_0 \frac{\sigma(p_0,p)}{p_0}\,. \tag{2}$$

In the free limit, at infinite temperature, for all meson channels and massless quarks the spatial correlation functions are known to decay with screening masses of $2\pi T$ at $z \to \infty$, see *e.g.* [7]. First order perturbative corrections have been computed to be positive and the same for all channels [8].

In the following, we present results obtained from two data sets, both from simulations with $2+1$ dynamical staggered quarks. Data set A is based on configurations with temporal extents $N_\tau = 4, 6$ and 8 generated within the p4 discretization scheme at light quark masses which correspond to a Goldstone pion mass value of 220 MeV. The second data set B utilizes configurations with $N_\tau = 8$ generated with the HISQ action at light quark masses which lead to $m_\pi = 160$ MeV at zero temperature. In both cases, the strange quark mass was kept at its physical value and the aspect ratios N_σ/N_τ have always been 4. While data set B so far covers temperatures in the vicinity of $T_{\rm c}$, set A extends up to $T \simeq 800$ MeV. Note that at finite lattice spacing the critical temperature depends on the discretization, on N_τ and on the quark masses. In the following plots for set A, the results have been shifted in T such that the "common" $T_{\rm c}$ has a value of 196 MeV which is the critical temperature for $N_\tau = 6$. For set B, $T_{\rm c}$ amounts to 163 MeV. The results of set A are published [9], those of set B are preliminary.

In Fig. 1 (left) we show our results for the screening mass in the transverse vector channel, normalized to the free theory value of $2\pi T$. The comparison of the values obtained at different temporal extents N_τ, corresponding to different lattice spacings at a given temperature, $a = 1/(N_\tau T)$, reveals the presence of discretization effects. At high temperatures, the $N_\tau = 6$ and 8 data already overshoot 1 such that the continuum extrapolation clearly will be above the free theory number, in qualitative accord with [8]. On the right-hand side, the ratio of transverse axialvector to vector is shown. This ratio is much less affected by discretization effects. It is seen that right at the transition temperature, determined from the behaviour of the chiral condensate, the ratio is compatible with 1, *i.e.* the chiral partners become degenerate at this temperature. The same ratio from set B is shown in Fig. 2 with better resolution at low temperatures. The left part depicts results in the light $\bar{u}d$ flavour combination where the data indicate chiral symmetry restoration within the error bars. The right plot gives the ratio in the strange ($\bar{s}s$) sector, showing the sensitivity of the ratio, and of chiral symmetry restoration, to the quark mass.

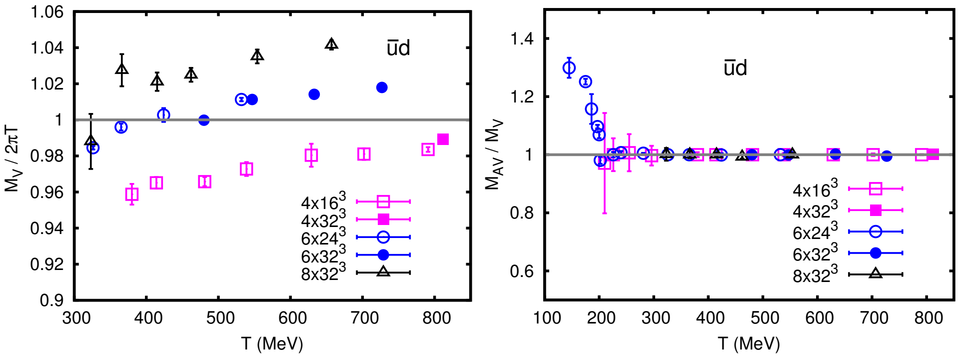

Fig. 1. Vector screening masses (left) and the ratio of axialvector to vector masses (right) as a function of temperature from data set A.

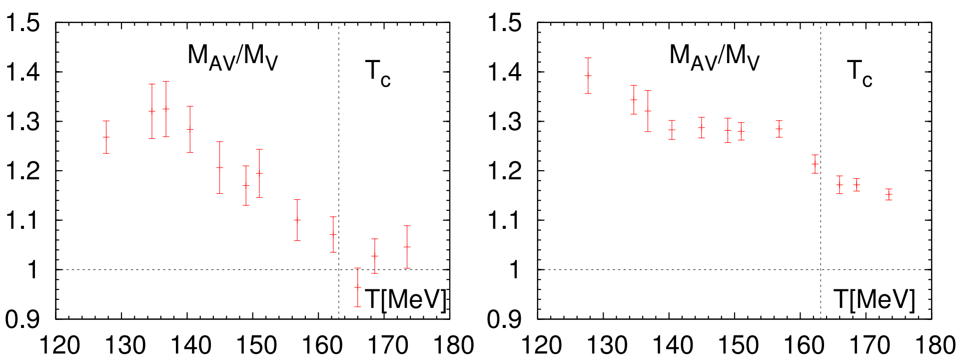

Fig. 2. Ratio of axialvector to vector screening mass for the $\bar{u}d$ (left) and the $\bar{s}s$ flavour combination (right), from data set B. The vertical line denotes T_c at $N_\tau = 8$ for the HISQ discretization.

In Fig. 3 we turn to the pseudoscalar channel. The left part exhibits the temperature dependence of the corresponding screening mass, showing a strong rise above T_c. On the right-hand side, we plot its ratio to the vector screening mass. Although in both plots finite lattice spacing effects are visible, it remains clear that pseudoscalar and vector do not become degenerate in the investigated temperature range.

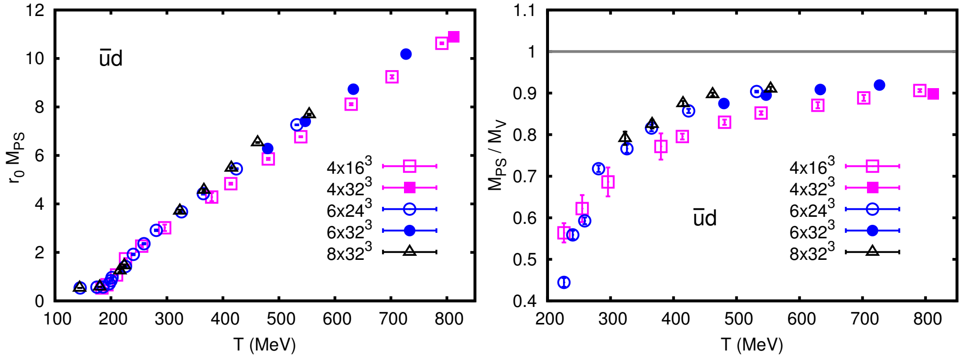

Fig. 3. Pseudoscalar screening masses in units of $r_0 = 0.469$ fm (left) and their ratio to the vector results (right), from data set A.

In Fig. 4 results for pseudoscalar and scalar isovector from set B are presented. The left plot summarizes the pseudoscalar screening masses for three different flavour combinations, the pion, the kaon and a hypothetical $\bar{s}s$ pseudoscalar. The figure highlights a slight temperature dependence of the masses below T_c which is remarkably well described by an exponential behaviour as $a + b\exp(T/c)$. In the right part, the temperature dependence of the scalar isovector is compared with the pion. It is clearly seen that the scalar screening mass drops in the vicinity of T_c but does not become degenerate with the pion at the transition

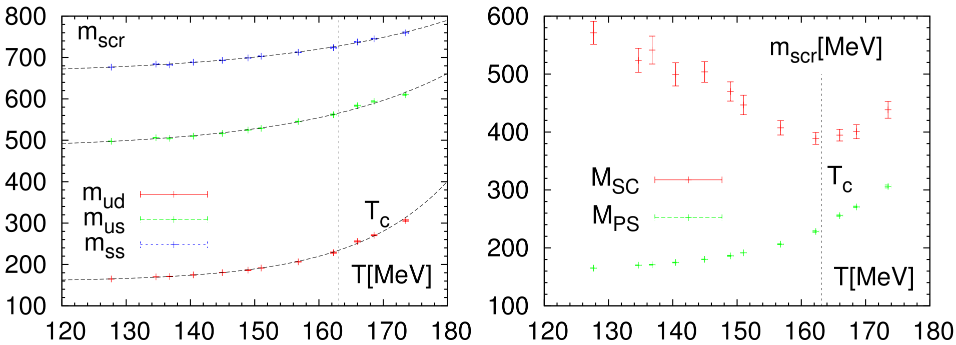

Fig. 4. Pseudoscalar screening masses for three different flavour combinations (left); scalar and pseudoscalar for the $\bar{u}d$ channel; both from data set B.

The (non-)degeneracy of scalar and pseudoscalar is further depicted in Fig. 5. Here, we plot their ratio, at low temperatures below 175 MeV from set B and for temperatures above T_c only from set A. At present, this ratio deviates from 1 at temperatures up to about 1.25 T_c. Although the comparison of the $\bar{u}d$ with the $\bar{u}s$ combination does not indicate a strong quark mass dependence it must be pointed out that the subtle chiral extrapolation of the ratio could not yet been carried out. Taken at face value the present results however do favour that the $U_A(1)$ is not restored effectively at the chiral transition temperature.

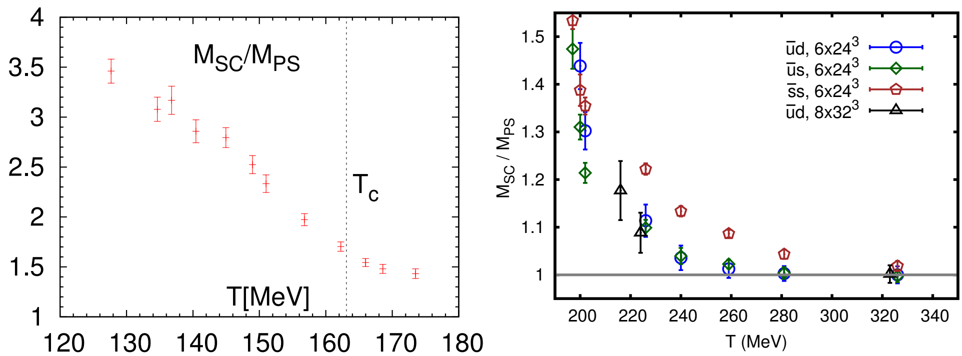

Fig. 5. Scalar to pseudoscalar screening mass ratios from set B (left) and A (right).

3. Low eigenmodes of the Dirac operator

The question of whether a_0 and π become degenerate can also be attacked through an analysis of the density $\rho(\lambda)$ of low lying eigenvalues λ of the Dirac matrix. Their relation to the integrated π and a_0 correlators, the generalized susceptibilities χ_{π,a_0}, is given by the difference

$$\chi_\pi - \chi_{a_0} = \lim_{m_l \to 0} \int_0^{+\infty} d\lambda \frac{4m_l^2}{(\lambda^2 + m_l^2)^2}\, \rho(\lambda) \tag{3}$$

in the chiral limit for the light quark masses m_l. The vanishing of this difference signals effective $U_A(1)$ restoration. A sufficient condition for this to happen would be that $\rho(\lambda)$ develops a gap around $\lambda = 0$.

Because of the importance of this issue for the nature of the chiral QCD transition, investigations of the Dirac spectrum have seen renewed interest recently. For two flavour QCD with dynamical overlap fermions, albeit on small lattices and with configurations restricted to the $Q = 0$ sector (where Q is the topological charge), it was reported that $U_A(1)$ may be restored near the chiral transition [10]. With the HISQ action on larger and finer lattices, it was reported that $U_A(1)$ may be broken even at 1.5 T_c [11]. Recent simulations with dynamical domain wall fermions [12] also suggest that $U_A(1)$ is broken in and above the chiral cross-over region.

Here, we use the overlap operator as a tool to analyze the same $32^3 \times 8$ gauge field configurations generated with the HISQ action which have been used in Sec. 2. The advantages of the overlap operator are its chiral properties, the possible presence of exact zero modes and the applicability of the index theorem $Q = n_+ - n_-$, relating the topological charge to the number $n_\pm$ of zero modes with chirality ± 1, all valid even at non-vanishing lattice spacing.

The distribution of the lowest eigenvalues per configuration is shown in Fig. 6 for two temperatures, just above T_c and for $1.5\,T_c$. Even at $1.5\,T_c$ there is a considerable number of exact zero modes, which, in addition, distinguish themselves by their chiralities of ± 1, shown as the grey/red contribution to the lowest eigenvalue bin. This indicates that also above T_c there is quite a number of topologically non-trivial configurations which would lift the a_0 mass. The figure also shows that the density of small eigenvalues decreases with rising temperature. However, even at the high temperature there are near-zero modes remaining, with a smeared $\delta(\lambda)$ like distribution. It remains to be studied whether the weight of this distribution scales with the quark mass as $m_l^\alpha, \alpha > 1$, in which case the observed distribution accommodates a vanishing chiral condensate and $\mathrm{U_A}(1)$ breaking at the same time.

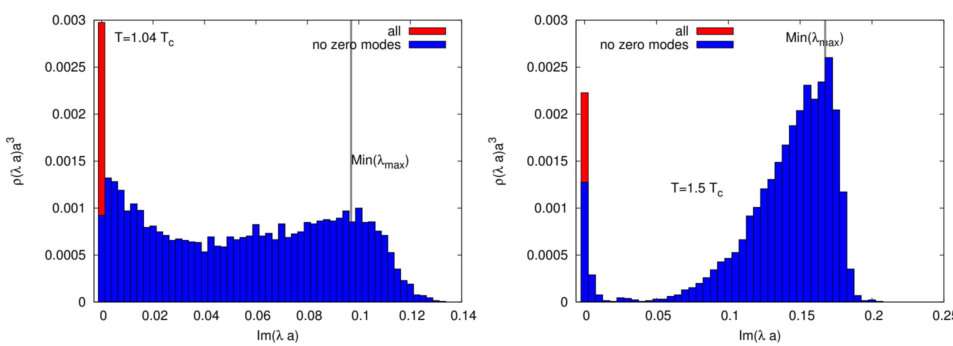

Fig. 6. Distribution of the 100 (at $T = 1.04\,T_c$, left) and 50 (at $T = 1.5\,T_c$, right) lowest eigenvalues computed on every configuration. The grey/red contribution to the lowest eigenvalue bin stems from exact zero-modes. The vertical line denotes the minimal value of the largest eigenvalue obtained on every configuration; beyond this value the distribution cannot be trusted because of the truncation in the number of eigenvalues computed.

4. Conclusions

Lattice simulations of QCD with two light and a strange quark based on two different improved discretization schemes have given evidence that at the transition temperature where the chiral condensate drops rapidly and its susceptibility develops a peak, the vector and axialvector screening masses become degenerate, which is an additional signal for the restoration of chiral symmetry. In the same temperature region, the current results for the pseudoscalar and scalar isovector screening masses favour that the $\mathrm{U_A}(1)$ symmetry is not effectively restored. This is supported by an analysis of the spectrum of the Dirac matrix in the overlap discretization. However, performing the thermodynamic, the continuum and the chiral limit is a difficult problem which can hopefully be addressed in future computations.

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